ON THE DETERMINATION OF THE DYNAMIC COEFFICIENT OF VISCOSITY OF THE ASTHENOSPHERE

MECHANICS

1966

SovietRxiv

Abstract

Full Text

UDC 550.312

MECHANICS

S. S. GRIGORYAN, M. S. KRASS, V. L. LEITES, S. A. USHAKOV

ON THE DETERMINATION OF THE DYNAMIC COEFFICIENT OF VISCOSITY OF THE ASTHENOSPHERE

(Presented by Academician D. I. Shcherbakov, January 4, 1966)

In the light of contemporary studies of Antarctica and Greenland ([1,2]), it may be asserted that regions of recent glaciations–Canada, Fennoscandia, and Spitsbergen–can be used to estimate the rate of the process of restoration of equilibrium. Estimates of the viscosity of the substratum from the rate of postglacial uplift of Fennoscandia have been made by various researchers ([3–6]), both under the assumption that the earth' s crust does not impede the restoration of equilibrium and under the assumption that the crust is an elastic shell. As a result of these estimates, the value $\mu = 10^{21}$—10^{22} poise was obtained. However, as was pointed out in ([3]), this conclusion is insufficiently substantiated, since relaxation processes in the crust are not taken into account in it. Moreover, in these estimates no account is taken of the circumstance that the flow occurs not in a semi-infinite space but, apparently, in a rather thin subcrustal layer–a waveguide. Neglect of the latter circumstance leads to a solution in which the long waves of the anomalous gravitational field must decay much faster than the short ones, which contradicts the available information on the figure of the Earth ([7]). Therefore it becomes necessary to consider the case of uplift of the earth' s crust under the condition that flow occurs in a thin subcrustal layer (asthenosphere), which is the Gutenberg waveguide ([3,10]).

If it is assumed that: 1) the earth' s crust does not impede the restoration of equilibrium; 2) the flow of the asthenosphere is close to the flow of a viscous fluid; 3) the flow of mantle material below the asthenospheric layer of finite thickness may be neglected, then the problem of uplift of the earth' s crust after deglaciation reduces to the problem of uplift of a depression pressed into a layer of viscous fluid of finite thickness and infinite extent resting on a rigid base. The shape of glaciation regions is usually close to a circle; the shape of the daily surface of the ice along the diameter of the continental glaciation is close to a semiellipse ([8]), strongly elongated along the major axis ($a/b = 1/100 \div 1/450$). We shall therefore consider the axisymmetric case of the uplift of a depression whose initial form is a disk of thickness a.

Fig. 1. Decay of a disturbed surface at $k = 0.3$ and $k = 1.0$

Figure 1: Fig. 1. Decay of a disturbed surface at $k = 0.3$ and $k = 1.0$

Thus, it is assumed that a layer of viscous fluid of infinite extent and thickness h floats on a rigid base. The initial disturbance of the free surface of this layer (Fig. 1), in cylindrical coordinates under the condition that the z-axis is directed downward, is

$$\zeta(r, 0) = \begin{cases} a, & r < R_0, \\ 0, & r \geqslant R_0, \end{cases} \qquad a \ll h. \tag{1}$$

It is required to determine the formula of the free surface at any time t.

As is known ([9]), the components of the stress tensor (of which we need only two) for a viscous incompressible fluid in cylindrical coordinates have the form

$$\sigma_{zz} = -p + 2\mu \partial v_z / \partial z, \qquad \sigma_{rz} = \mu(\partial v_r/\partial z - \partial v_z/\partial r), \tag{2}$$

where p is the pressure of the liquid at the given point in space; μ is the coefficient of dynamic viscosity; v_r, v_z are the velocity projections.

For the case of large values of the kinematic viscosity $\eta = \mu/\rho$ that is of interest to us, i.e., small values of the Reynolds number Re, as an estimate of the terms of the Navier–Stokes equations shows when they are reduced to dimensionless form, the inertial terms may be discarded while retaining a high degree of accuracy. Then the system of Navier–Stokes equations without inertial terms takes the form

$$\frac{\partial}{r\partial r}\left(r\frac{\partial v_r}{\partial r}\right) - \frac{v_r}{r^2} + \frac{\partial^2 v_r}{\partial z^2} = \frac{1}{\mu}\frac{\partial \bar{p}}{\partial r}$$

$$\frac{\partial}{r\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{\partial^2 v_z}{\partial z^2} = \frac{1}{\mu}\frac{\partial \bar{p}}{\partial z}, \tag{A}$$

where $\bar{p} = p - p_0 g z$; p_0 is the specific density of the viscous fluid (in our case, the Earth' s mantle). Adding to system (A) the continuity equation:

Fig. 1. Decay of a disturbed surface at $k = 0.3$ and $k = 1.0$

$$\frac{\partial}{r\partial r}(r v_r) + \frac{\partial v_z}{\partial z} = 0, \tag{3}$$

we obtain a system of three differential equations for the unknown functions $\bar{p}, v_r, v_z$.

