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ON THE THEORY OF THE SINGLE-PULSE OPERATING REGIME OF OPTICAL QUANTUM GENERATORS

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In works devoted to the theoretical consideration of the single-pulse operating regime of optical quantum generators (OQG), the processes occurring after an instantaneous increase in the resonator Q are considered mainly ([1–6]). In the present work, the process of energy accumulation in the active medium is also included in the consideration. At a considerable population inversion there occurs amplification of spontaneous emission, leading to a substantial decrease in the lifetime of the excited state ([7,8]) and being, along with bleaching of the active medium, one of the main factors limiting the magnitude of the accumulated excitation energy and, correspondingly, the limiting power and energy of generation.

1. **Accumulation of excitation energy in the active medium.** Taking into account that, in order to attain the limiting value of the population inversion, excitation of the active medium is necessary for a time greater than the effective lifetime of the excited state τ_e, we shall restrict ourselves to consideration of the stationary problem. For a four-level medium with a frozen-out lower level of the working transition, under the validity of the usual assumptions about the probabilities of nonradiative transitions ([9]), the inverted population N is determined by the relation

$$\eta_1(N_0 - N)B_n u_n = \frac{N}{\tau}\gamma, \quad (1)$$

where $\gamma = \tau/\tau_e = 1 + \eta_2 N_B/N_c$; N_c and N_B are the numbers of acts of spontaneous and stimulated (caused by quanta of spontaneous transitions) radiation corresponding to the working transition; the remaining notation is the same as in ([9]). The quantity $\xi = N_B/N_c$, and correspondingly γ, depends on the degree of inversion and the geometry of the active rod.

If the generation threshold corresponds to the population value N_p and the pump radiation density u_{np}, then with the aid of relation (1) it is not difficult to obtain

$$N = \frac{nN_{\mathrm{p}}}{\gamma + (N_{\mathrm{p}}/N_0)(n-\gamma)}, \tag{2}$$

where $n = u_{\mathrm{n}}/u_{\mathrm{n}p}$ is the excess over threshold in power. The threshold value of γ is taken equal to 1.

Similarly, for a three-level scheme we find

$$N = N_0 \, \frac{n - \gamma + (N_{\mathrm{p}}/N_0)(n+\gamma)}{n + \gamma + (N_{\mathrm{p}}/N_0)(n-\gamma)}. \tag{3}$$

Let us proceed to the calculation of the quantity $\xi = N_{\mathrm{B}}/N_{\mathrm{c}}$. Obviously,

$$\xi = \frac{\int_v K_\xi(v)\,dv}{\int_v K_v\,dv}, \tag{4}$$

where $\xi(\nu)$ is the number of stimulated quanta caused by one spontaneous quantum of frequency ν, and K_ν is the gain coefficient at the frequency ν. To estimate $\xi(\nu)$, one may use the concept of an effective length l_e, equal to the mean path length traversed by a spontaneous quantum before leaving the sample ([7,8]). In this case

$$\xi(\nu) = \frac{K_\nu}{K_\nu - \rho}\left[\exp(K_\nu - \rho)l_e - 1\right], \tag{5}$$

where ρ is the loss coefficient.

For a cylindrical sample of length l with polished walls, $l_e \approx l$ ([7]). If a cylindrical sample of length l and diameter D is surrounded by an immersion jacket of diameter D_0 with a polished outer surface, then, for $l/D \geqslant 10$,

$$l_e \approx \frac{2}{3}D + \frac{4}{3\pi}\frac{l\ln\chi}{D_0/D - D/8D_0}$$

(χ is the refractive index of the medium). For a cylindrical sample with a frosted or matted lateral surface, l_e for $l/D > 5$ is almost independent of l: the mean path of photons up to their first incidence on the wall of the sample is $\sim 2/3\,D$; photons scattered isotropically from the wall into the sample traverse, on the average, a path $\sim D$ before the next incidence on the wall; and $l_e \simeq D(2/3 + \alpha/(1-\alpha))$, where α is the fraction of radiation isotropically scattered back upon incidence from inside on the surface of the sample (for a matted surface it is hereafter taken that $\alpha = 0.5$, and for a frosted one $\alpha = 0$). A more correct calculation of the number of stimulated-emission events leads in this case to the expression

$$\xi(\nu) = \frac{K_\nu}{K_\nu - \rho}\left\{\frac{(1-\alpha)\exp[^2/_3 D(K_\nu - \rho)]}{1 - \alpha\exp[D(K_\nu - \rho)]} - 1\right\}. \tag{6}$$

