

# ON SELF-SUSTAINING FRACTURE OF A STRESSED BRITTLE BODY

THEORY OF ELASTICITY

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**Abstract**

**Full Text**

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*THEORY OF ELASTICITY*

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## **ON SELF-SUSTAINING FRACTURE OF A STRESSED BRITTLE BODY**

§ 1. The phenomenon of self-sustaining fracture of a brittle body is well known. Let us imagine that a piece of brittle material, located in a field of compressive stresses such that no development of initial shear microcracks takes place, suddenly finds itself under conditions in which its surface has become free of loads. Immediately, a unloading wave will begin to propagate from the surface into the interior of the piece. If the potential energy of elastic compression stored by the body is sufficiently large, then the shear microcracks situated at the front of the unloading wave become unstable. Their dynamic growth leads to the destruction of the body. In the case of a very large number of such unstable microcracks, one may speak of a fracture wave, meaning by this the boundary separating the fractured material from the unfractured material. It should be emphasized at once that the velocity of propagation of the fracture wave does not depend on the velocity of propagation of individual cracks and, in particular, is not determined by the limiting velocity of propagation of an isolated crack, equal to the velocity of Rayleigh waves. The self-sustaining mechanism of fracture, consisting in the conversion of the potential elastic energy of a brittle body into the surface and kinetic energy of individual particles of the fractured body, resembles the self-sustaining mechanism of propagation of a detonation wave, in which the shock wave is sustained at the expense of the reserve of chemical energy in the body. Self-sustaining fracture of a brittle body is accompanied by a loud sound and by scattering of particles of the destroyed substance, which externally resembles a weak explosion.

Thus, the principal factor determining the ability of a brittle body to undergo self-sustaining fracture is the reserve of potential elastic energy in the unfractured body. The greatest reserve of elastic energy in a body (practically unlimited) can be created under all-around compression or along some loading path close to all-around compression, when the body remains unfractured by transverse-shear cracks. An important role in the possibility of creating a reserve of potential elastic energy in a brittle body is played by the strength of the material. Removal of surface microcracks, or their compression by internal stresses, and homogenization of the material as a result of certain technological operations increase the strength, other conditions being equal, and thereby

make it possible to attain a larger value of the elastic energy of the body before its fracture. In strong glasses, characterized by the absence of surface microcracks or by large internal compressive stresses in the surface layer and by a very homogeneous volume structure, it is possible to observe self-sustaining fracture not only under compression, but also under bending, and even under tension. A significant role here is also played by the difference in the specific surface energy under static and dynamic loading, which will be considered below.

The necessary reserve of elastic energy in a body may be created by internal compressive stresses technologically ( “Batavian tear—”

”). In places of concentration of compressive stresses, a large reserve of elastic energy can likewise be attained (mine workings, rock burst).

On the basis of what has been said, the following hypothesis seems natural.

A brittle body, initially in a homogeneous stressed state and then suddenly finding itself under conditions in which its surface becomes free of load, undergoes self-sustaining fracture if the potential elastic energy per unit volume of the body exceeds a certain critical value that is a constant of the material (for the same technology, the same temperature, and other equal conditions). This critical value apparently is of the order  $\frac{1}{2E}\sigma_+^2$ , where  $E$  is Young’s modulus and  $\sigma_+$  is the compressive strength of the material.

The surface energy of the body is obviously equal to the product of the area of the entire surface of the body (including cracks) and the specific surface energy  $T$ . It is necessary to note the following circumstance. The specific surface energy depends on the path and manner of loading and therefore, generally speaking, is not a material constant, but characterizes at the same time the degree of development of plastic, viscous, and other irreversible deformations in the surface layer. Therefore the experimentally obtained values of  $T$  depend on the experimental procedure. The material constant is the smallest of the possible values of  $T$ , which characterizes the purely surface properties of the material. Therefore, under a suddenly applied load, when cracks propagate with limiting speed and plastic, viscous, and other irreversible deformations do not have time to develop in the surface layer, the value of  $T$  will be close to the smallest. The fact that the value of  $T$  under static loading of a brittle body is greater than under dynamic loading plays a large role in explaining the causes of self-sustaining fracture (especially under tension and bending).

