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Abstract

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MAGNETIC PERMEABILITY, ELECTRICAL CONDUCTIVITY, DIELECTRIC PERMITTIVITY, AND THERMAL CONDUCTIVITY OF A MEDIUM CONTAINING SPHERICAL AND ELLIPSOIDAL INCLUSIONS

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In connection with the development of geophysical prospecting methods, much attention is now being paid to the study of the laws governing changes in the physical properties of rocks and ores. The question of the dependence of the magnetic permeability μ, electrical conductivity σ, dielectric permittivity ε, and thermal conductivity λ of rocks and ores on the parameters of their constituent components (minerals) and on the ratio between the latter is acquiring great importance. The theories of this question lag behind practical needs. Even the simplest theoretical problem—to find the dependence of μ, σ, ε, or λ of a system consisting of spherical inclusions chaotically dispersed in a cementing medium, on the parameters of the cement material μ_1, σ_1, ε_1, or λ_1 and of the inclusions μ_2, σ_2, ε_2, or λ_2, and on the volume content of inclusions v_2—has been rigorously solved only for the case of small values of v_2.

In the present paper a solution is proposed for the stated problem, valid for any contents of inclusions, and a generalization of the solution to the case of ellipsoidal grain shape is given.

In view of the equivalence of the calculations of μ, σ, ε, and λ for complex systems, it is sufficient to consider the solution of the stated problem using magnetic permeability as an example.

Two solutions of the indicated simplest problem are known.

Maxwell ([1]), and after him A. V. Veshev and I. K. Ovchinnikov ([2]), derived for an aggregate with spherical inclusions the following dependence of μ on μ_1, μ_2, and v_2:

$$\mu = \mu_1 \frac{\mu_2 + 2\mu_1 + 2(\mu_2 - \mu_1)v_2}{\mu_2 + 2\mu_1 - (\mu_2 - \mu_1)v_2} \quad \text{or} \quad \frac{\mu - \mu_1}{\mu + 2\mu_1} = v_2 \frac{\mu_2 - \mu_1}{\mu_2 + 2\mu_1}. \tag{1}$$

D. S. Shteinberg and F. Ollendorf ([3]) arrived at formulas analogous to (1) for $\mu_1 = 1$.

V. I. Odelevskii ([4]) and E. I. Kondorskii ([5]) (the latter for the case $\mu_1 = 1$) derived for μ of the mixture under consideration the equation

$$(1 - v_2)\frac{\mu - \mu_1}{\mu + 2\mu_1} = v_2\frac{\mu_2 - \mu}{\mu_2 + 2\mu}. \tag{2}$$

For small values of v_2, both solutions for μ can be represented in the form of a power series in v_2, which, to within the first two terms of the expansion, has the form

$$\mu = \mu_1 + \frac{3\mu_1(\mu_2 - \mu_1)}{\mu_2 + 2\mu_1}v_2. \tag{3}$$

Experimental studies ([6]) have shown that Maxwell' s solution (1), and consequently also expression (3), is certainly valid

only for small v_2. At significant volume concentrations v_2 and larger values of μ_2/μ_1, both solutions (1), (2) do not give satisfactory agreement with experimental data and are theoretically insufficiently justified*.

The paths to the solution proposed below, valid for any concentrations of inclusions, were outlined in the works of E. I. Kondorskii and A. S. Semenov. E. I. Kondorskii ([7]) proposed an effective principle for taking account of the interaction of inclusions: neighboring grains affect the individual grain under consideration in the same way as a homogeneous medium possessing the average properties of the mixture. A. S. Semenov ([8]), in calculating the electrical conductivity of a mixture, applied the method of successive (portionwise) filling of a medium with inclusions.

We shall use an integral method to calculate the μ of the mixture. We shall gradually (i.e., in contrast to A. S. Semenov, in infinitely small portions) fill the medium with inclusions until the final concentration is obtained. At intermediate concentrations we shall assume that the influence of neighboring grains on the next portion of inclusions is analogous to the influence of a homogeneous medium with the corresponding intermediate value of μ.