A more exact method for determining $\xi(\nu)$ for samples with a frosted or matted surface is a method based on calculating the spectral density of illumination E_ν (on the inside of) the surface of the sample by amplified spontaneous radiation ([8]). With an inversion constant throughout the volume, for E_ν one can obtain the integral equation

$$E_\nu(s) = \int_\Sigma \left\{\frac{\alpha(s')E_\nu(s')}{\pi}\exp[(K_\nu - \rho)l_{ss'}] + \frac{N_{c\nu}h\nu}{4\pi V(K_\nu - \rho)}\left[\exp[(K_\nu - \rho)l_{ss'}] - 1\right]\right\}\frac{\cos\varphi_1\cos\varphi_2}{l_{ss'}^2}\,ds', \tag{7}$$

where s and s' are arbitrary points on the surface of the sample; $N_{c\nu}$ is the spectral density of the quantity N_c; V and Σ are the volume and surface of the sample; $l_{ss'}$ is the length of the segment ss'; and φ_1 and φ_2 are the angles between the segment ss' and the normals to the surface of the sample at the points s and s'.

Having found $E_\nu(s)$ from equation (7), it is then not difficult to obtain the value of $\xi(\nu)$ by means of the relation defining the spectral density of the radiation leaving the sample:

$$\int_\Sigma E_\nu(s)[1 - \alpha(s)]\,ds = h\nu N_{c\nu}\left[1 + \frac{K_\nu - \rho}{K_\nu}\xi(\nu)\right].$$

Ultimately, the value of $\xi(\nu)$, as was to be expected, depends only on K_ν, ρ, and the geometry of the sample. For a cylindrical sample with frosted end faces, (7) becomes a linear integral equation readily solvable by numerical methods.

The results of calculations of the quantities $\xi(\nu)$ and $\bar{\xi}$ for $\rho = 0$ and a Lorentzian line shape are presented in Fig. 1. As is seen from the data obtained, for small inversion the quantity $\bar{\xi}$ is determined mainly by the value KD and depends only weakly on the sample length. The results show that the use-

use for estimating ξ from relations (5) and (6) is quite permissible for not very large inversions. For matted samples, as follows from Fig. 1, there exists a limiting attainable value of the inversion, determined approximately by the relation $\alpha\exp[D(K - \rho)] = 1$, which is the condition for compensation of losses by gain (see (6)).

Using formulas (2)–(6) and the data given in Fig. 1, for a specified excess above threshold n one can determine the inverse population and the gain coefficient of the active medium, related by

Fig. 1

Figure 1: Fig. 1

$$N/N_{\text{th}} = K/K_{\text{th}}$$

(K_{th} is the threshold gain coefficient).

Fig. 1. Matted lateral surface: **1, 2, 3**—according to (7), l/D respectively $20, 10, 5$; **4**—according to (6). Dimensionless sheath: **5**—according to (5), $l/D = 10$, $D/l = \chi = 1.5$. Translucent lateral surface: **6, 7, 8**—according to (7), l/D respectively $20, 10, 5$; **9**—according to (5).

2. Generation regime. In what follows it is assumed that the following conditions are fulfilled:

1) The duration of the generation pulse T is much greater than the transit time of a quantum in the resonator,

$$\Delta t = \frac{l_{\text{r}} + l_{\text{k}}(\chi - 1)}{C}$$

(l_{r} and l_{k} are the lengths of the resonator and the active sample).

2) The gain coefficient is constant in the active sample, and the distribution of radiation density in the resonator does not differ substantially from the distribution under stationary generation.

3) The relaxation time of the lower working level (four-level scheme) is $\ll T$. Then for a four-level active medium one can write:

$$\frac{du}{dt} = \frac{h\nu}{C\Delta\nu}\,\frac{\chi l_{\text{k}}}{\Delta t}\,NBu - \frac{K_{\text{th}} l_{\text{k}}}{\Delta t}\,u, \qquad \frac{dN}{dt} = -NBu, \tag{8}$$

where u is the density of the generated radiation in the active rod; B is the Einstein coefficient for the working transition; $\Delta\nu$ is the luminescence line width;

$$K_{\text{th}} = \sigma_0 + \frac{1}{2l_{\text{k}}}\left(\delta + \ln\frac{1}{R}\right);$$

σ_0 is the threshold loss per 1 cm length of the active rod (8); δ is the loss in the shutter; R is the reflection coefficient of the partially transparent mirror.

