ON STATIONARY SOLUTIONS OF MIXED PROBLEMS ARISING IN THE STUDY OF CERTAIN CHEMICAL PROCESSES

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Abstract

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I. The process inside a flat catalyst grain in the case of a single reaction, under certain assumptions, is described by the following system of equations:

$$\frac{\partial\theta}{\partial t} = \Pi\frac{\partial^2\theta}{\partial\xi^2} + kQ_1 e^{\frac{\theta}{1+b\theta}}(1-x), \tag{1}$$

$$\frac{\partial x}{\partial t} = \Pi\frac{\partial^2 x}{\partial\xi^2} + ke^{\frac{\theta}{1+b\theta}}(1-x),$$

considered for prescribed $\theta|_{t=0}$ and $x|_{t=0}$, and also under the boundary conditions

$$\theta'|_{\xi=0} = x'|_{\xi=0} = \theta|_{\xi=1} = 0; \qquad x|_{\xi=1} = \beta \tag{2}$$

or

$$\theta'|_{\xi=0} = x'|_{\xi=0} = 0; \qquad \theta'|_{\xi=1} = \gamma(\theta-\theta_x)|_{\xi=1}; \qquad x|_{\xi=1} = \beta, \tag{3}$$

where θ is the temperature; x is the concentration of the reacting substance; Π, k, Q_1, γ, θ_x, b, $0 < \beta < 1$ are positive constants characterizing the catalyst and the reacting substance. For applications it is important to study the question of the number of stationary (time-independent) solutions. Such solutions, evidently, can be determined from the equation (for problem (1), (2))

$$\frac{d^2\theta}{d\xi^2} = \frac{k}{\Pi}(\theta + \beta Q_1 - Q_1)e^{\frac{\theta}{1+b\theta}} \tag{4}$$

under the boundary conditions

$$\theta'|_{\xi=0} = \theta|_{\xi=1} = 0. \tag{5}$$

The function x, evidently, depends linearly on θ. Introduce the notation

$$\theta|_{\xi=0} = \theta_0, \qquad \theta|_{\xi=1} = \theta_1. \tag{6}$$

It is easy to verify that the solutions of problem (4), (5) are monotone functions of ξ, $0 \le \theta(\xi) \le Q_1(1-\beta)$, and the solutions in this case can be found as solutions of the Cauchy problem, if θ_0 is determined from the functional equation

$$f(\theta_0) = \int_0^{\theta_0} \frac{d\theta_2}{\sqrt{\int_{\theta_0}^{\theta_2} [\theta_1 + \beta Q_1 - Q_1]\, e^{\frac{\theta_1}{1+b\theta_1}}\, d\theta_1}} = \sqrt{\frac{2k}{\Pi}}. \tag{7}$$

It is also easy to prove that

$$f(\theta_0) \xrightarrow[\theta_0\to 0]{} 0, \qquad f(\theta_0) \xrightarrow[\theta_0\to Q(1-\beta)]{} \infty, \qquad \frac{df}{d\theta_0} \xrightarrow[\theta_0\to Q(1-\beta)]{} \infty.$$

Hence it follows that, for any set of admissible parameters, there exists at least one stationary solution. Let us investigate the stationary solutions of problems (1), (2) and (1), (3) in the case $b = 0$. For problem (4), (5) we have

$$\theta'' = \lambda(\theta - Q)e^{\theta}, \tag{8}$$

$$\theta'(0) = \theta(1) = 0, \tag{9}$$

where $\lambda = \dfrac{k}{\Pi}$, $Q = Q_1(1-\beta)$. Let us find the bifurcation points of problem (8), (9).

Let λ_0 be a bifurcation point, and let $\theta = \varphi(\lambda_0, \xi)$ be the corresponding solution of problem (8), (9). The substitution $z = \theta - \varphi$ leads us to the problem

$$z'' = \lambda_0(z + \varphi - Q)e^{z+\varphi} - \varphi'', \tag{10}$$

$$z'(0) = z(1) = 0. \tag{11}$$

For problem (10), (11), every sufficiently small solution z is identically zero by our assumption. By a known theorem, λ_0 must be an eigenvalue of the problem

$$y'' = \lambda_0(1 + \varphi - Q)e^{\varphi}y, \tag{12}$$

$$y'(0) = y(1) = 0. \tag{13}$$

Since φ is monotone, we make the substitution $\varphi(\lambda_0, \xi) = w$. Problem (12), (13) is then transformed into the problem

