DETERMINATION OF THE LUNAR GRAVITATIONAL FIELD FROM THE MOTION OF THE ARTIFICIAL LUNAR SATELLITE "LUNA-10"

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DETERMINATION OF THE LUNAR GRAVITATIONAL FIELD FROM THE MOTION OF THE ARTIFICIAL LUNAR SATELLITE “LUNA-10”

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The motion of the artificial lunar satellite Luna-10 in the noncentral gravitational field of the Moon is investigated, taking into account the gravitational influence of the Earth, the Sun, and the planets.

If the gravitational field of the Moon were central, and the influence of external bodies were absent, the satellite’ s motion would proceed along an unperturbed orbit (a Keplerian ellipse), whose shape and dimensions would remain unchanged in absolute space. The noncentrality of the lunar gravitational field and the action of external bodies, the principal ones being the Earth and the Sun, cause perturbations in the motion of the satellite. Under the action of perturbations the satellite’ s orbit evolves with time.

The perturbing action of the Earth and the Sun on the motion of a lunar satellite is well known. Of greatest interest is the evolution of the satellite’ s orbit that arises owing to the unknown noncentrality of the lunar gravitational field. Knowledge of this evolution makes it possible to determine the parameters of the lunar gravitational field.

To reveal the evolution of the orbit of the artificial satellite Luna-10, measurements of the trajectory of the satellite’ s motion were used, carried out throughout the entire period of its active existence (April 3–May 30, 1966). The trajectory measurements were subjected to statistical processing for the purpose of jointly determining the parameters of the lunar gravitational field and the elements of the satellite’ s orbit. The measurement-processing method was based on an analytical theory of the motion of a lunar satellite, which made it possible to encompass in a single computation the entire two-month interval of the satellite’ s motion.

The description of the selenocentric motion of the satellite and of the motion of the Moon about its own center of mass was carried out in a Cartesian rectangular selenocentric coordinate system XYZ. The plane XY of this system coincides

Fig. 1. Perturbations in the longitude Ω of the ascending node and in the angular distance ω of the pericenter from the node of the orbit. Thick lines–due to the noncentrality of the Moon' s gravitational field; thin lines–due to the gravitational influence of the Earth and the Sun.

Figure 1: Fig. 1. Perturbations in the longitude Ω of the ascending node and in the angular distance ω of the pericenter from the node of the orbit. Thick lines –due to the noncentrality of the Moon' s gravitational field; thin lines–due to the gravitational influence of the Earth and the Sun.

with the plane of the mean equator of the Moon, and the plane XZ with the plane of its prime meridian at epoch t_0. The directions of the axes of the system are fixed relative to the stars. The X-axis of the system is directed toward the Earth, the Z-axis toward the north pole of the Moon, and the Y-axis completes the system to a right-handed one. To describe the satellite' s motion, the elements of its orbit were used: semimajor axis a, eccentricity e, inclination i, longitude of the ascending node Ω, angular distance of the pericenter from the node ω, and time of passage through the node T_Ω. (The angular orbital elements i, Ω, ω are reckoned from the plane of the mean equator of the Moon and from its prime meridian at epoch t_0, in the manner customary in celestial mechanics.) It is assumed that the rotation of the Moon about its own center of mass takes place in accordance with Cassini' s laws–uniformly about the fixed axis OZ of the introduced coordinate system. The motion of the Moon' s center of mass (the origin of the coordinate system) in the geoequatorial coordinate system with the mean equinox of epoch 1960.0 is given by Brown' s theory.

The noncentrality of the Moon' s gravitational field is an essential factor determining the evolution of the orbit of the Luna-10 satellite. The perturbations of the satellite' s orbit arising from the noncentrality of the Moon' s field of attraction are manifested especially clearly in the evolution of the longitude Ω of the ascending

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node of the satellite and the angular distance ω of the pericelenium from the node. This evolution of the elements Ω, ω of the satellite' s orbit during the time of its active existence is shown in Fig. 1 as a function of the number of revolutions of the satellite. The evolution of the parameters Ω, ω contains a pronounced secular departure, leading to regression of both parameters. A noticeable periodic perturbation is superposed on the secular departure of the parameter ω. Over 460 revolutions of the satellite, the perturbations of the parameters Ω, ω caused by the noncentrality of the Moon' s gravitational field reach the values $\Delta\Omega = -7^\circ.7$; $\Delta\omega = -11^\circ.8$. The perturbations of the inclination i of the satellite' s orbit and of its eccentricity e are mainly periodic in character and have amplitudes $\Delta i \approx 0^\circ.15$; $\Delta e \approx 0.003$.

The perturbations of the satellite' s orbit due to the noncentrality of the Moon' s gravitational field lead to perturbations of its coordinates amounting, over one revolution of the satellite, to $|\Delta\bar{r}| \approx 0.75$ km.

