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FREQUENCIES OF FREE VIBRATIONS OF BRANCHED CHAINS

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Fig. 1. Branched chains of diatomic molecules

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Abstract

Full Text

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FREQUENCIES OF FREE VIBRATIONS OF BRANCHED CHAINS

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Branched chains of diatomic molecules may be of two types (see Fig. 1).

Fig. 1. Branched chains of diatomic molecules

We choose the coordinate system in such a way that the x-axis is parallel to the chain. The y-axis is perpendicular to the x-axis and lies in the plane of the chain, and the z-axis is perpendicular to the x- and y-axes. The components of the displacements of atoms from their positions of stable equilibrium along the x-axis are U_n, along the y-axis V_n, and along the z-axis W_n.

Small free vibrations can be divided into two groups: 1) vibrations in which the atoms do not leave the plane of the chain; 2) vibrations in which the atoms leave the plane of the chain. The equations of motion for small free vibrations of the U_n components of a branched chain of the first type (Fig. 1a) have the form

$$\begin{aligned}
M_1\ddot{U}_1 &= \beta_1(U_2 - U_1),\\
M_2\ddot{U}_2 &= \alpha_2(U_4 - U_2) + \beta_1(U_1 - U_2) - \alpha_2 U_2,\\
\\
M_1\ddot{U}_3 &= \beta_1(U_4 - U_3),\\
&\cdots\\
M_1\ddot{U}_{2n+1} &= \beta_1(U_{2n+2} - U_{2n+1}),\\
M_2\ddot{U}_{2n+2} &= \alpha_2(U_{2n+4} + U_{2n} - 2U_{2n+2}) + \beta_1(U_{2n+1} - U_{2n+2}),\\
&\cdots\\
M_1\ddot{U}_{N-1} &= \beta_1(U_N - U_{N-1}),\\
M_2\ddot{U}_N &= \alpha_2(U_{N-2} - U_N) + \beta_1(U_{N-1} - U_N) - \alpha_2 U_N.
\end{aligned} \tag{1}$$

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We seek the solution of system (1) in the form

$$U_n = |U_n| e^{i\omega t}.$$

For the vibration frequencies we obtain the equation

$$\begin{vmatrix}
a_1 & -\beta_1 & 0 & 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
-\beta_1 & a_2 & 0 & -\alpha_2 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
0 & 0 & a_1 & -\beta_1 & 0 & 0 & \cdots & \cdots & \cdots & 0 \\
0 & -\alpha_2 & -\beta_1 & a_2 & 0 & -\alpha_2 & \cdots & \cdots & \cdots & 0 \\
0 & 0 & 0 & 0 & a_1 & -\beta_1 & \cdots & \cdots & \cdots & 0 \\
0 & 0 & 0 & -\alpha_2 & -\beta_1 & a_2 & \cdots & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & a_1 & -\beta_1 & 0 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & -\beta_1 & a & 0 & -\alpha_2 & \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & 0 & a_1 & -\beta_1 & \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & -\alpha_2 & -\beta_1 & a_2 &
\end{vmatrix} = 0, \qquad (2)$$

where

$$a_1 = -M_1\omega^2 + \beta_1, \qquad a_2 = -M_2\omega^2 + 2\alpha_2 + \beta_1.$$

The polynomials ${}^2n_N(a_1, a_2, \beta_1, \alpha_2)$ satisfy the recurrence relations

$${}^2n_N - (a_1a_2 - \beta_1^2)^2 n_{N-2} + a_1^2 a_2^{2 2} n_{N-4} = 0, \qquad (3)$$

therefore they can be represented in the form of functions of the chain parameters ([1])

$${}^2n_N = (a_1a_2)^{N/2} \frac{\operatorname{sh}(N+2)\gamma}{\operatorname{sh} 2\gamma} \qquad (4)$$

for $a_1a_2 - \beta_1^2 = 2a_1a_2 \operatorname{ch} 2\gamma$.

