PLANE EXPLOSION IN AN ELECTRICALLY CONDUCTING GAS IN AN OBLIQUE MAGNETIC FIELD

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V. P. KOROBEINIKOV

PLANE EXPLOSION IN AN ELECTRICALLY CONDUCTING GAS IN AN OBLIQUE MAGNETIC FIELD

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The problem of a plane explosion in a gas was considered earlier in works ([1−3]), both for the case of finite conductivity and for infinite conductivity of the gas. In these works, in solving problems for magnetic fields of arbitrary intensity, only the cases were studied in which the magnetic field is parallel to the plane of the explosion and to the front of the shock wave.

Below we consider the problem of a plane explosion in an ideally conducting gas, when the intensity vector of the initial magnetic field has arbitrary magnitude and is directed at a certain angle to the plane of the explosion.

1. Let an instantaneous release of energy along a plane occur in an unbounded ideally conducting gas, i.e., let a plane "point" explosion occur. We denote the energy released per unit area by E_0. We assume that in the initial state the gas was at rest, and that the pressure p_∞ and the initial density ρ_∞ are constant. The initial magnetic field H_∞ is constant in magnitude and is directed at a certain angle α_0 to the plane of the explosion. Without loss of generality, it may be assumed that the field has components H_x, H_y, where the y-axis is parallel to the plane of the explosion and the x-axis is perpendicular to it. It follows from the symmetry of the problem that it will be one-dimensional, and all the initial characteristics of the flow will depend only on the time t and the coordinate x. The arising plane shock wave propagates in the direction of the x-axis. Behind the shock-wave front there arises a nonstationary flow, for the description of which we shall use the equations of magnetohydrodynamics for a nonviscous, non-heat-conducting perfect gas. For the flows under consideration we have the system of equations

$$\frac{\partial \rho v_x}{\partial t} = -\frac{\partial}{\partial x}\left(\rho v_x^2 + p^*\right), \qquad \frac{\partial \rho v_y}{\partial t} = -\frac{\partial}{\partial x}\left(\rho v_y v_x - \frac{H_x}{4\pi} H_y\right),$$

$$\frac{\partial \rho}{\partial t} = -\frac{\partial}{\partial x}(\rho v_x), \qquad \frac{\partial p^{1/\gamma}}{\partial t} = -\frac{\partial}{\partial x}(v_x p^{1/\gamma}), \tag{1}$$

$$\frac{\partial H_y}{\partial t} = -\frac{\partial}{\partial x}(v_x H_y - H_x v_y), \qquad H_x = H_{x\infty} = \text{const.}$$

Here v_x, v_y are the components of the gas velocity,

$$p^* = p + \frac{1}{8\pi} H_y^2.$$

From (1) and the thermodynamic relations one can obtain the energy equation in the form

$$\frac{\partial \varepsilon^*}{\partial t} = -\frac{\partial}{\partial x}\left[v_x(\varepsilon^* + p^*) - \frac{1}{4\pi} H_x v_y H_y\right], \qquad \varepsilon^* = \frac{1}{2}\rho v^2 + \frac{p}{\gamma - 1} + \frac{H_y^2}{8\pi}.$$

The conditions at the shock wave have the form (4)

$$(v_{xn} - D)H_{yn} - H_{xv_{yn}} = -DH_{y\infty}, \qquad D = \frac{dx_n}{dt},$$

$$-D\rho_\infty v_{yn} - \frac{1}{4\pi} H_{xH_{yn}} = -\frac{1}{4\pi} H_{xH_{y\infty}}, \qquad (v_{xn} - D)\rho_n = -D\rho_\infty, \tag{2}$$

$$(v_{xn} - D)\varepsilon_n^* + v_{xn} p_n^* - \frac{H_x}{4\pi} H_{yn} v_{yn} = -D\left(\frac{p_\infty}{\gamma - 1} + \frac{H_{y\infty}^2}{8\pi}\right),$$

$$-D\rho_\infty v_{xn} + p_n^* = p_\infty^*.$$

Here the subscript n denotes the parameters at the front of the shock wave, and D is the velocity of the shock wave. In addition to conditions (2), we have the usual condition at the center of symmetry of the flow

$$v_x(0, t) = 0. \tag{3}$$

To solve the problem it is necessary to find the functions $v_x, v_y, p, \rho, H_y, x_n(t)$ satisfying system (1) and the boundary conditions (2), (3).

For $H_x = 0$ the magnetic field is parallel to the wave front. The problem corresponding to this case was considered in ([3]), where a method for its numerical solution was given. The solution of the problem for $H_x \neq 0$ is complicated by the presence of the new unknown function v_y and by the increase in the number of differential equations. For its complete solution it is necessary to use numerical and special approximate methods.

2. Let us note some general properties of the solution of the problem.

System (1) and conditions (2) are invariant with respect to the transformation $v'_y = -v_y,\ v'_x = -v_x,\ x' = -x$. This means, in particular, that the function v_y is odd in x, and under the condition of continuity of the solution in a neighborhood of $x = 0$ we have the equality $v_y(0, t) = 0$. Thus, at $x = 0$ the gas velocity is zero.

Let us next consider the question of the law of decay of the shock wave at large distances from the explosion site. As the shock wave propagates from the center of the explosion it will decay, and the overall zone of disturbed motion will increase. Between the center of the explosion and the leading front of the wave there will occur a gas flow accompanied by a complex pattern of interaction of magnetohydrodynamic compression and rarefaction waves. At large distances the shock wave becomes weak. Since the entropy changes little in passing through a weak shock wave ([5]), in some neighborhood of the wave front the flow may be regarded as isentropic. By analogy with gas dynamics one may assume ([6]) that the flow in some neighborhood behind the front of a magnetohydrodynamic shock wave is an isentropic Riemann wave ([4]). Then, applying, for example, the method for obtaining asymptotic laws of shock-wave decay considered in ([6]), one can find the asymptotic law of variation of v_{xn} as a function of x_n. Similarly to gas dynamics, or to the case $H_x = 0$, we have

$$v_{xn} = C x_n^{-1/2}, \tag{4}$$

where C is a certain constant.