$$y''_{ww}\, 2\big[(w-Q-1)e^{w} - (w_0-Q-1)e^{w_0}\big] + (w-Q)e^{w} y'_w = (1+w-Q)e^{w} y, \tag{14}$$

$$y\big|_{w=0} = 0, \qquad y'_w\big|_{w=w_0} = \frac{1+w_0-Q}{w_0-Q}\, y\big|_{w=w_0}, \tag{15}$$

where $w_0 = \varphi(\lambda_0, 0)$. In finding the conditions (15), the indeterminacies that arise are easily resolved. A particular solution of equation (12) is, obviously, the function φ_ξ. By lowering the order of this equation, it is easy to find a second particular solution, linearly independent of the first. Thus the general solution of equation (12) has the form

$$y = C_1\sqrt{F(w)-F(w_0)} - \frac{2C_2}{F'(w)} - 2C_2\sqrt{F(w)-F(w_0)}\times$$

$$\times \int_{w_0}^{w} \frac{F''(\eta)}{F'(\eta)^2\sqrt{F(\eta)-F(w_0)}}\, d\eta, \tag{16}$$

where $F(w) = (w-Q-1)e^{w}$. For the existence of a nontrivial solution of problem (14), (15), it is necessary that the equality

$$A = \frac{1}{Q} + \sqrt{-1-Q-(w_0-Q-1)e^{w_0}} \int_0^{w_0} \frac{F''(\eta)\, d\eta}{F'(\eta)^2[F(\eta)-F(w_0)]^{1/2}} = 0. \tag{17}$$

As was to be expected, the expression for A differs from

$$-\left.\frac{df}{d\theta_0}\right|_{\theta_0=w_0}$$

only by a positive factor.

For $0 < \eta < w < Q$ the inequalities hold

$$\frac{F''(\eta)}{F'(\eta)^2\sqrt{F(\eta)-F(w)}} \geq \frac{32}{27}\, \frac{F'(\eta)}{F(\eta)^2\sqrt{F(\eta)-F(w)}}, \tag{18}$$

and for $0 < \eta < w < Q - 2$,

$$\frac{F'(\eta)}{F^2(\eta)\sqrt{F(\eta) - F(w)}} \geq \frac{F''(\eta)}{F'(\eta)^2\sqrt{F(\eta) - F(w)}}. \tag{19}$$

Using these inequalities for A, we obtain

$$A \geq \frac{1}{Q} + \frac{32}{27}\frac{1}{|F(w_0)|} + \frac{16}{27|F(w_0)|}\sqrt{1 - \frac{F(0)}{F(w_0)}} \times$$
$$\times \ln\left|\frac{\sqrt{F(0) - F(w_0)} - \sqrt{|F(w_0)|}}{\sqrt{F(0) - F(w_0)} + \sqrt{|F(w_0)|}}\right| - \frac{32}{27(Q+1)}, \tag{20}$$

$$A \leq \frac{1}{Q} - \frac{1}{Q+1} + \frac{1}{|F(w_0)|} + \frac{1}{2}\sqrt{1 - \frac{F(0)}{F(w_0)}}\frac{1}{|F(w_0)|} \times$$
$$\times \ln\left|\frac{\sqrt{F(0) - F(w_0)} - \sqrt{|F(w_0)|}}{\sqrt{F(0) - F(w_0)} + \sqrt{|F(w_0)|}}\right|. \tag{21}$$

Putting

$$\frac{F(0)}{F(w_0)} = \frac{4e^{-\rho}}{(1 + e^{-\rho})^2}$$

with $\rho = 4$, for $Q \geq 14.5$ and some $w_0 \leq 4$, we have $A < 0$. From (20), for $Q \leq 5$ we obtain $A > 0$. Thus, for $Q < 5$ and all $\lambda = \frac{k}{\Pi}$, the stationary solution is unique. For $Q \geq 14.5$ and some λ, several solutions exist. Calculations show that uniqueness may already be violated for $Q > 5$. From comparison theorems it is easy to obtain that, for

$$Q < 2 + \ln\frac{\pi^2}{4\lambda},$$

the stationary solution is unique.

In the case of problem (1), (3), we obtain the equations for stationary solutions ($b = 0$):

$$\frac{d^2\theta}{d\xi^2} = \frac{k}{\Pi}(\theta - \theta_1 - Q)e^{\theta}, \tag{22}$$

$$\theta'(0) = 0, \qquad \theta'(1) = \gamma(\theta_1 - \theta_\chi), \tag{23}$$

where $\theta_1 = \theta(1)$.

