DYNAMIC EFFECT OF ATMOSPHERIC-PRESSURE DISTURBANCES ON OCEAN CURRENTS

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Abstract

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S. S. LAPPO

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The theory of steady wind-driven currents has explained the main features of the mean multi-year motion of water. Consideration, from observations, of the temporal variability of currents shows that, for most regions of the ocean, the steady current velocities are only a background against which numerous processes occur, accompanied by significant variations in current velocities. For example, oscillations of current velocities have been noted whose amplitude reaches 50 cm/sec and changes almost not at all with depth. The periods of these oscillations vary from 10 hours to 10 days, without coinciding with the inertial period ([1,2]).

The synchronism of changes in current velocities at different depths permits one to suppose that such oscillations are not random and are caused by a single force. Analysis of observational materials leads to the conclusion that the cause of such motions is variations in atmospheric pressure. The most intense barotropic oscillations of current velocities have been recorded after the passage of rapidly moving cyclones.

The pressure field over the oceans is characterized by substantial variations ([3]). In some tropical cyclones the pressure drop exceeds 100 mbar. The mean speed of motion of cyclones is 30–40 km/hr, and hurricanes sometimes move at speeds greater than 100 km/hr. The energy of individual pressure formations in the atmosphere is enormous. Thus, a tropical cyclone 700 km in diameter releases during its lifetime an amount of energy equal to $26 \cdot 10^{20}$ J, which is equivalent to the energy that the Bratsk Hydroelectric Power Station will produce in 26,000 years.

There is no theory of currents that takes into account the motion of a pressure formation.

As a first approximation, an idea of the effect of a moving cyclone on currents in the ocean can be obtained by considering the following simple schemes.

From the solution of the problem of the motion of a wave of pressure relief over an unbounded ocean of constant depth and homogeneous density, it follows that, as the speed of motion of the pressure formation increases, the amplitude of the oscillations of current velocities caused by the atmospheric-pressure gradient increases, whereas the component of the current velocity due to wind stress decreases as the speed of displacement of the wind-stress wave increases. At ordinary speeds of motion of pressure formations, the wind component of the velocity of the gradient current is 100 times smaller than the component caused by the atmospheric-pressure gradient. On the basis of this estimate, in what follows we shall neglect the effect of wind on deep currents in a moving pressure formation.

In a stationary pressure formation, the effect of the atmospheric-pressure gradient is compensated by the slope of the sea surface. In a moving pressure formation, according to the work of Miyazaki ([5]), the sea level has a form different from its equilibrium form. This is also shown by individual observations of the form of the level in a cyclone ([4]).

It does not seem possible to assume any particular form for the sea-level surface in a moving baric formation, since this question has not been sufficiently studied. In fact, in order to obtain the form of the level in a moving cyclone, it is necessary to take into account the entire prehistory of its formation.

Let us consider the problem of currents associated with a moving atmospheric-pressure gradient, assuming that the sea level does not change.

The assumption that the sea level remains unchanged as a pressure wave passes implies that the water flow produced by the pressure gradient adjusts to the Earth' s rotation by changing the direction of motion. This does not contradict observations, according to which the velocity vector of the current rotates after the passage of a cyclone. In order for the motion to be quasi-steady, i.e., for the flow to have time to adjust to motion in the field of the Coriolis force, the pressure oscillations must be long-period (of the order of a day).

Let us find the amplitude of the oscillation of current velocities when a pressure wave moves over an unbounded ocean. The water is assumed homogeneous in density and incompressible. Bottom friction is neglected. We seek the solution for the region of the ocean that moves together with the pressure. For this purpose we pass to a new coordinate system $x = x_1 - Vt,\ y = y_1$, where V is the velocity of displacement of the pressure over the ocean. The initial system of hydrodynamic equations may be linearized, bearing in mind that the velocity of displacement of the pressure is 100 times greater than the velocities of the water currents. For the established motion of the pressure wave over the ocean, neglecting small quantities, we obtain the system of equations

$$V\frac{\partial u}{\partial x} - \Omega v = -\frac{1}{\rho}\frac{\partial P_a}{\partial x},$$

$$V\frac{\partial v}{\partial x} + \Omega u = -\frac{1}{\rho}\frac{\partial P_a}{\partial y}, \tag{1}$$

where the x-axis is directed eastward, and the y-axis northward; u, v are the components of the current velocity; $\Omega = 2\omega \sin\varphi$; P_a is atmospheric pressure; ω is the angular velocity of the Earth' s rotation; φ is latitude.

