SCATTERING OF MESONS IN A MODEL WITH A FIXED NUCLEON

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SCATTERING OF MESONS IN A MODEL WITH A FIXED NUCLEON

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In papers (1-3) a model of a meson-nucleon system with three possible states of a fixed nucleon was considered. In the present note this model is used to describe the scattering of mesons by a fixed source.

We write the Hamiltonian of the meson-nucleon system in the form

$$H = m_{A0}A^+A + m_{B0}B^+B + m_C C^+C + \int dk^3 \omega a^+(k)a(k) +$$
$$+\lambda_{01}\int dk^3 u(\omega)[A^+Ba(k) + \text{h. c.}] + \lambda_{02}\int dk^3[B^+Ca(k) + \text{h. c.}], \qquad (1)$$

where $\omega = \omega_k = \sqrt{k^2+\mu^2}$; $u(\omega)$ is a real cutoff function (for a point source $u(\omega) = 1/\sqrt{2\omega}$); $A^+(A)$, $B^+(B)$, $C^+(C)$, and $a^+(k)(a(k))$ are the creation (annihilation) operators of fermions A, B, C and of the boson θ. All quantities with the subscript 0 are unrenormalized.

In paper (3) the Schrödinger equation was solved exactly for one-nucleon states belonging to the discrete energy spectrum. Now the Schrödinger equation

$$H|X\rangle = E|X\rangle \qquad (2)$$

is solved for states of the continuous spectrum $|X\rangle$, containing one A-particle—the states of meson scattering on nucleons.

We seek the state $|X\rangle$ in the form:

$$|X\rangle = \{\psi(E)A^+ + \int dk^3\psi_1(E,\omega_k)B^+a^+(k) +$$
$$+\frac{1}{\sqrt{2!}}\int dk^3dl^3\psi_2(E,\omega_k,\omega_l)C^+a^+(k)a^+(l)\}|0\rangle, \qquad (3)$$

where $|0\rangle$ is the vacuum state. Equation (2) is equivalent to the following relations between the wave functions:

$$(E - m_{A0})\psi(E) = \lambda_{01} \int dk^3 \psi_1(E, \omega_k) u(\omega_k), \tag{4}$$

$$\begin{gathered}(E - m_{B0} - \omega_k)\psi_1(E, \omega_k) = \\ = \lambda_{01} u(\omega_k)\psi(E) + \sqrt{2}\lambda_{02} \int dl^3 u(\omega_l)\psi_2(E, \omega_k, \omega_l),\end{gathered} \tag{5}$$

$$\begin{gathered}(E - m_C - \omega_k - \omega_l)\psi_2(E, \omega_k, \omega_l) = \\ = \frac{1}{\sqrt{2}}\lambda_{02}[u(\omega_k)\psi_1(E, \omega_l) + u(\omega_l)\psi_1(E, \omega_k)].\end{gathered} \tag{6}$$

Among the states $|X\rangle$ we choose the states corresponding to an incident plane wave $B^+ a^+(k_0)|0\rangle$ and an outgoing scattered wave. These states $|B\theta k_0\rangle_+$ describe the scattering of mesons by B-particles. For them

$$\psi_1(E, \omega_k) = \psi_1(\omega_0, \omega_k) = \delta^3(k_0 - k) + \varphi_{1+}(\omega_k, \omega_0), \tag{7}$$

where $\omega_0 = E - m_B$ is the energy of the scattered meson; m_B is the renormalized mass of the B-particle. Equations (5) and (6) reduce to an integral

equation

$$\begin{gathered}h(\omega_0 + b - \omega)\varphi_{1+}(\omega, \alpha_0) = \lambda_{01} Z_{Bu}(\omega)\psi(\omega_0) - \\ -\gamma u(\omega)u(\omega_0)\frac{1}{\omega - b} - \gamma \int \frac{\varphi_{1+}(\omega_l)u(\omega_l)\, dl^3}{\omega + \omega_l - \omega_0 - b - i\varepsilon},\end{gathered} \tag{8}$$

in which

$$h(z) = (z - b)\left[1 + 4\pi(z - b)\gamma \int_\mu^\infty \frac{\sqrt{\omega^2 - \mu^2}\, \omega u^2(\omega)\, d\omega}{(\omega - b)^2(\omega - z - i\varepsilon)}\right], \tag{9}$$