Using relations (2), one can establish the corresponding asymptotic formulas also for the remaining unknown functions.

3. Let us consider some methods for solving the problem.

For a numerical solution of the problem one may use the method of characteristics. This method would make it possible to describe with sufficient accuracy the system of magnetohydrodynamic waves in the flow region, at any rate up to the moment of formation in the flow of secondary magnetohydrodynamic shock waves. However, this method is rather laborious and inconvenient for computation on calculating machines. Below we shall indicate simpler methods of calculation.

For $H_{x\infty} < H_{y\infty}$, i.e., when the angle α_0 is small, neglecting in (1), (2) terms of order $v_y H_x$ in comparison with $H_y v_x$, we find that the problem of determining v_x, p, ρ, H_y separates from the problem of determining v_y. If the problem of a plane explosion for the case $H_x = 0,\ \alpha_0 = 0$ has been solved, then, using the results of its solution, v_y can be found by integrating the second equation of

Figure 1. Dependences of τ and p_n/p_∞ on R_n. a–$\beta = 1$; b–$\beta = 0.5$; v–$\beta = 0$

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Figure 2. Dependences of V_{yn} and G_n on R_n. a–$\beta = 1$; b–$\beta = 0.5$; v–$\beta = 0$

Figure 2: Figure 2. Dependences of V_{yn} and G_n on R_n. a–$\beta = 1$; b–$\beta = 0.5$; v –$\beta = 0$

system (1). In [3], for the calculation of H_y, v_x, p, ρ, the method of integral relations was used. If one uses the computational results obtained by this method, then it is expedient to apply it also to the second equation of system (1) for determining v_y in the case of small α_0.

For calculating the problem at arbitrary α_0, it appears expedient to use either the method of straight lines or the method of integral relations, analogous to the methods developed in [3, 7]. Here by the method of straight lines is meant that variant of the method of integral relations in which, between the points of subdivision of the spatial coordinate into strips, the integrand expressions containing the sought functions are approximated by linear functions, i.e., the integrals over space are in fact evaluated by the trapezoidal rule. The advantage of applying the method of straight lines is that the system of approximating ordinary differential equations has a simple structure.

Important characteristics of the flow are the quantities v_n, p_n, ρ_n, H_{yn} and the law of motion of the shock wave $x_n(t)$. Let us consider an approximate method for determining these quantities independently of the calculation of the entire flow field.

Since at the initial instants of time a finite magnetic field weakly affects the motion of the gas, the dependence $v_n(x_n)$ for small x_n is close to the gasdynamic one and has the form [8]

Fig. 1. Dependences of τ and p_n/p_∞ on R_n.
a–$\beta = 1$; b–$\beta = 0.5$; v–$\beta = 0$

Fig. 2. Dependences of V_{yn} and G_n on R_n. a–$\beta = 1$, b–$\beta = 0.5$, v–$\beta = 0$

$$V_{xn} = \frac{4}{3}\frac{1}{\gamma + 1}\frac{1}{\sqrt{\gamma\varkappa R_n}}, \tag{5}$$

where $V_{xn} = v_{xn}/a_\infty,\ R_n = x_n p_\infty / E_0,\ a_\infty^2 = \gamma p_\infty/\rho_\infty,\ \varkappa = \text{const}.$

Dependence (5) is analogous to formula (4), and it is natural to assume [8] that $V_{xn}(R_n)$ can be approximated by formula (5) for -

of a free range of values x_n. Using (2), (5), one can determine all the sought dimensionless quantities at the wave front as functions of R_n or of the dimensionless time $\tau = E_0^{-1}\rho_\infty^{-1/2}p_\infty^{3/2}t$. We note that investigation of this method in application to the problem with $H_x = 0$ and to a problem of gas dynamics has shown that it provides sufficiently high accuracy.

The results of the calculations are presented in Figs. 1 and 2. In the problem under consideration, besides γ, there are two dimensionless independent parameters $\beta^2 = H_x^2/4\pi\gamma p_\infty,\ G_\infty^2 = H_{y\infty}^2/4\pi\gamma p_\infty$.

In Fig. 1 the graphs of $\tau(R_n)$ and $\dfrac{p}{p_\infty}(R_n)$ are given for $\gamma = 1.4,\ G_\infty = 0.5$, and various β. In Fig. 2 the dependences of V_{yn} on R_n and G_n on R_n are given for the same values of $G_\infty,\ \beta$, where $V_y = v_y/a_\infty,\ G^2 = H_y^2/4\pi\gamma p_\infty$.

It follows from the results presented that the component v_y behind the shock-wave front behaves nonmonotonically, increasing in absolute value from zero to a certain maximum at the beginning of the explosion and falling to zero at the late stage of the explosion. This means that gas particles located near the center of the explosion will acquire velocities directed almost along the x-axis. For particles lying outside a certain neighborhood of the center, however, the initial velocities will be directed at a certain angle to the x-axis. The gasdynamic parameters $p_n,\ \rho_n$ and the velocity component v_{xn} behave qualitatively in the same way as the corresponding quantities in the problem of an explosion without taking into account the influence of the magnetic field.

Steklov Mathematical InstituteAcademy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.