PEOPLE OF SOVIET SCIENCE

1966

SovietRxiv

Abstract

Full Text

PEOPLE OF SOVIET SCIENCE

ON THE 50TH ANNIVERSARY OF A. V. BITSADZE

On May 22, it will be 50 years since the birth, and 26 years of scientific and pedagogical activity, of Corresponding Member of the Academy of Sciences of the USSR Andrei Vasil' evich Bitsadze, one of the leading mathematicians in the field of the theory of partial differential equations.

A. V. Bitsadze was born in 1916 in the village of Tskhrukveti, Chiatura district, Georgian SSR. In 1931 he graduated from the Chiatura Pedagogical Technicum, and from the end of 1932 until entering a higher educational institution (in 1935) he taught mathematics and physics at a school. In 1940 he graduated with distinction from the Faculty of Physics and Mathematics of Tbilisi State University, specializing in mathematics. From 1940 to 1948 he worked at the Tbilisi Mathematical Institute of the Academy of Sciences of the Georgian SSR. In 1948 he was sent to the doctoral program of the V. A. Steklov Mathematical Institute of the Academy of Sciences of the USSR, where he remained, after defending his doctoral dissertation in 1951, as a senior research associate. From 1959 to the present, A. V. Bitsadze has worked at the Institute of Mathematics of the Siberian Branch of the Academy of Sciences of the USSR as head of a department. In 1958 Andrei Vasil' evich was elected a Corresponding Member of the Academy of Sciences of the USSR.

The first scientific investigations of Andrei Vasil' evich pertain to the mathematical theory of elasticity. In work [2] he found, in quadratures, the solution of the generalized Hertz problem on local deformations under compression of two plane elastic bodies.

A significant part of Andrei Vasil'evich's works from the bibliography given below is devoted to the theory of boundary-value problems for elliptic equations. The results he obtained in this direction are set forth in his monograph *Boundary-Value Problems for Second-Order Elliptic Equations* [42].

In particular, for the elliptic system

$$\Delta u + a\frac{\partial u}{\partial x} + b\frac{\partial u}{\partial y} + cu = 0, \tag{1}$$

whose coefficients a, b, c are real analytic $m \times m$ matrices, Andrei Vasil' evich proved the Fredholm property of the Dirichlet boundary-value problem (the first boundary-value problem) in a finite domain D of the plane of the variables x, y, bounded by a Lyapunov curve S, and showed that the positive definiteness of the matrix

$$\frac{\partial a}{\partial x} + \frac{\partial b}{\partial y} - 2C$$

in the domain D guarantees the uniqueness of the solution of this problem. To him also belongs the proof of the Noether property of the Poincaré problem

$$p^1 \frac{\partial u}{\partial x} + p^2 \frac{\partial u}{\partial y} + qu = f(x, y), \qquad (x, y) \in S, \tag{2}$$

$$\det(p^1 + ip^2)_S \neq 0 \tag{3}$$

for system (1).

Under condition (3) he showed that the difference κ between the dimension l of the space of solutions of the corresponding homogeneous problem (1)–(2) and the number l' of solvability conditions for problem (1)–(2) (the index of the Poincaré problem) is equal to $2(p + m)$, where p denotes the increment of

$$\frac{1}{\pi} \arg\det(p^1 - ip^2)$$

under a single traversal of the contour S in the positive direction, and m is the number of equations in system (1).

Whereas for a single elliptic equation of second order in a finite domain the Dirichlet problem is always Fredholm, the requirement of ellipticity for a second-order system, as Andrei Vasil'evich showed, may guarantee neither the Fredholm property nor the Noether property of this problem, nor even its normal solvability in the sense of Hausdorff. He also indicated a class of so-called weakly coupled elliptic systems for which the Dirichlet and Poincaré problems are always Noether.

In the theory of non-Fredholm planar elliptic boundary-value problems (the Dirichlet, Poincaré problems, etc.), a very important role is played by the well-developed theory of one-dimensional singular integral equations with kernels of Cauchy type.

In recent years, in the works of a whole series of mathematicians, much attention has been devoted to developing the theory of multidimensional singular integral equations.

