ON THE THEORY OF INDUCED TRANSITIONS IN THE RADIATION OF A LUMINOUS ELECTRON

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ON THE THEORY OF INDUCED TRANSITIONS IN THE RADIATION OF A LUMINOUS ELECTRON

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Let us consider the radiation of an electron moving in a constant and homogeneous magnetic field, occurring under the influence of an incident external electromagnetic wave. As is known ([1]), the probability of induced transitions of an electron from the state E_n to the state $E_{n'}$ is given by the expression:

$$W_{nn'} = \frac{2\pi N(\vec{\varkappa})e^2 c}{\hbar L^3 \varkappa} \{(\vec{\alpha}^+\vec{\alpha}) - (\vec{\alpha}^+\vec{\varkappa}^0)(\vec{\alpha}\vec{\varkappa}^0)\} \left| \int_0^t e^{-ict(\varkappa_{nn'}-\varkappa)}\, dt \right|^2, \quad (1)$$

where $\omega_{nn'} = c\varkappa_{nn'} = |E_n - E_{n'}|/\hbar$ is the resonance transition frequency; $N(\vec{\varkappa})$ is the number of incident quanta with momentum $\hbar\vec{\varkappa} = \hbar\varkappa\vec{\varkappa}^0$; α_μ is the matrix element of the Dirac matrix, related to the wave functions of the electron in the magnetic field by the relation

$$\alpha_\mu = \int \psi^+_{n'} e^{-i\vec{\varkappa}\vec{x}} \alpha_\mu \psi_n \, d^3x. \quad (2)$$

In the case of damping, caused in particular by the finite time (on the average equal to $\tau/2$) of the electron' s stay at the initial level, the right-hand side of equality (1) can be generalized by introducing the decaying time factor $\exp(-t/\tau)$. Then the integral

$$I = \left| \int_0^t e^{-ict[\varkappa_{nn'}-\varkappa]} e^{-t/\tau}\, dt \right|^2 = \frac{1 + e^{-t/\tau} - 2e^{-t/\tau}\cos(\varkappa_{nn'} - \varkappa)t}{c^2(\varkappa_{nn'} - \varkappa)^2 + 1/\tau^2} \quad (3)$$

for a time interval $t \gg \tau$ goes over into the expression

$$I = \tau g(\omega) = \tau \frac{\tau}{1 + \tau^2(\omega_{nn'} - \omega)^2}, \quad (4)$$

in which $\omega = c\varkappa$ is the frequency of the incident quanta. Therefore, for large time intervals we may introduce the mean probability per unit time of quantum transitions

$$w_{nn'} = 2\frac{W_{nn'}}{\tau} = \frac{2\pi N(\vec{\varkappa})e^2c}{\hbar\varkappa L^3}\left\{\vec{\alpha}^+\vec{\alpha} - (\vec{\alpha}^+\vec{\varkappa}^0)(\vec{\alpha}\vec{\varkappa}^0)\right\} g(\omega). \tag{5}$$

The wave functions of an electron moving in a constant and homogeneous magnetic field and satisfying the Dirac equation were given in our earlier works ([2], see also [1]). The energy spectrum of the electron is then given by the expression:

$$E_n = c\hbar K_n = c\hbar\sqrt{k_0^2 + 4n\gamma + k_z^2}, \tag{6}$$

in which $k_0 = m_0c/\hbar$, n is the principal quantum number ($n = 0, 1, 2\ldots$); $\gamma = e_0H/2c\hbar$, k_z is the momentum of the electron along the field.

Determining the frequency of the forced resonant transitions, we find

$$\omega_{n,n\mp\nu} = \nu\Omega\,\frac{m_0c^2}{E}\left\{1 \pm \frac{\beta^2\nu}{4n}\right\} = \nu\Omega\,\frac{m_0c^2}{E}\left\{1 \pm \nu\,\frac{\hbar\Omega}{2m_0c^2}\left(\frac{m_0c^2}{E}\right)^2\right\}. \tag{7}$$

Here ν is the harmonic number; $\Omega = e_0H/m_0c$ is the cyclotron frequency. In this formula and in all subsequent calculations we shall retain, in the expansions, terms of order ν/n no higher than the first, which, as is evident from (7), is equivalent to an expansion in Planck' s constant $\hbar$.

Let us now suppose that the incident electromagnetic wave is linearly polarized, with the electric-field vector lying in the plane of the electron' s orbit of rotation and directed along the radius toward its center (the so-called σ-component ([3], see also [1], §28). For simplicity we shall also assume that the angle of incidence of the external electromagnetic wave in the spherical coordinate system is $\theta \sim \pi/2$, i.e., the momentum vector also lies in the plane of the electron' s orbit of rotation. Then for the transition probability (5) we obtain (see [2])

$$w_{nn'} = \frac{2\pi Ne^2c}{\hbar\chi L^3}\,\bar{\alpha}_1^+\bar{\alpha}_1\,g(\omega), \tag{8}$$

where

$$\bar{\alpha}_1^+\bar{\alpha}_1 = \frac{KK' - k_0^2}{4KK'}\left(I_{n,n'-1}^2 + I_{n-1,n'}^2\right) - 2\,\frac{4\gamma\sqrt{nn'}}{4KK'}\,I_{n,n'-1}I_{n-1,n'}. \tag{9}$$