Let, as a result of filling the medium with grains up to some intermediate concentration v, a medium with permeability μ_v be obtained. Let us uniformly add to each unit volume of this intermediate medium a small volume dn of spherical inclusions with magnetic permeability μ_2. According to equation (3) (it may be used, since dn is small) and the principle of E. I. Kondorskii, the increment of the magnetic permeability μ caused by the portion of grains dn is

$$d\mu_v = \frac{3\mu_v(\mu_2 - \mu_v)}{\mu_2 + 2\mu_v}\,dn. \tag{4}$$

Fig. 1

Figure 1: Fig. 1

When a unit volume is filled with a portion of inclusions dn, a part of them, equal in volume to $v\,dn$, will replace grains already present in the medium. Therefore the increment of the volume content dv is related to dn by the equality

$$dv = (1 - v)dn. \tag{5}$$

Substituting the value of dn from (5) into (4) and separating the variables, we obtain the differential equation

$$\left(\frac{1}{3\mu_v} + \frac{1}{\mu_2 - \mu_v}\right) d\mu_v = \frac{dv}{1 - v}. \tag{6}$$

Integrating equation (6) over the limits from $v = 0,\ \mu_v = \mu_1$ to $v = v_2,\ \mu_v = \mu$, we obtain the following equation characterizing the relation of μ to μ_1, μ_2, and v_2:

$$(1 - v_2)\left(\frac{\mu}{\mu_1}\right)^{1/3} = \frac{\mu_2 - \mu}{\mu_2 - \mu_1}. \tag{7}$$

It is seen from Fig. 1 that the theoretical plots of the dependence of μ on v_2, constructed with the aid of the equation (7) obtained by us, deviate from the experimentally observed data ([9],[10]) for magnetodielectrics to a lesser degree than the plots constructed from equations (1) and (2). It is not legitimate to demand exact coincidence of the empirical plots presented with equation (7), since the shape of ferromagnetic inclusions in magnetodielectrics differs substantially from spherical. For example, permalloy grains have the form of flakes ([10]). In rocks and ores

* The limited range of applicability of equations (1) and (2) follows from the assumptions made in their deriv

inclusions also, as a rule, have not an isometric, but a flattened or elongated shape. Therefore, for practical purposes it is of interest to generalize the solution to the case of ellipsoidal grain shape.

With strict orientation of ellipsoidal grains by their principal axes along the direction of measurement (an anisotropic aggregate), the proposed

Fig. 1. Dependence of μ on v_2 for powders of various ferromagnets cemented with a nonmagnetic material: a—permalloy [(10)], b—molybdenum permalloy [(9)], c—alsifer [(10)], d—carbonyl iron [(10)]. 1—experimental data of J. I. Parkhin, L.

J. Graham, and Guimond; 2, 3, 4, 5—theoretical curves: 2—constructed by Maxwell' s formula (1); 3—constructed from the equation of V. I. Odelevskii-E. I. Kondorskii (2); 4—constructed from equation (7); 5—constructed from equation (9) at $\beta = 0.5$.

method of solution leads to an equation of the form (7), with the sole difference that the exponent of the ratio μ/μ_1 on the left-hand side of the equation, equal to 1/3, is replaced by the quantity $N/4\pi$, where N is the demagnetizing factor of the grains in the direction of measurement.

With random orientation of grains in the form of ellipsoids of revolution (an isotropic aggregate), formula (4) takes the form

$$d\mu_v = \frac{\mu_2 - \mu_v}{3}\left(\frac{1}{1 + \frac{L}{4\pi}\frac{\mu_2-\mu_v}{\mu_v}} + \frac{2}{1 + \frac{M}{4\pi}\frac{\mu_2-\mu_v}{\mu_v}}\right), \tag{8}$$

where L and M are, respectively, the demagnetizing factor in the direction of the axis of revolution of the ellipsoid (grain) and perpendicular to it. Introducing the notation $L - M = 4\pi\beta$, using the equality $L + 2M = 4\pi$, and carrying out calculations analogous to those given above, we arrive at the equation

$$(1 - v_2)\left(\frac{\mu}{\mu_1}\right)^{1/3-2^2/3(1-\beta)}\left[\frac{(2-\beta)\mu + (1+\beta)\mu_2}{(2-\beta)\mu_1 + (1+\beta)\mu_2}\right]^{2^2/(1-\beta)(2-\beta)} = \frac{\mu_2 - \mu}{\mu_2 - \mu_1}. \tag{9}$$

Theoretical curves constructed from equations (9) at $\beta = 0.5$ (grains—oblate ellipsoids of revolution with an axial ratio

1 : 3.5), show satisfactory agreement with the experimental data, which confirms the correctness of the proposed solution.

The solution found for μ can be applied to the study of σ, ε, and λ of analogous systems. For this it is sufficient, in equations (7) and (9), to replace the symbol μ by the symbol σ, ε, or λ.

The equations obtained may be used to determine the physical parameters of minerals from measurements of the properties of powders or specimens with a known content of the mineral under study, to estimate the content of particular minerals in rocks (ores) from measurements of their physical properties, and to solve other petrophysical problems. The solution found may also be used in developing the technology for manufacturing a number of electrical-engineering and heat-engineering materials with specified properties.

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Note: Figure translations are in progress. See original paper for figures.