In works [19, 20, 21] Andrei Vasil' evich studied certain classes of multidimensional singular integral equations with special matrix kernels of the form

$$M(P, Q) = -D^* \left(\frac{\partial}{\partial \xi}, \frac{\partial}{\partial \eta}, \frac{\partial}{\partial \zeta} \right) \frac{1}{\rho} \cdot D(\alpha, \beta, \gamma),$$

where

$$D^*(X,Y,Z) = \begin{Vmatrix} 0 & X & Y & Z \\ X & 0 & Z & -Y \\ Y & -Z & 0 & X \\ Z & Y & -X & 0 \end{Vmatrix}, \qquad D(\alpha,\beta,\gamma) = \begin{Vmatrix} 0 & \alpha & \beta & \gamma \\ \alpha & 0 & -\gamma & \beta \\ \beta & \gamma & 0 & -\alpha \\ \gamma & -\beta & \alpha & 0 \end{Vmatrix},$$

$\rho(P,Q)$ is the distance between the points $P(x,y,z)$ and $Q(\xi,\eta,\zeta)$ on the Lyapunov surface S, and α,β,γ are the cosines of the exterior normal to S. An analogue of the well-known Poincaré–Bertrand formula here proved to be the following formula for the interchange of singular integrals:

$$\int_S M(P,Q)\,ds_Q \int_S M(Q,Q_1)\varphi(Q_1,Q)\,ds_{Q_1} = 4\pi^2\varphi(P,P) +$$

$$+\int_S ds_{Q_1} \int_S M(P,Q)M(Q,Q_1)\varphi(Q_1,Q)\,ds_Q,$$

where the vector $\varphi = (\varphi_1,\varphi_2,\varphi_3,\varphi_4)$ prescribed on S satisfies a Hölder condition.

The works of Andrei Vasil' evich [36–40] are devoted to the problem of the oblique derivative for harmonic functions in multidimensional domains and to one class of multidimensional singular integral equations closely connected with it.

In the monographs [16 and 30] Andrei Vasil' evich set forth the results he obtained on various problems for model equations of mixed type of M. A. Lavrent' ev, F. Tricomi, and S. A. Chaplygin. He had to overcome serious difficulties in proving the existence and uniqueness of solutions of the generalized Tricomi problem and of the Frankl problem under fairly broad assumptions concerning the data of these problems.

Finally, it should be noted that in his creative activity Andrei Vasil'evich devotes considerable attention to the training of highly qualified scientific personnel. Since 1941 he has carried out teaching work at universities in Tbilisi, Moscow, and Novosibirsk.

A. V. Bitsadze is at present in the prime of his creative powers. We wish him great scientific success.

S. L. SOBOLEV, A. N. TIKHONOV, N. P. ERUGIN

BIBLIOGRAPHY OF THE SCIENTIFIC WORKS OF A. V. BITSADZE

1. Tangential derivative of the simple-layer potential (1941), in the book by N. I. Muskhelishvili, *Singular Integral Equations*. Moscow—Leningrad, 1946, pp. 34—37.

2. On local deformations under compression of elastic bodies. Communications of the Academy of Sciences of the Georgian SSR, vol. III, No. 5, 1942.

3. On the general representation of solutions of elliptic differential equations. Communications of the Academy of Sciences of the Georgian SSR, vol. IV, No. 7, 1943.

4. Boundary value problem for a system of linear differential equations of elliptic type. Communications of the Academy of Sciences of the Georgian SSR, vol. V, No. 8, 1944.

5. General representation of solutions of elliptic systems of differential equations and some of its applications. Candidate dissertation, Academy of Sciences of the Georgian SSR, 1944, 62 pp.

6. On some applications of the general representation of solutions of elliptic equations. Communications of the Academy of Sciences of the Georgian SSR, vol. VII, No. 6, 1946.

7. The problem of the oscillation of a uniformly compressed thin elastic plate. Proceedings of Tbilisi State University, 30-a, 1947.

8. General representation of solutions of linear elliptic systems of differential equations. In the book by I. N. Vekua, *New Methods for Solving Elliptic Equations*. Moscow, §§ 28—30, 1948.

9. On the uniqueness of the solution of the Dirichlet problem for elliptic equations with partial derivatives. UMN, 3 : 6 (28), 241—242, 1948.