From (8) one can find the quantities of interest to us: the maximum pulse power P, the generation energy W, and the duration of the generation pulse T:

$$P = \frac{E\ln(1/R)}{2\Delta t}\left[1 - \frac{K_{\text{th}}}{K}\left(1 + \ln\frac{K}{K_{\text{th}}}\right)\right], \tag{9}$$

$$W = \frac{E\ln(1/R)}{2l_{\text{k}}K_{\text{th}}}\left(1 - \frac{K_1}{K}\right), \tag{10}$$

$$T \simeq \frac{W}{P} = \frac{\Delta t(1 - K_1/K)}{l_{\text{k}}K_{\text{th}}\left[1 - (K_{\text{th}}/K)(1 + \ln K/K_{\text{th}})\right]}, \tag{11}$$

where K and K_1 are the gain coefficients in the active medium at the initial and final moments of generation, respectively; $E = h\nu V_{kN} = CV_k\Delta\nu K/\chi B$ is the excitation energy stored in the active sample. The quantity K_1 can be found from the relation $(K - K_1)/K_p = \ln(K/K_1)$. For $K/K_p \geq 2.5$, $K_1 \simeq K\exp(K/K_p)$, and the term K_1/K in (10) and (11) may be neglected in the first approximation.

To determine the reflection coefficient of the mirror R_0 at which the generation power is maximal (P_0), equation (9) was solved numerically for various values of R and K/K_p. The calculation results showed that, for $K/K_p \geq 1.5$, the following interpolation expressions may be written for R_0 and P_0:

$$\ln(1/R_0) \simeq 0.57\,K_{p0}l_k\,(K/K_{p0} - 0.88); \tag{12}$$

$$P_0 \simeq 0.1\,\frac{E_{p0}K_{p0}l_k}{\Delta t}\,\frac{(K/K_{p0} - 1)^3}{K/K_{p0} + 1}, \tag{13}$$

where K_{p0} is the threshold gain coefficient at $R = 1$; $E_{p0} = h\nu V_{kNp0}$ is the excitation energy stored in the active sample at threshold inversion and $R = 1$.

The error in determining R_0 and P_0 from (11) and (12) does not exceed 10 and 15%, respectively.

For a three-level active medium, expressions analogous to (9)–(13) can be obtained; in this case it is only necessary to add a factor of 2 to the denominator of expressions (9), (10), and (13).

The results obtained make it possible to find the limiting parameters of a generator operating in the monopulse regime. As an example, a calculation was carried out of the dependence of the maximum generation power on the magnitude of the threshold excess for matted cylindrical samples of active media operating according to three- and four-level schemes.

Fig. 2. $l_k = 8$ cm; $l_p = 40$ cm; $D = 0.8$ cm; $K_{p0} = 0.1$ cm^{-1}; $\eta_r = 1$. **1** and **2**—four-level scheme, $N_0/N_{p0} = 2 \cdot 10^2$; **3** and **4**—three-level scheme, $N_0/N_{p0} = 4$; **1** and **3**—without taking into account shortening of τ, **2** and **4**—with it taken into account.

Figure 2

Figure 2: Figure 2

The calculation results are shown in Fig. 2 (the radiation power for $n \to \infty$ was taken as the conventional unit of power). The results of a calculation performed without taking into account the decrease in the lifetime of the excited state ($\gamma = 1$) are also shown there.

As is seen from the data presented, the influence of amplification of spontaneous emission is very significant for four-level active media, and the limiting power is restricted precisely by this factor. In the case of a three-level active medium, the main factor limiting the limiting radiation power for the selected parameters of the optical quantum generator is bleaching of the active medium.

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CITED LITERATURE

1. A. M. Prokhorov, *Radiotekhnika i elektronika*, **8**, 1073 (1963).
2. A. A. Wuylsteke, *J. Appl. Phys.*, **34**, 1615 (1963).
3. W. G. Wagner, B. A. Landuel, *J. Appl. Phys.*, **34**, 2040 (1963).
4. A. L. Mikaelyan, Yu. T. Turkov, *Radiotekhnika i elektronika*, No. 4, 743 (1964).
5. M. Menat, *J. Appl. Phys.*, **36**, 73 (1965).
6. T. V. Galadze, I. K. Krasyuk et al., *ZhETF*, **48**, 106 (1965).
7. B. A. Ermakov, A. V. Lukin, A. A. Mak, *Optika i spektroskopiya*, **18**, 353 (1965).
8. Yu. A. Anan' ev, A. A. Mak, B. M. Sedov, *ZhETF*, **48**, issue 7 (1965).
9. Yu. A. Anan' ev, V. F. Egorova et al., *ZhETF*, **44**, 1884 (1963).

Note: Figure translations are in progress. See original paper for figures.