System (1) is reduced to the equation

$$V^2\frac{\partial^2 u}{\partial x^2} + \Omega^2 u = -\frac{1}{\rho}V\frac{\partial^2 P_a}{\partial x^2} - \frac{1}{\rho}\Omega\frac{\partial P_a}{\partial y}. \tag{2}$$

Let

$$P_a = P_0 \cos kx = P_0 R\{e^{ikx}\}, \tag{3}$$

where $k = 2\pi/l$, l is the wavelength. We seek the solution for the current velocity in the form

$$u = u_0 e^{ikx}. \tag{4}$$

Substituting (4) into (2), we find the amplitude of the oscillations of the component of the current velocity:

$$u_0 = \frac{4\pi^2}{\rho l^2}\frac{P_0 V}{\Omega^2 - V^2 4\pi^2/l^2} \tag{5}$$

and from (1)

$$v_0 = i\frac{2\pi}{l\rho}\frac{P_0\Omega}{V^2 4\pi^2/l^2 - \Omega}. \tag{6}$$

We shall now construct a model of oscillations closer to the observed variability of currents after the passage of a cyclone. Let us consider the problem of how current velocities will change in the presence of horizontal friction, if at the initial moment there were oscillations of the form (4) with amplitu-

(5), while the pressure remained constant.

The equation for the pulsation of velocity along the x-axis takes the form

$$\frac{\partial u}{\partial t} + V\frac{\partial u}{\partial x} - \Omega v - A_l\frac{\partial^2 u}{\partial x^2} = 0, \tag{7}$$

along the y-axis

$$\frac{\partial v}{\partial t} + \Omega u = 0, \tag{8}$$

where A_l is the coefficient of horizontal turbulent friction.

From (7) and (8) follows the equation

$$\frac{\partial^2 u}{\partial t^2} - A_l \frac{\partial^3 u}{\partial t\, \partial x^2} + V \frac{\partial^2 u}{\partial t\, \partial x} - \Omega^2 u = 0. \tag{9}$$

The initial condition for (9) is

$$u\Big|_{t=0} = u_0 e^{ikx}. \tag{10}$$

From the presence of friction it follows that

$$u\Big|_{t\to\infty} \to 0. \tag{11}$$

Equation (9) can be solved by the method of separation of variables. In accordance with the initial conditions, we shall seek the solution in the form

$$u = u_0 \exp\left[-\alpha t + i\frac{2\pi}{l}x\right]. \tag{12}$$

Substituting (12) into (9), we obtain the equation for α

$$\alpha^2 - A_l \frac{4\pi^2}{l^2}\alpha - iV\frac{2\pi}{l}\alpha + \Omega^2 = 0, \tag{13}$$

whence

$$\alpha = A_l \frac{2\pi^2}{l^2} + iV\frac{\pi}{l} \pm \sqrt{\left(A_l \frac{2\pi^2}{l^2} + iV\frac{\pi}{l}\right)^2 - \Omega^2}. \tag{14}$$

The real part of the expression under the radical is negative for the usual dimensions of baric waves, their propagation velocities, and $A_l = 10^8$ cm^2/sec.

Let us represent this root in the form

$$\sqrt{\left(A_l \frac{2\pi^2}{l^2} + iV\frac{\pi}{l}\right)^2 - \Omega^2} = iB \exp\left(i\frac{\varphi}{2}\right), \tag{15}$$

where

$$B = \frac{1}{l^2}\sqrt{(\Omega^2 l^4 + V^2\pi^2 l^2 - 4\pi^4 A_l^2)^2 + 16A_l^2 V^2 \pi^6}, \tag{16}$$

$$\operatorname{tg}\varphi = 4A_l\pi^3 lV/(l^4\Omega^2 + V^2 l^2\pi^2 - 4A_l^2\pi^4).$$