10. On the so-called area-monogenic functions. DAN SSSR, 59, 1385—1388, 1948.

11. On one system of functions. UMN, 5 : 4 (38), 154—155, 1950.

12. On the uniqueness of the solution of the general problem for an equation of mixed type. Communications of the Academy of Sciences of the Georgian SSR, 11, 1950, pp. 205—210.

13. On the problem of equations of mixed type. DAN SSSR, **70**, 373-376, 1950 (jointly with M. A. Lavrent' ev).

14. On certain problems of mixed type. DAN SSSR, **70**, 561-564, 1950.

15. On the general problem of mixed type. DAN SSSR, **78**, 621-624, 1951.

16. On the problem of equations of mixed type. Trudy Mat. in-ta im. V. A. Steklova AN SSSR, **61**, 1953, 60 pp.

17. On one equation of mixed type. UMN, 8 : 1 (59), 174-175, 1953.

18. A spatial analogue of the Cauchy-type integral and some of its applications. Izv. AN SSSR, ser. matem., **17**, 525-538, 1953.

19. A spatial analogue of the Cauchy-type integral and some of its applications. DAN SSSR, **93**, 389-392, 1953 (see also the corrections, DAN SSSR, **94**, No. 6, 1954).

20. Generalization of one system of singular integral equations. DAN SSSR, **93**, 595-597, 1953.

21. On two-dimensional Cauchy-type integrals. Soobshcheniya AN GruzSSR, **16**, 177-184, 1955.

22. On one Frankl problem. DAN SSSR, **109**, 1091-1094, 1956.

23. On the problem of equations of mixed type in multidimensional domains. DAN SSSR, **110**, 901-902, 1956.

24. Equations of mixed type. Proceedings of the 3rd All-Union Mathematical Congress, vol. 2. Moscow, 1956.

25. On the uniqueness of the solution of the Frankl problem for Chaplygin' s equation. DAN SSSR, **112**, 375-376, 1957.

26. On elliptic systems of partial differential equations of second order. DAN SSSR, **112**, 983-986, 1957.

27. On one elementary method for solving certain boundary-value problems for holomorphic functions and the special integral equations connected with them. UMN, 12 : 5 (77), 185-190, 1957.

28. Some linear problems for linear partial differential equations (in Chinese). Beijing, 1958.

29. Incorrectness of the Dirichlet problem for equations of mixed type. DAN SSSR, **122**, 167-170, 1958.

30. Equations of mixed type. Itogi nauki (phys.-math. sciences), 2, AN SSSR, 1959.

31. On the theory of equations of mixed-composite type. Siberian Mathematical Journal, vol. II, No. 1, 1961, 7-19 (jointly with M. S. Salakhitdinov).

32. On equations of mixed-composite type. Some Problems of Mathematics and Mechanics, AN SSSR, Siberian Branch, 1961, pp. 47-49.

33. On the theory of harmonic functions. Trudy Tbilisi State University, **84**, 1961, pp. 35-38.

34. On equations of mixed type in three-dimensional domains. DAN SSSR, **143**, 1017–1019, 1962.

35. On one three-dimensional analogue of the Tricomi problem. Siberian Mathematical Journal, **3**, 642–644, 1962.

36. On a homogeneous problem of the oblique derivative for harmonic functions in three-dimensional domains. DAN SSSR, **148**, 749–752, 1963.

37. On the oblique derivative problem for harmonic functions in three-dimensional domains. Materials of the Joint Soviet-American Symposium on Partial Differential Equations. Siberian Branch, AN SSSR, Novosibirsk, 1963.

38. On one frequently occurring case of the oblique derivative problem for harmonic functions in three-dimensional domains. DAN SSSR, **155**, 730–731, 1964.

39. The oblique derivative problem with polynomial coefficients. DAN SSSR, **157**, 1273–1276, 1964.

40. On one class of multidimensional singular integral equations. DAN SSSR, **159**, 955–957, 1964.

41. Normally solvable elliptic boundary-value problems. DAN SSSR, **164**, 1218–1220, 1965.

42. Boundary-value problems for elliptic equations of second order. Moscow, Nauka Publishing House, 1966.

Note: Figure translations are in progress. See original paper for figures.