The boundary conditions of our problem are as follows:

1. On the free surface the components of the stress tensor are equal to zero:

$$\sigma_{zz}\Big|_{z=\xi(r,t)} = 0, \tag{4}$$

$$\sigma_{rz}\Big|_{z=\xi(r,t)} = 0. \tag{5}$$

2. On the rigid base—the no-slip condition:

$$v_r\Big|_{z=h} = 0, \tag{6}$$

$$v_z\Big|_{z=h} = 0. \tag{7}$$

Since the depth of the step $a \ll h$, to a first approximation one may assume that the equation of the free surface is $z = 0$. Therefore, with a high degree of accuracy, equalities (4) and (5) may be replaced by

$$\sigma_{zz}\Big|_{z=0} = 0, \tag{4'}$$

$$\sigma_{rz}\Big|_{z=0} = 0. \tag{5'}$$

To solve the problem we shall also assume that the particle remains on the surface of the moving fluid, which, by virtue of the reasons indicated above, may be written in the form

$$\frac{\partial}{\partial t}\xi(r,t) = v_z\Big|_{z=0}. \tag{8}$$

Solving system (A), (3), using the boundary conditions (4′)–(7) with allowance for (1), (2), and (8), and also reducing the formula for the free surface to dimensionless form, introducing for this purpose the dimensionless quantities $u = \lambda R_0$; $\tau = t/T_0$; $2\mu/\rho g R_0 = T_0$–the relaxation time; $\beta = r/R_0$; $k = h/R_0$;
$\bar{\xi}(\beta,\tau) = \xi(r,t)/R_0$, we obtain

$$\bar{\xi}(\beta,\tau) = \bar{a}\int_0^\infty e^{\tau f(u)} J_1(u) J_0(\beta u)\, du, \tag{9}$$

where $\bar{a} = a/R_0$;

$$f(u) = \frac{1}{u}\frac{1+\gamma_1}{1-\gamma_1}; \qquad \gamma_1 = \frac{2(2ku - e^{2ku} - 2k^2u^2 - 1)}{2e^{-2ku} + 1 + (1+2ku)^2}.$$

Since $f(u) < 0$, the exponential character of the decay of the surface uplift is evident from (9), which agrees with the factual data for Fennoscandia.

The integral in expression (9) was computed on an electronic computer with an absolute accuracy of 10^{-4} for two values of k (0.3 and 1.0). For each value of τ, the parameter β was varied with a variable step from 0.15 to 3.5 (Fig. 1). As was to be expected, with increasing k the decay of the disturbed surface intensifies. Thus, at $k = 1$ it proceeds about an order of magnitude faster than at $k = 0.3$, from which it follows that, for one and the same h (the thickness of the asthenospheric layer), those anomalies whose radius of initial disturbance is smaller will decay faster.

For Fennoscandia one may take $k \approx 0.3$, since $R_0 \approx 700$ km, and $h \approx 100$–150 km ([4,10]). Therefore, in all calculations performed without taking into account the finite thickness of the asthenospheric layer (i.e., based on the solution of the equation for the spreading of a viscous fluid in a semi-infinite space), the value of the dynamic coefficient of viscosity was overestimated, at least by an order of magnitude. It is therefore most probable that for the asthenosphere $\mu = 10^{20}$–10^{21} poise.

Moscow State University
named after M. V. Lomonosov

Received
3 I 1966

CITED LITERATURE

[1] S. A. Ushakov, *Geophysical Investigations of the Structure of the Earth' s Crust in East Antarctica*, Publishing House of the Academy of Sciences of the USSR, 1963.
[2] A. Joset, J. J. Holtscherer. Ann. Geophys., **10**, No. 4 (1954).
[3] V. A. Magnitskii, *Internal Structure and Physics of the Earth*, Moscow, 1965.
[4] N. A. Haskell, Am. J. Sci., **33**, 22 (1937).
[5] F. A. Vening-Meinesz, Proc. Koninkl. Nederl. Akad. Wet., **40**, No. 8 (1937).
[6] T. Niskanen, Publ. Isostatic Inst. Intern. Assoc. Geodesy, Helsinki, No. 2 (1948).
[7] H. Takeuchi, J. Geophys. Res., **68**, No. 8 (1963).
[8] P. A. Shumskii, *Collection of Seismic and Glaciological Investigations during the IGY*, No. 2, Publishing House of the Academy of Sciences of the USSR, 1959.
[9] I. Sneddon, *Fourier Transforms*, IL, 1955.
[10] B. Gutenberg, *Physics of the Earth' s Interior*, IL, 1963.

Note: Figure translations are in progress. See original paper for figures.