$\operatorname{tg}\varphi$ is a small quantity; therefore we shall approximately replace it by:

$$\cos\varphi/2 = 1, \qquad \sin\varphi/2 = \frac{2A_l\pi^3 Vl}{l^4\Omega^2 + V^2 l^2\pi^2 - 4A_l^2\pi^4}. \tag{17}$$

The solution for the pulsation of velocity will now be written in the form

$$u = u_0 \exp\left\{-\left[\frac{2A_l\pi^2}{l^2} \pm B\frac{2A_l\pi^3 lV}{l^4\Omega^2 + V^2 l^2\pi^2 - 4A_l^2\pi^4}\right] - i\left(\frac{\pi}{l}V \pm B\right)\right\} t \exp i\frac{2\pi}{l}x. \tag{18}$$

We take the initial current velocity relative to the fixed coordinate system to be equal to zero.

Neglecting small quantities, we write the real part of solution (18) in the form

$$u = u_0 \exp\left[A_l\frac{2\pi^2}{l^2}t\right]\left\{\cos\left(V\frac{\pi}{l} \mp B\right)t\cos\frac{2\pi}{l}x + \right.$$

$$\left. + \sin\left(V\frac{\pi}{l} \mp B\right)t\sin\frac{2\pi}{l}x\right\}. \tag{19}$$

For cyclones of size about $5 \cdot 10^7$ cm, approximately

$$B = V\pi/l + \Omega, \tag{20}$$

respectively

$$u = u_0 \exp\left[-A_l\frac{2\pi^2}{l^2}t\right]\left\{\cos\left[\frac{2\pi}{l}\left(V + \Omega\frac{l}{2\pi}\right)t - \varphi(x)\right] + \cos[\Omega t - \varphi(x)]\right\}. \tag{21}$$

It follows from this that after the passage of cyclones of the indicated size, oscillations of the velocities of currents arise, which are a superposition of oscillations of the known inertial period (6) and forced oscillations with the period

$$T_{\text{forced}} = l/(V + \Omega l/2\pi). \tag{22}$$

Pulsations of velocity along the y axis can be obtained from (8). The amplitude of oscillations along the y axis is expressed through the amplitude of oscillations of the velocity along the x axis by

$$v_0 = \frac{T_{\text{forced}}}{T_{\text{inert}}} u_0, \quad \text{where } T_{\text{inert}} = \frac{2\pi}{\Omega}. \tag{23}$$

For a quantitative estimate of the parameters of the velocity oscillations according to formula (21), we use the conditions under which, on the 17th cruise of the R/V "Mikhail Lomonosov," long-period oscillations of current velocities were observed in the layer from 15 to 1600 m. The cyclone was moving northward at a speed of 40 km/h; the pressure at the point where currents were recorded decreased by 25 mbar; the period of oscillations of the current velocities was 10 h; the amplitude of the current velocity (projection onto the meridian) was 25 cm/s. The amplitude of the velocity oscillations calculated by formula (5) is 19 cm/s; the size of the region of pressure drop according to (22) is $5 \cdot 10^7$ cm, which corresponds to the size of the cyclone taken from the baric map. Consequently, it may be considered that the observed oscillations of current velocities are barogradient currents. Equating the damping decrement determined from the observations to its expression from the solution, we determine the value of the coefficient of horizontal turbulent exchange. Such a method of comparing theory and observations makes it possible to take into account a whole series of observations. For our example $A_l = 5 \cdot 10^7$ cm^2/s. It follows from the observations that the product of the damping decrement by the oscillation period is a constant quantity for the given place. Hence follows the formula for A_l:

$$A_l = l(V + \Omega l/2\pi)S/2\pi^2, \tag{24}$$

where $S = 0.1$ according to observations.

In the case of inertial oscillations, the formula for A_l has the form

$$A_l = l^2 \Omega S/4\pi^3. \tag{25}$$

Formula (25) relates the coefficient of horizontal exchange to planetary vorticity. To a certain extent, this formula makes it possible to understand the physical cause of the coexistence in the equatorial zone of the oceans of oppositely directed flows with high velocities.

Moscow State University
named after M. V. Lomonosov

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Note: Figure translations are in progress. See original paper for figures.