$\gamma = \lambda_{02} Z_B = \lambda_2^2$ is the square of the renormalized coupling constant of the $BC\theta$-interaction; Z_B is the renormalization constant of the B-particle, coinciding with the renormalization constant of the V-particle in the Lee model ([4]). We shall restrict ourselves to the case $0 < Z_B \leq 1$ and regard the B-particle as stable: $b = m_B - m_C < \mu$. By the substitution

$$\varphi_{1+}(\omega, \omega_0) = \gamma \frac{u(\omega)u(\omega_0)\varphi(\omega, \omega_0)}{h(\omega_0 + b - \omega)} \tag{10}$$

equation (9) is transformed into the form

$$\varphi(\omega,\omega_0) = -\frac{1}{\omega - b} + K - \frac{1}{\pi}\int_\mu^\infty \frac{\operatorname{Im} h(\omega')\varphi(\omega',\omega_0)\,d\omega'}{h(b+\omega_0-\omega)(\omega'+\omega-\omega_0-b-i\varepsilon)}, \quad (11)$$

$$K = \frac{\lambda_{01} Z_B}{\gamma}\frac{\psi(\omega_0)}{u(\omega_0)}.$$

An integral equation with such a kernel for the inhomogeneous term $-1/(\omega - b)$ in the case $b = 0$ was solved in works ([5],[6]), and for a constant inhomogeneous term in work ([3]). Generalizing the solution ([5]) to the case $b \neq 0$ by the method used in ([3]), we find

$$\varphi(\omega,\omega_0) = \frac{h(\omega_0+b-\omega)}{\omega_0-\omega+i\varepsilon}\left[f(\omega,\omega_0) + KZ_B^{-1} j(\omega,\omega_0)\right], \quad (12)$$

where

$$f(\omega,\omega_0) = -\left[\frac{\omega-b}{h(\omega)}\frac{1}{\omega-b} + \frac{2A(\omega)}{1-h(\omega)A(\omega_0)}\right], \quad (13)$$

$$j(\omega,\omega_0) = \frac{1+h(\omega_0)[A(\omega)-A(\omega_0)]}{1-h(\omega)A(\omega_0)}, \quad (14)$$

$$A(\omega) = \frac{1}{\pi}\int_\mu^\infty \frac{(\omega'-b)\,d\omega'}{(\omega'+\omega-\omega_0-b-i\varepsilon)h(\omega_0+b-\omega')}\operatorname{Im}\frac{1}{h(\omega')}. \quad (15)$$

Returning to $\varphi_{1+}(\omega,\omega_0)$, we obtain

$$\varphi_{1+}(\omega,\omega_0) = \frac{u(\omega)}{\omega_0-\omega+i\varepsilon}\left[\gamma u(\omega_0) f(\omega,\omega_0) + \lambda_{01}\psi(\omega_0) j(\omega,\omega_0)\right]. \quad (16)$$

Substituting $\varphi_{1+}(\omega,\omega_0)$ into equation (4), we determine $\psi(\omega_0)$

$$\psi(\omega_0) = \lambda_{01}\frac{u(\omega_0)[1+L(\omega_0)]}{g(\omega_0)}, \quad (17)$$

where

$$L(\omega_0) = \frac{1}{\pi}\int_\mu^\infty \frac{d\omega\,\operatorname{Im} h(\omega) f(\omega,\omega_0)}{\omega_0-\omega+i\varepsilon}, \quad (18)$$

$$g(\omega_0) = m_B + \omega_0 + i\varepsilon - m_{A_0} - \frac{\lambda_{01}^2}{\gamma} M(\omega_0), \tag{19}$$

$$M(\omega_0) = \frac{1}{\pi} \int_\mu^\infty \frac{d\omega \ \mathrm{Im}\, h(\omega) j(\omega, \omega_0)}{\omega_0 - \omega + i\varepsilon}. \tag{20}$$

The zeros of the function $g(\omega_0)$ determine the values of the renormalized mass of the A-particle [3].

Thus, the state $|B\theta k_0\rangle_+$ is completely determined ($\psi_2(\omega_0, \omega_k, \omega_l)$ is immediately found from equation (6)). Taking into account the normalization, the $|B\theta k_0 \mathrm{in}\rangle$-state has the form

$$|B\theta k_0 \mathrm{in}\rangle = Z_B^{1/2} |B\theta k_0\rangle_+. \tag{21}$$

The last independent solution of the Schrödinger equation (2), corresponding to the scattering of two mesons by a C-particle $|C2\theta k_0 l_0\rangle_+$, contains the plane wave

$$\frac{1}{\sqrt{2}} C^+ a^+(k_0) a^+(l_0) |0\rangle$$

and the outgoing scattered wave $\varphi_{2+}(\omega_{k_0}, \omega_{l_0}, \omega_k, \omega_l)$ (the notation $E = m_C + \omega_{k_0} + \omega_{l_0}$ is adopted). The main technical difficulty in determining this state consists in solving the integral equation

$$\eta(\omega) = -\frac{1}{\omega - \omega_{k_0}} + \frac{1}{\pi} \int_\mu^\infty \frac{\mathrm{Im}\, h(\omega')\eta(\omega')\, d\omega'}{(\omega' + \omega - \omega_{k_0} - \omega_{l_0} - i\varepsilon)\, h(\omega_{k_0} + \omega_{l_0} - \omega')}, \tag{22}$$

in which the pole of the inhomogeneous term does not coincide with the zero of the function $h(z)$. Such an equation for $b = 0$ was solved in Refs. [2,7]. Using the method of Ref. [3], it is easy to generalize the solution to the case $b \neq 0$. Because of the cumbersome formulas, the explicit expression for the state $|C2\theta k_0 l_0\rangle_+$ is not given. The state $|C2\theta k_0 l_0\rangle_+$ is normalized to a plane wave and will be denoted by $|C2\theta k_0 l_0 \mathrm{in}\rangle$.

