ON THE THEORY OF UNDAMPED INTENSITY PULSATIONS OF QUANTUM GENERATORS

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Abstract

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ON THE THEORY OF UNDAMPED INTENSITY PULSATIONS OF QUANTUM GENERATORS

In solving problems connected with the dynamics of quantum generators, one encounters a nonlinear equation of the form ([1,2])

$$\ddot{x} + x(x-1) - \dot{x}^2/x = \varepsilon f(x, \dot{x}), \tag{1}$$

where x is the radiation intensity, ε is a dimensionless parameter, and $f(x, \dot{x})$ is a nonlinear function whose form depends on the particular problem. For solid-state quantum generators, as a rule, $\varepsilon \ll 1$; therefore the solution of equation (1) is close to the solution of the conservative equation

$$\ddot{x} + x(x-1) - \dot{x}^2/x = 0. \tag{2}$$

The stable limit cycle (2) corresponds to undamped oscillations of the radiation intensity of a quantum generator.

Our problem is to find the solution of (2) in analytic form, which makes it possible to calculate the characteristics of a number of regimes that arise in lasers.

Equation (2) has the integral of motion

$$\dot{x}^2 = x^2[C + 2\ln x - 2x], \tag{3}$$

which determines the trajectories in the phase plane $(x, \dot{x})$. The family of periodic solutions (3), $x_0 = x_0(\theta + \theta_0, C)$, depends on two arbitrary constants C and θ_0. The maximum and minimum values of the amplitude of the periodic process are the roots of the equation

$$C + 2\ln x - 2x = 0, \tag{4}$$

which is solved either numerically or graphically under the initial conditions $x_0(0, C) = x_{\min}$, $\dot{x}_0(0, C) = 0$. The function $x_0(t)$ is determined by the equation

$$\int_{x_{\min}}^{x_0} \frac{dx}{\pm x\sqrt{C + 2\ln x - 2x}} = \theta + \theta_0, \tag{5}$$

and the period is

$$T_0 = 2\int_{x_{\min}}^{x_{\max}} \frac{dx}{x\sqrt{C + 2\ln x - 2x}}. \tag{6}$$

In the general case, the solution (5) for x_0 is not expressible in elementary functions. Let us investigate the most interesting case of pulsations with a large modulation depth, corresponding to values $C \gg 1$.

From (3) one can obtain asymptotic solutions: for $0 < x \ll 1$

$$\dot{x} = \pm x\sqrt{C + 2\ln x},$$

and for $x \gg 1$

$$\dot{x} = \pm x\sqrt{C - 2x}.$$

Let us divide the phase plane $(x, \dot{x})$ by the straight line $x = 1$ into two parts. For $x_{\min} \le x \le 1$ we shall describe the motion by the equation

$$\dot{x}_1 = \pm x_1\sqrt{\frac{C-2}{C}}\sqrt{C + 2\ln x_1}, \tag{7}$$

and for $1 \le x \le x_{\max}$ by the equation

$$\dot{x}_2 = \pm x_2\sqrt{C - 2x_2}. \tag{8}$$

For $x = 1$, $\dot{x}_1 = \dot{x}_2 = \dot{x}_0$, and therefore the solutions $x_1(t)$ and $x_2(t)$ join at $x = 1$, while the value of the first derivative coincides with its exact value. In addition, the slopes of the phase trajectories $d\dot{x}_1/dx_1$ and $d\dot{x}_2/dx_2$ at $x = 1$ differ little from $d\dot{x}_0/dx_0$ for $C \gg 1$. Therefore, for large C, equations (7) and (8) approximate the exact phase trajectories sufficiently well. Equations (7) and (8) are integrated.

In the upper half-plane, for $x_{\min} \le x \le 1$,

$$x_1(t) = \exp\left\{\frac{C}{2}\left[\frac{t^2}{C-2} - 1\right]\right\}, \tag{9}$$

Fig. 1

Figure 1: Fig. 1

$$x_{\min} = e^{-C/2}, \qquad 0 \le t \le t_1 = \sqrt{C-2}.$$

For $1 \le x \le x_{\max}$,

$$x_2(t) = \frac{C}{2}\frac{1}{\operatorname{ch}^2 \frac{\sqrt{C}}{2}(t_2 - t)}, \qquad x_{\max} = \frac{C}{2},$$

$$t_1 \le t \le t_1 + \frac{2}{\sqrt{C}} \operatorname{Ar\,th} \sqrt{\frac{C-2}{C}}. \tag{10}$$

Fig. 1

It is seen that the solution is divided into two stages. The first stage is slow motion, during which active particles accumulate, while the amplitude changes comparatively slowly from $x = 1$ to $x_{\min}$ and again to $x = 1$ ($x = 1$ is the stationary value of the amplitude in the absence of pulsations). The duration of this section is $T_0' = 2\sqrt{C-2}$. Then there follows a rapid increase of the amplitude to $x_{\max}$, and all the stored energy is emitted in a narrow pulse. The duration of the second stage, when the amplitude changes from $x = 1$ to $x_{\max}$ and back to $x = 1$, is $T_0'' = 2\ln 2(C-1)/\sqrt{C}$. The duration of the radiation pulse at half-power is $T_{0\,\mathrm{em}} = 3.52/\sqrt{C}$.

Let us now apply the relations found to the calculation of the scheme in Fig. 1, proposed in ([3]) for obtaining short pulses of light. A semiconductor quantum generator Γ is closed on itself through a delay circuit, which may include an amplifier y. The signal again enters the generator with a delay by a time τ. Let $P_1/2$ and $P_2/2$ be the numbers of photons, and T_1 and T_2 their lifetimes in two mutually perpendicular resonators. To simplify the analysis, let us assume that $T_1 \gg T_2$. Then, for any regimes whose oscillation frequency is less than T_2^{-1},

$$P_2 = kP_1(t-\tau), \tag{11}$$

where k is the fraction of energy re-entering the resonator through the delay circuit. The quantity k depends on the transmission of the resonator and the gain coefficient of the amplifier.