The number of distinct solutions of problem (22), (23) is equal to the number of distinct solutions of the functional equations

$$\pm\sqrt{\frac{2k}{\Pi}}\sqrt{-Q-1-(z_0-Q-1)e^{z_0}} = \gamma(\theta_1-\theta_x)e^{-\frac{\theta_1}{2}}, \tag{24}$$

$$\pm\int_{z_0}^{0}\frac{dy}{\sqrt{(y-Q-1)e^y-(z_0-Q-1)e^{z_0}}} = \sqrt{\frac{2k}{\Pi}}e^{\frac{\theta_1}{2}}, \tag{25}$$

where $z_0 = \theta(0)-\theta(1) = \theta_0-\theta_1$. The fact that, to each pair θ_0, θ_1 for which (24), (25) hold, there corresponds a stationary solution of problem (1), (3) is established analogously to the preceding case. Eliminating θ_1 from (24), (25), we obtain the equation

$$\Phi(z_0) = f(z_0)e^{\frac{1}{2\gamma}\sqrt{F(0)-F(z_0)}\,f(z_0)} = \sqrt{\frac{2k}{\Pi}}e^{-\frac{\theta_x}{2}}. \tag{26}$$

Here we have used the notation introduced in (7) and (16). $\Phi(z_0)$ assumes all positive values; therefore a solution of equation (26) always exists and $z_0 > 0$. Differentiating the left-hand side of (26), we obtain

$$\Phi'(z_0) = -\frac{F'(z_0)}{\sqrt{F(0)-F(z_0)}}\left\{\frac{f(z_0)}{4\gamma}+Af(z_0)\times\right.$$

$$\left.\times\frac{\sqrt{F(0)-F(z_0)}}{2\gamma}\right\}\exp\left\{\frac{\sqrt{F(0)-F(z_0)}}{2\gamma}f(z_0)\right\}, \tag{27}$$

where A is the same as in formula (17). From (27) we conclude: for $Q < 5$ the solution of the problem is unique. For large γ, when $Q > 14.5$, (27) may have several solutions. A brief account of the results established by us may be found in [1]. Formula (7) and the existence of several stationary regimes for concrete parameter values (the graph of $f(z_0)$ is constructed for one set of parameters) were independently established in [2].

II. The question of the number of possible stationary regimes inside a flat catalyst grain, if the thermal conductivity of the gas is taken into account, leads to determining the number of solutions of the problem

$$\frac{1}{p}\theta''-\theta'+kQ_1(1-x)e^{\theta} = 0, \qquad \frac{1}{p}x''-x'+k(1-x)e^{\theta} = 0, \tag{28}$$

$$\frac{1}{p}\theta'|_{\xi=0} = \theta|_{\xi=0}, \qquad \theta'|_{\xi=1} = 0, \qquad \frac{1}{p}x'|_{\xi=0} = x|_{\xi=0}, \qquad x'|_{\xi=0} = 0; \tag{29}$$

here k, p, Q_1 are positive parameters.

This system is equivalent to the following problem:

$$\frac{1}{p}\theta'' - \theta' + k(Q - \theta)e^{\theta} = 0, \tag{30}$$

$$\frac{1}{p}\theta'|_{\xi=0} = \theta|_{\xi=0}, \qquad \theta'|_{\xi=1} = 0. \tag{31}$$

The problem (30), (31) was studied in [3], [4] by numerical methods.

We shall indicate for which p one can guarantee the existence of one (or several) solutions. We prove that the solution of problem (30), (31) is monotone and $0 < \theta(\xi) < Q$. Indeed, let $\theta_0 = \theta(0) > Q$. Then $\theta'|_{\xi=0} > 0$ and $\theta''|_{\xi=0} > 0$, i.e., θ' increases and, consequently, θ increases. In this case θ' does not vanish for any $\xi > 0$, which leads us to a contradiction. Now let $\theta|_{\xi=0} < 0$, $\theta'|_{\xi=0} < 0$, $\theta''|_{\xi=0} < 0$. Then θ' decreases, $\theta' < 0$, and can never become zero, since θ, while decreasing, remains negative for all ξ. Thus, a solution of problem (30), (31) is possible only if $0 < \theta|_{\xi=0} < Q$. In this case the following are possible:

$$\text{a)}\ \ \theta''|_{\xi=0} \le 0; \qquad \text{b)}\ \ \theta''|_{\xi=0} > 0.$$