In this expression the Laguerre function, for $n > n'$,*

$$I_{nn'}(y) = \frac{1}{\sqrt{n!\,n'!}}\, e^{-y/2} y^{(n-n')/2} Q_{n'}^{n-n'}(y) \tag{10}$$

depends on the argument $y = \chi^2/4\gamma$, where χ is the frequency of the incident photon. Expanding the coefficients in formula (9) with respect to ν/n, we obtain

$$\bar{\alpha}_1^{+}\bar{\alpha}_1 = \frac{\gamma}{K_n^2}\left\{n \mp \frac{\nu}{2}(1-\beta^2)\right\}\left\{\begin{array}{l} R^2_{n,n-\nu}, \\ R^2_{n,n+\nu} \end{array}\right. \tag{11}$$

where

$$R_{n,n'} = I_{n,n-1}(y) - I_{n-1,n'}(y). \tag{12}$$

Let us now write the expression for the energy absorbed per unit time by the electron in resonant transitions (the absorption power)

$$P_n = \hbar\omega_{n,n+\nu} w_{n,n+\nu} - \hbar\omega_{n,n-\nu} w_{n,n-\nu} = \frac{4\pi N(\vec{\chi})\hbar\omega}{L^3}\,\nu\,\frac{\Omega^2}{\omega^2}\,\frac{e^2}{2m_0}\left(\frac{m_0c^2}{E}\right)^3 \tau\Phi, \tag{13}$$

where

$$\Phi = \frac{\left[n + \frac{1}{2}\nu(1-\frac{3}{2}\beta^2)\right] R^2_{n,n+\nu}}{1+\tau^2(\omega_{n,n+\nu}-\omega)^2} - \frac{\left[n - \frac{1}{2}\nu(1-\frac{3}{2}\beta^2)\right] R^2_{n,n-\nu}}{1+\tau^2(\omega_{n,n-\nu}-\omega)^2}. \tag{14}$$

We transform the denominator in the following way:

$$\tau(\omega_{n,n-\nu}-\omega) = \tau(\omega_{n,n+\nu}-\omega) + \tau(\omega_{n,n-\nu}-\omega_{n,n+\nu}) = x + \frac{\nu}{2n}Q\beta^2, \tag{15}$$

where $Q = \omega\tau$ and the quantity $\omega \simeq \nu\Omega m_0 c^2/E$ is close to the resonant frequency.

Next we consider the expression for $R^2_{n,n\pm\nu}$. As was shown earlier (see, for example, ([2])), the Laguerre functions $I_{nn'}$ admit a good approxi-

* In the case $n' > n$, we must interchange the indices n and n' and multiply the entire expression by $(-1)^{n-n'}$.

···mation through Bessel functions

$$I_{nn'}(y) \simeq J_\nu(2\sqrt{yn}) - \frac{(\nu-1)\sqrt{4ny}}{4n}\, J'_\nu(2\sqrt{yn}), \tag{16}$$

where $\nu = n - n'$.

Retaining terms of order ν/n no higher than the first, we find, with the aid of this approximation,

$$R^2_{n,n\pm\nu} = 4J'^2_\nu(\nu\beta)\left\{1 - \frac{1\mp\nu}{n}\sqrt{yn}\,\frac{J''_\nu(\nu\beta)}{J'_\nu(\nu\beta)}\right\}, \tag{17}$$

where the second derivative J''_ν is not difficult to eliminate with the aid of Bessel' s equation.

Substituting this entire expansion into formula (14), we find that

$$\Phi = \frac{4\nu J'^2_\nu(\nu\beta)}{1+x^2}\left\{\frac{\nu(1-\beta^2)}{\beta}\,\frac{J_\nu(\nu\beta)}{J'_\nu(\nu\beta)} - \frac{3}{2}\beta^2 + \frac{\beta^2 xQ}{1+x^2}\right\}. \tag{18}$$

Finally, the absorbed energy (13) can be expressed in terms of the electric-field strength of the incident wave. Indeed, proceeding from the expression for the energy density $\mathcal{E}^2/4\pi = \hbar\omega N/L^3$, we obtain

$$P_n = \frac{e^2\mathcal{E}^2\tau}{2m_0}\,\frac{m_0c^2}{\nu E}\,\Phi. \tag{19}$$

The formula obtained is applicable for any harmonic number ν and has no restrictions associated with the velocity of the electron.

From (18), in the particular case of a nonrelativistic electron $\beta \ll 1$, for dipole transitions, when in (18) one may put

$$J_1(\nu\beta) = \nu\beta/2, \qquad J'_1(\nu\beta) = 1/2, \qquad E \sim m_0c^2,$$

we find the result obtained in (4):

$$P_n = \frac{e^2\mathcal{E}^2\tau}{2m_0}\,\frac{1}{1+x^2}\left[1 + \beta^2\frac{Qx}{1+x^2}\right]. \tag{20}$$

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Note: Figure translations are in progress. See original paper for figures.