The scattering states make it possible to determine the amplitudes for scattering of a θ-particle by a B-particle, $T(\omega)$, and for production of two θ-particles in $B - \theta$ collisions, $T(\omega_p, \omega_q)$, from the formulas

$$(\mathrm{out}\ B\theta k' | B\theta k\ \mathrm{in}) = \delta^3(k - k') - 2\pi i \delta(\omega - \omega') T(\omega), \tag{23}$$

$$(\mathrm{out}\ C2\theta pq | B\theta k\ \mathrm{in}) = -2\pi i \delta(\omega_p + \omega_q - \omega_k - b) T(\omega_p, \omega_q). \tag{24}$$

Calculating the scalar products, we obtain

$$T(\omega) = u^2(\omega)\left\{\gamma f(\omega,\omega) + \lambda_{01}^2 j(\omega,\omega) g^{-1}(\omega)[1+L(\omega)]\right\}, \tag{25}$$

$$\begin{aligned} T(\omega_p,\omega_q) = \frac{1}{\sqrt{2}}\lambda_2 u(\omega_p)u(\omega_q)u(\omega_1)\Bigg\{ \gamma\left[\frac{f(\omega_q,\omega_1)}{\omega_p-b} + \frac{f(\omega_p,\omega_1)}{\omega_q-b}\right] \\ + \lambda_{01}^2\frac{[1+L(\omega_1)]}{g(\omega_1)}\left[\frac{j(\omega_q,\omega_1)}{\omega_p-b} + \frac{j(\omega_p,\omega_1)}{\omega_q-b}\right]\Bigg\}, \end{aligned} \tag{26}$$

$$\omega_1 = \omega_p + \omega_q - b.$$

With the aid of equalities (13), (14), the $B-\theta$ scattering amplitude is transformed to the form

$$T(\omega) = u^2(\omega)\frac{1}{1-h(\omega)A_0(\omega)}\left\{\lambda_{01}^2\frac{[1+L(\omega)]}{g(\omega)} - \gamma\,\frac{1+h(\omega)A_0(\omega)}{h(\omega)}\right\},$$

$$A_0(\omega) = \frac{1}{\pi}\int_\mu^\infty \frac{d\omega'}{h(\omega+b-\omega')}\,\mathrm{Im}\,\frac{1}{h(\omega')}. \tag{27}$$

The amplitude $T(\omega)$, as expected, for $\lambda_{02}=0$ becomes the $N\theta$-scattering amplitude of Lee' s model [4], and for $\lambda_{01}=0$ the $V\theta$-scattering amplitude of the same model [8].

It should be noted that at incident-meson energies $\omega \le 2\mu - b$, scattering of θ-particles by a B-particle is purely elastic, whereas at energies $\omega > 2\mu - b$ the competing process of production of two mesons with probability amplitude (26) is switched on, and the scattering becomes inelastic.

The cross section for elastic scattering of θ-particles is proportional to $|T(\omega)|^2$. It may have two maxima associated with resonance denomina-

by $1-h(\omega)A_0(\omega)$ and $g(\omega)$. The first denominator leads to a maximum of the cross section when the interaction constant γ is such that, with the $AB\theta$-interaction switched off, no bound $B\theta$-states are formed ($V\theta$-bound states in the Lee model), i.e., for $Z_B > 1/2$ [9]. The second denominator gives a resonance when the constant γ is so small that no second one-particle state of the A-particle is formed [3]. As shown in [3], the resonance energy corresponding to the minimum of $|g(\omega)|$, for fixed coupling constants λ_{01}, λ_2, is greater than the resonance energy of the first denominator. Therefore, as γ decreases from

$$\gamma_{\text{crit}} = \left[\int \frac{dk^3 u^2(\omega)}{(b-\omega)^2}\right]^{-1}$$

to 0, first the second resonance appears, then the first; the resonance energies increase, while the distance between the maxima of the cross section decreases—they merge into a single asymmetric maximum. The amplitude for the production of two mesons, $T(\omega_p, \omega_q)$, has the same features.

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Note: Figure translations are in progress. See original paper for figures.