Consider case a). Since $\theta'|_{\xi=0} > 0$ and θ' decreases, while θ increases, either θ' decreases to zero and then increases, but then $\theta'' = 0$, $\theta' = 0$, $\theta = Q$ at some point $0 < \xi^* < 1$, and, consequently, $\theta \equiv Q$, which is impossible; or, having reached zero, it continues to decrease: $\theta'(\xi^*) = 0$, $\theta'' < 0$, and $\theta'(\xi) < 0$ for $\xi > \xi^* < 1$. But then, if $\theta'(1) = 0$, there exists $\bar{\xi}$ such that $\theta''(\bar{\xi}) = 0$ and $\xi^* < \bar{\xi} < 1$, i.e.

$$\theta'(\bar{\xi}) = k(Q - \theta)e^{\theta}\big|_{\xi=\bar{\xi}}.$$

But this is possible only when $\theta(\bar{\xi}) > Q$, since $\theta'(\bar{\xi}) < 0$. On the interval $(0, \xi^*)$, $\theta(\xi) < Q$ (otherwise, owing to $\theta' > 0$, θ'' would also be positive for such ξ); hence it follows that on the interval $(\xi^*, \bar{\xi})$, $\theta'(\xi) > 0$, $\theta'(\bar{\xi}) > 0$, since θ, changing from a number smaller than Q to a number larger than Q, must increase. The contradiction obtained proves that in case a) $\theta'(\xi) > 0$ for $0 \le \xi \le 1$. It is now easy to conclude that in this case $0 < \theta(\xi) < Q$.

Consider case b). Since $\theta'(0) > 0$, $\theta(0) > 0$, it follows that $\theta(\xi)$ increases in some neighborhood of zero. Let $\bar{\xi}, \xi_1$ be the first points such that $\theta''(\bar{\xi}) = 0$ and $\theta'(\xi_1) = 0$, $1 > \xi_1 > \bar{\xi} > 0$. Then

$$\theta'(\bar{\xi}) = k(Q-\theta)e^{\theta}\big|_{\xi=\bar{\xi}},$$

and, consequently, $\theta(\bar{\xi}) < Q$. Since $\theta'(\xi_1) = 0$, $\theta''(\xi_1) < 0$, it follows that $\theta(\xi_1) < Q$. Suppose $\theta'(\xi_2) = 0$, $\xi_1 < \xi_2$, and $\theta'(\xi) < 0$ for $\xi_1 < \xi < \xi_2$; then at some point $\xi_1 < \bar{\xi} < \xi_2$ we have $\theta''(\bar{\xi}) = 0$ and

$$\theta'(\bar{\xi}) = k(Q-\theta)e^{\theta}\big|_{\xi=\bar{\xi}}.$$

Since $\theta'(\bar{\xi}) < 0$, we have $\theta(\bar{\xi}) > Q$, but this is impossible, since $\theta(\xi_1) < Q$, $\theta'(\xi) < 0$ for $\xi_1 < \xi < \xi_2$. Hence it follows that also in case b) the solution is monotone in ξ. It is now easy to reduce the problem to the equations

$$\frac{1}{p}yy' - y + k(Q-\theta)e^{\theta} = 0, \tag{32}$$

$$y\big|_{\theta=\theta_0} = p\theta_0, \tag{33}$$

where $y = \theta'$, $\theta_0 = \theta(0)$. If $y(\theta, \theta_0)$ is the solution of problem (32), (33), then, in order to determine θ_0, we obtain the equation

$$\int_{\theta_0}^{\theta_1} \frac{d\theta}{y(\theta, \theta_0)} = 1,$$

where θ_1 is determined from the equation $y(\theta_1, \theta_0) = 0$. For the numerical solution of these equations it is convenient to transform the integral by integration by parts using (32). Analogously to (17), one can obtain an equation for determining the bifurcation point, but, because of its cumbersomeness, it is apparently more convenient to solve it by numerical methods.