§ 2. We shall consider the problem of self-sustaining fracture of a brittle body using the following one-dimensional model. Suppose that in an infinite elastic space, which is in a homogeneous field of compressive principal stresses  $N_1, N_2$ , and  $N_3$ , the plane  $x = 0$ , free of stresses, has suddenly formed. Let us consider the brittle half-space  $x > 0$ . We assume that the stresses  $N_1, N_2$ , and  $N_3$  are such that they do not cause the propagation of shear microcracks and thereby fracture of the solid body (strictly speaking, this imposes a restriction mainly on the difference of the principal stresses). However, we suppose that the stresses

$N_1, N_2, N_3$  are sufficiently large, so that, according to the hypothesis, the reserve of elastic energy accumulated by the body exceeds a certain value critical for the given material. Then, in the half-space  $x > 0$ , a fracture wave will travel with some velocity  $D$ , which we shall represent in the form of the rupture plane  $x = Dt$ , separating the fractured material ( $0 < x < Dt$ ) from the unfractured material ( $x > Dt$ ). The fractured material consists of a set of separate particles moving with velocity  $\mathbf{u}$  relative to the quiescent unfractured part of the body. The fracture wave obviously has a thickness of the order of the characteristic particle size. Let us write the laws of conservation of mass, momentum, and energy at the fracture discontinuity. We have

$$\rho_0 D = \rho_1 (D - u_x),$$

$$\rho_0 D u_x = -\sigma_1 + \sigma_0, \quad \rho_0 D u_t = -\tau_1 + \tau_0, \quad (1)$$

$$\frac{1}{2} D^2 + U = \frac{1}{2} [(D - u_x)^2 + u_t^2] + \Pi.$$

Here  $\rho$  is the density;  $\sigma$  is the normal stress acting on an area parallel to the plane of fracture;  $\tau$  is the greatest tangential stress acting on the same area. The subscript 0 refers to the unfractured material ahead of the jump, and the subscript 1 to the fractured material behind the jump. The quantities  $u_x$  and  $u_t$  are the normal and tangential components of the velocity vector  $\mathbf{u}$ . By  $U$  is denoted the elastic energy of unit mass of the unfractured substance, equal, for a Hookean body, to

$$U = \frac{1}{2E\rho_0} [(1 + \nu)(N_1^2 + N_2^2 + N_3^2) - \nu(N_1 + N_2 + N_3)^2] \quad (2)$$

( $E$  is Young' s modulus,  $\nu$  is Poisson' s ratio), and by  $\Pi$  the surface energy of the fractured material per unit mass. As is customary in strength of materials, compressive stress is considered negative.

Obviously, the velocity of propagation of the fracture jump  $D$  cannot be less than the velocity of propagation of longitudinal elastic waves  $c$  in the continuous material (if  $\sigma_0 \neq 0$ ). Otherwise the material would be fractured in the elastic wave traveling ahead of the fracture jump, which is impossible by the definition of a fracture wave.

We shall adopt the following hypothesis.

The velocity of propagation of the fracture jump is equal to the velocity of propagation of longitudinal elastic waves in the continuous material

$$D = c. \quad (3)$$

For the stresses usually encountered, much smaller than Young' s modulus, this hypothesis raises no doubt. For compressive stresses  $N_1, N_2, N_3$  of the order of (or greater than) Young' s modulus, the adopted hypothesis also appears quite natural, since the fracture wave is a certain analogue of a rarefaction wave in a compressed gas.