The equations describing the generation process in such a system can be written as follows:

$$\dot{P}_1 = -\frac{1}{T_1}P_1 + \alpha n P_1,$$

$$\dot{n} = -\frac{1}{T}n - \alpha n\left[P_1 + kP_1(t-\tau)\right] + I_0, \tag{12}$$

where n is the number of active particles in the generator; the quantity a is related to the Einstein coefficient for stimulated emission; I_0 is the exciting current divided by the electron charge; T is the time of interband recombination of carriers.

Equation (12) has the stationary solution

$$n_0 = \frac{1}{aT_1}, \qquad P_{10} = \frac{I_0 - I_{\text{th}}}{1+k}\,T_1; \tag{13}$$

$I_{\text{th}} = 1/aTT_1$ is the threshold-current value.

As shown in ([3]), for certain values of the parameters k and τ, the stationary solution (13) is unstable: the radiation will be amplitude-modulated. In what follows we shall regard the delay τ as small in comparison with all characteristic times of the self-modulation regime that arises when solution (13) is unstable. Then

$$P_1(t-\tau) = P_1(t) - \tau\dot{P}_1(t) + \frac{\tau^2}{2}\ddot{P}_1(t) + \cdots \tag{14}$$

It is not difficult to show that system (12) is equivalent to equation (1), in which

$$\varepsilon f = \frac{1}{\sqrt{\frac{T}{T_1}(\beta-1)}}\,\dot{x}\left[\frac{k\tau(\beta-1)}{(1+k)T_1} - (\beta-1)x - 1\right] + $$

$$+\frac{k\tau(\beta-1)}{(1+k)T}\dot{x}^2 - \sqrt{\frac{\beta-1}{TT_1}}\,\frac{k\tau^2(\beta-1)}{2(1+k)T}\,\dot{x}\,\ddot{x} - \frac{k\tau^2}{2(1+k)TT_1}\,x\ddot{x}, \tag{15}$$

where $x = P_1(t)/P_{10}$; $\beta = I_0/I_{\text{th}}$; differentiation is with respect to the dimensionless time $\theta = \Omega_0 t$; $\Omega_0 = [(\beta-1)/TT_1]^{1/2}$. In (15) the dimensionless small parameter ε has been introduced in order to reflect the fact that the right-hand side of (15) is small and the solution of (1) is close to the solution of (2).

In ([3]) it was obtained that, when the condition

$$\tau > \tau_{\text{crit}}, \qquad \tau_{\text{crit}} = \frac{\beta}{\beta-1}\,\frac{1+k}{k}\,T_1 \tag{16}$$

is satisfied, the stationary solution (13) is unstable and pulsations of the radiation intensity arise. We shall seek the general solution of (1) in the form of a series in powers of ε:

$$x(t) = x_0(t) + \varepsilon x_1(t) + \varepsilon^2 x_2(t) + \cdots \tag{17}$$

In order that equation (1) admit a periodic solution (17), which for $\varepsilon = 0$ turns into the generating solution of the conservative equation (2), $x_0(\theta)$, belonging to the family (3), it is necessary that the integral of motion C satisfy the equation

$$P(C) = \int_0^{T_0} f(x_0, \dot{x}_0) \frac{\partial C}{\partial \dot{x}_0} \, d\theta = 0, \tag{18}$$

where the partial derivatives of the function C are calculated for the generating solution, and T_0 is the period of the pulsations.

Evaluation of (18) with the right-hand side (15) for pulsations with a large modulation depth ($C \gg 1$) leads to the following value of C:

$$C = \frac{5(1+k)TT_1}{(\beta-1)^2 k\tau^2} \left[\frac{k\tau(\beta-1)}{(1+k)T_1} - \beta \right]. \tag{19}$$

Knowing $x_0(\theta)$, one can also calculate the subsequent approximations $x_1, x_2, \ldots$. From (19) we obtain that pulsations exist only when (16) is satisfied, and

analysis shows that the self-modulation regime that arises is stable.

Let us give a numerical example. For $T_1 = 10^{-12}$ sec., $\tau = 10^{-11}$ sec., $T = 1.5 \cdot 10^{-8}$ sec., $k = 0.2$, $\gamma = k\tau(\beta-1)/(1+k)T_1 - \beta = 2 \cdot 10^{-2}$, and $\beta \simeq 2.5$, the right-hand side of equation (1) is $\sim 10^{-2} \div 10^{-3}$. For these parameter values, $\Omega_0 = 10^{10}$ sec., $C \simeq 40$, the pulsation period is $T_p = 1.4 \cdot 10^{-9}$ sec., and the duration of the radiation pulse at half power is $T_{\text{izl}} = 5.6 \cdot 10^{-11}$ sec. The modulation depth differs little from 100%. The calculations carried out are valid if the pulsations of the population inversion arising in the self-modulation regime are small, i.e. $(n_{\max} - n_0)/n_0 \ll 1$, or, equivalently, $x_{\max} = C/2 \ll 1/\varepsilon^2 \sim 10^4$. This condition is always fulfilled in semiconductor quantum generators and can be violated only when the quantum generator operates in the regime of modulated Q.

We note that, as shown in ([2]), for pulsations with a large modulation depth the field intensity at the instants corresponding to the minimum of the radiation may be below the level of spontaneous emission, and therefore field fluctuations may affect the magnitude of the pulses and the spacing between them.

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Note: Figure translations are in progress. See original paper for figures.