From (2) we obtain the following values of the vibration frequencies of the chain:

$$\omega^2 = \frac{(M_1+M_2)\beta_1 + 4M_1\alpha_2 \sin^2 k\pi/(N+2)}{2M_1M_2} \pm \left\{ \left[\frac{(M_1+M_2)\beta_1 + 4M_1\alpha_2 \sin^2 k\pi/(N+2)}{2M_1M_2} \right]^2 - 4\alpha_2\beta_1 \sin^2 \frac{k\pi}{N+ \ldots}\right.$$

$$k = 1, 2, \ldots, \frac{N}{2}. \qquad (5)$$

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The frequencies of the free vibrations making up V_n and W_n are obtained from (5) by replacing α_2, β_1 by β_2, α_1 and by β_2, β_1, respectively.

The frequencies of the free vibrations of a chain of the second type (Fig. 1b) are determined from the equation

$$^4n_N(a_1, a_2, a_3, a_4, \beta_1, \alpha_1, \beta_1, \alpha_1) =$$

$$= \begin{vmatrix} a_1 & -\beta_1 & 0 & 0 & 0 & 0 & \dots & \dots & \dots & 0 \\ -\beta_1 & a_2 & 0 & -\alpha_1 & 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & 0 & a_3 & -\beta_1 & 0 & 0 & \dots & \dots & \dots & 0 \\ 0 & -\alpha_1 & -\beta_1 & a_4 & 0 & -\alpha_1 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & 0 & a_1 & -\beta_1 & \dots & \dots & \dots & 0 \\ 0 & 0 & 0 & -\alpha_1 & -\beta_1 & a_2 & \dots & \dots & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & \dots & \dots & \dots & \dots & a_1 & -\beta_1 & 0 & 0 \\ 0 & \dots & \dots & \dots & \dots & -\beta_1 & a_2 & 0 & -\alpha_1 & \\ 0 & \dots & \dots & \dots & \dots & 0 & 0 & a_3 & -\beta_1 & \\ 0 & \dots & \dots & \dots & \dots & 0 & -\alpha_1 & -\beta_1 & a_4 & \end{vmatrix} = 0. \quad (6)$$

In equation (6) it is assumed that

$$a_1 = -M_1\omega^2 + \beta_1, \qquad a_2 = -M_2\omega^2 + 2\alpha_1 + \beta_1,$$

$$a_3 = -M_2\omega^2 + \beta_1, \qquad a_4 = -M_1\omega^2 + 2\alpha_1 + \beta_1. \quad (7)$$

The polynomials ^4n_N satisfy a recurrence relation of the form

$$^4n_N - \left[(a_1a_2 - \beta_1^2)(a_3a_4 - \beta_1^2) - 2a_1a_3\alpha_1^2\right] {}^4n_{N-4} + a_1^2a_3^2\alpha_1^4\,{}^4n_{N-8} = 0. \quad (8)$$

If

$$(a_1a_2 - \beta_1^2)(a_3a_4 - \beta_1^2) - 2a_1a_3\alpha_1^2 = 2a_1a_3\alpha_1^2 \operatorname{ch} 4\gamma, \quad (9)$$

then

$$^4n_N = (a_1a_3\alpha_1^2)^{N/4} \frac{2\operatorname{sh}(N+2)\gamma \operatorname{ch} 2\gamma}{\operatorname{sh} 4\gamma}. \quad (10)$$

From (6) the following values for γ are obtained:

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$$\gamma = k\pi i/(N+2), \qquad k = 1, 2, \ldots, N/4. \tag{11}$$

Knowing γ, from equation (8) one can determine the values of the frequencies.

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CITED LITERATURE

1. A. F. Pozubenko, *Tr. Gos. optich. inst. im. S. I. Vavilova*, **30**, no. 159, 127 (1963).

Note: Figure translations are in progress. See original paper for figures.

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