On the basis of hypothesis (3), the material ahead of the fracture jump is at rest. Then the boundary conditions imply the relations

$$\begin{aligned}\tau_0^2 &= (N_1^2\alpha^2 + N_2^2\beta^2 + N_3^2\gamma^2) - (N_1\alpha^2 + N_2\beta^2 + N_3\gamma^2)^2, \\ \sigma_0 &= N_1\alpha^2 + N_2\beta^2 + N_3\gamma^2, \quad \sigma_1 = \tau_1 = 0.\end{aligned}\quad (4)$$

Here  $\alpha, \beta, \gamma$  are the cosines of the angles that the  $x$ -axis makes with the principal stress axes (respectively with the first, second, and third). According to (4), the potential elastic energy in the fractured material is zero, which is reflected in the energy equation (1). Using formulas (3) and (4), from the first three equations of conservation of mass and momentum (1) we obtain the value of the density of the fractured material  $\rho_1$  and of the velocity components of motion of its particles  $u_x$  and  $u_t$

$$\rho_1 = \rho_0(1 - \sigma_0/\rho_0c^2)^{-1}, \quad u_x = \sigma_0/\rho_0c, \quad u_t = \tau_0/\rho_0c. \quad (5)$$

The energy equation (1) gives an expression for the surface energy  $\Pi$  of the fractured material

$$\Pi = U - \frac{1}{2}(\sigma_0^2 + \tau_0^2)/\rho_0^2c^2 - 2\sigma_0/\rho_0, \quad (6)$$

where  $U, \sigma_0,$  and  $\tau_0$  are determined by formulas (2) and (4).

Knowing the surface energy  $\Pi$ , one can, on the basis of physical and probabilistic considerations, calculate the geometrical dimensions of the particles of the fractured substance. We shall regard all particles as having the shape of a sphere. Let us note that the spherical form is energetically the most advantageous. The surface energy of a spherical particle of radius  $r$  is equal to

$4\pi Tr^2$ . We shall regard the specific surface energy  $T$  as a known constant of the material; this assumption is apparently well satisfied in the problem under consideration, since the development of individual cracks occurs at limiting or near-limiting velocities.

The radius of a particle  $r$  is a random quantity with some distribution density  $p(r)$ . Then the surface energy per unit mass of the fractured material  $\Pi$  can evidently be written in the form

$$\Pi = 3T \int_{-\infty}^{+\infty} r^2 p(r) dr / \rho_0 \int_{-\infty}^{+\infty} r^3 p(r) dr. \quad (7)$$

Here it is assumed that the density of each individual particle of the fractured substance is equal to the density of the material before the fracture wave. Thus, in its most general form, the energy equation gives a linear relation between the second and third moments of the distribution  $p(r)$ . Specific assumptions concerning the function  $p(r)$  make it possible to obtain more definite results. For example, the crudest assumption,

$$p(r) = \delta(r - r_0), \quad (8)$$

where  $\delta(r - r_0)$  is the delta function, makes it possible to give an approximate estimate of the characteristic particle size  $r_0$ :

$$r_0 = \frac{3T}{\rho_0 \Pi}. \quad (9)$$

If we assume that the distribution  $p(r)$  is normal,

$$p(r) = \frac{1}{\Delta \sqrt{2\pi}} \exp \left[ -\frac{(r - r_0)^2}{2\Delta^2} \right], \quad (10)$$

then, on the basis of (7), the variance of the distribution  $\Delta$  is expressed in terms of the mean particle size  $r_0$  as follows:

$$\Delta^2 = \frac{3T - \rho_0 \Pi r_0}{3\rho_0 r_0 \Pi - 3T} r_0^2. \quad (11)$$

Thus, the energy equation makes it possible to estimate the mean size of the particles of the fractured substance and even the parameters of their distribution.

§ 3. Let the brittle body now have some complex shape and be in an inhomogeneous stressed state. Suppose, further, that in some region  $D$  of this body the specific elastic energy exceeds the critical value before fracture of the body. If conditions are created for the onset of self-sustaining fracture, then the region  $D$  will probably be destroyed. The present study also makes it possible to formulate this problem quite precisely.

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