The gravitational influence of the Earth and the Sun on the motion of Luna-10 also leads to an evolution of its orbit. This influence causes regression of the node and pericenter of the satellite' s orbit, which are also shown in Fig. 1. During the active existence of the satellite, these perturbations amount to $\Delta\Omega = -1^\circ$; $\Delta\omega = -2^\circ$. Under the influence of the Earth and the Sun, the eccentricity of the satellite' s orbit decreases over the time interval considered. With an almost unchanged semimajor axis of the orbit, the decrease in eccentricity leads to an increase in the orbit' s pericenter. The perturbations of the coordinates of the Luna-10 satellite caused by the action of the Earth and the Sun over one revolution of the satellite do not exceed $|\Delta\bar{r}| \approx 0.11$ km.

The comparative estimates given show that, for the orbit of Luna-10, the perturbations due to the noncentrality of the Moon' s gravitational field exceed by a factor of 5–6 the perturbations caused by the gravitational influence of the Earth and the Sun. The planetary perturbations of the satellite' s motion over the time interval considered are small in magnitude and therefore were not included in the calculation.

Processing of trajectory measurements containing their common dynamical linkage over the two-month flight interval of the Luna-10 satellite made it possible to determine quantitative characteristics of the noncentrality of the Moon' s gravitational field.

The expression for the gravitational potential U of the Moon was taken in the form of an expansion in a series of spherical functions

$$U\{r,\psi,\lambda\} = \frac{\mu}{r}\left\{1 + \sum_{n=2}^{\infty}\sum_{m=0}^{n}\left(\frac{R}{r}\right)^{n}\left[c_{nm}\cos m\lambda + d_{nm}\sin m\lambda\right]P_n^m(\sin\psi)\right\},$$

where μ is the mass of the Moon, R is its mean radius; r, ψ, λ are the spherical coordinates of a point: r is the polar radius, ψ is latitude measured from the Moon' s mean equator, λ is longitude measured from the zero meridian of the epoch t_0; $P_n^m(\sin\psi)$ are the associated Legendre functions. The coefficients c_{nm}, d_{nm} of this expansion were used as the parameters to be determined for the Moon' s gravitational field.

As a result of processing the trajectory measurements, numerical values were obtained for eleven coefficients of the expansion of the Moon' s gravitational potential. These values, together with their maximum possible errors, are given below.

$$
\begin{aligned}
c_{20} &= (-0.206 \pm 0.022) \cdot 10^{-3}, \\
c_{21} &= (0.157 \pm 0.059) \cdot 10^{-4}, \\
d_{21} &= (0.361 \pm 0.358) \cdot 10^{-5}, \\
c_{22} &= (0.140 \pm 0.012) \cdot 10^{-4}, \\
d_{22} &= (-0.139 \pm 0.145) \cdot 10^{-5}, \\
c_{30} &= (-0.363 \pm 0.099) \cdot 10^{-4}, \\
c_{31} &= (-0.568 \pm 0.026) \cdot 10^{-4}, \\
d_{31} &= (-0.178 \pm 0.032) \cdot 10^{-4}, \\
c_{32} &= (0.118 \pm 0.047) \cdot 10^{-4}, \\
d_{32} &= (-0.702 \pm 4.595) \cdot 10^{-6}, \\
c_{40} &= (0.333 \pm 0.270) \cdot 10^{-4}.
\end{aligned}
$$

Fig. 2. Sections of the level surface of the Moon' s gravitational potential (radial deviations from the circle are magnified 1000 times). A—equatorial, —meridional ($\lambda = 0$), —meridional ($\lambda = 90^\circ$).

There is a substantial correlation between the errors in determining the parameters c_{20} and c_{40}. The correlation coefficient is $k = -0.99$. Mutual

the correlation between the errors of the remaining determined parameters does not exceed 0.4.

The numerical values given for the parameters c_{20} and c_{22} agree with their known values obtained from libration measurements [1].

The nonzero values of the coefficients c_{nm} with even index m and of the coefficients d_{nm} with even index m indicate that the gravitational field on the visible and invisible sides of the Moon, as seen from Earth, is asymmetric.

To illustrate the determined gravitational potential of the Moon, a level surface passing through the point with spherical coordinates $r = 1738$ km, $\psi = 0$, $\lambda = 0$ was considered. In Fig. 2 the level lines obtained by sections of this surface by the equatorial plane XY (Fig. 2 A), by the plane of the zero meridian XZ (Fig. 2 B), and by the meridional plane YZ corresponding to longitude $\lambda = 90^\circ$ (Fig. 2 C) are plotted. The radius of the circle in these figures corresponds to the mean radius of the Moon, equal to 1738 km. For clarity, the radial deviation of the level lines from the circle is plotted with a magnification of 1000 times.

The pear-shaped form of the potential surface, with an elongation on the far side of the Moon, is visible. The pear-shaped form is produced mainly under the influence of terms in the expansion of the potential containing the tesseral harmonic P_3^1 (with coefficient c_{31}) and the zonal harmonic P_2^0.

The results presented above are preliminary. Further processing of trajectory measurements of Luna-10 and analysis of the motion of subsequent lunar satellites will make it possible to refine and supplement the obtained parameters of the Moon’ s gravitational potential, as well as to refine the motion of its center of mass.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.