We shall examine in more detail the case of small p. Equations (30), (31) are equivalent to the integral equation

$$\theta = \int_0^{\xi} \left[e^{p(\xi-\eta)} - 1\right] k[\theta(\eta) - Q]e^{\theta(\eta)}\, d\eta - k\int_0^1 e^{p(\xi-\eta)}[\theta(\eta) - Q]e^{\theta(\eta)}\, d\eta. \tag{34}$$

For $p = 0$ we obtain

$$\theta(\xi) = k\int_0^1 (Q - \theta(\eta))e^{\theta(\eta)}\, d\eta. \tag{35}$$

It follows from (35) that

$$\frac{d\theta}{d\xi} = 0,$$

and therefore any solution of equation (35) satisfies the functional equation

$$\theta = k(Q-\theta)e^{\theta}. \tag{36}$$

For $Q > 4$ and

$$\frac{1}{k_2} < \frac{1}{k} < \frac{1}{k_1},$$

where

$$\frac{1}{k_i} = \left(\frac{Q}{2} + (-1)^i\sqrt{\frac{Q^2}{4} - Q} - 1\right) e^{\frac{Q}{2} - (-1)^i\sqrt{\frac{Q^2}{4} - Q}},$$

(36) has three solutions.

We shall use Theorem 2 (3.XVIII) from [5], which establishes the existence of nearby solutions for nonlinear integral equations that differ only slightly. Computing the Fréchet derivatives of the operator determined by the right-hand side of formula (34), returning to the boundary-value problem and solving it, one easily obtains the estimates:

$$\|P\theta_1\| \leq p\theta_0 = \eta', \qquad \int_0^1 |K|\, d\xi \leq k|\theta + 2 - Q|e^{\theta} = k'(\theta),$$

$$\mu = pk(Q - \theta_0 + 1)e^{\theta_0}, \qquad \int_0^1 |G|\, d\eta \leq \frac{\mu}{|\mu - p|} = B,$$

where G is the kernel of the resolvent of the operator $P'(\theta_0)$, θ_0 is a solution of equation (36); K is the kernel of the operator P''. Then, by the condition of the theorem, if

$$h = (1+B)^2 k'\eta' \leq \frac{1}{2} \quad \text{for} \quad |\theta - \theta_0| < 2(1+B)\eta', \tag{37}$$

then a solution of equation (27) exists and is unique in the domain

$$|\theta - \theta_0| \leq \frac{1}{(B+1)k'}. \tag{38}$$

Hence, in the case $Q > 4$, $k_1 < k < k_2$, the following result can be obtained: if

$$p < \frac{10}{e^Q\left(1 + \dfrac{\mu_i}{\mu_i - p}\right)\theta_i}, \tag{39}$$

where

$$\mu_i = pk(Q - \theta_i)e^{\theta_i}, \qquad \theta_i = k(Q - \theta_i)e^{\theta_i},$$

then problem (30), (31) has three solutions. However, in specific cases it is more convenient to compute B, k', η' and to use the estimates (37), (38), since estimate (39) is crude.

III. Let us consider the equation describing the process in a chemical reactor (in the presence of a catalyst and taking internal heat exchange into account), when a zero-order reaction occurs in it (the reaction rate does not depend on the amount of reacting substance):

$$\beta\frac{\partial\theta}{\partial t} + \frac{\partial\theta}{\partial\xi} = kQe^\theta - \gamma(\theta - \theta_x), \tag{40}$$

$$-\frac{\partial\theta_x}{\partial t} - \frac{\partial\theta_x}{\partial\xi} = \gamma(\theta - \theta_x) \tag{41}$$

under the conditions $\theta_x|_{\xi=0} = \theta|_{\xi=0}$; $\theta_x|_{\xi=1} = \theta_0$. Here k, Q, γ, θ_0 are positive parameters. If $\dfrac{\partial\theta}{\partial t} = \dfrac{\partial\theta_x}{\partial t} = 0$, then we arrive at the problem

$$\frac{\partial\theta}{\partial\xi} = kQe^\theta - \gamma(\theta - \theta_x); \qquad \frac{\partial\theta_x}{\partial\xi} = -\gamma(\theta - \theta_x), \tag{42}$$

$$\theta_x|_{\xi=0} = \theta|_{\xi=0}; \qquad \theta_x|_{\xi=1} = \theta_0. \tag{43}$$

Putting $\theta - \theta_x = v$, we obtain the equations

$$\frac{dv}{d\xi} = kQe^\theta; \qquad \frac{d\theta}{d\xi} = Qke^\theta - \gamma v,$$

whence

$$\theta(\xi) = v(\xi) + \theta(1) - v(1) + \gamma\int_\xi^1 v(\eta)\,d\eta$$

and

$$\frac{dv}{d\xi} = kQe^{v(\xi)+\gamma\int_\xi^1 v(\eta)\,d\eta}e^{\theta_0} = \lambda e^{v(\xi)+\gamma\int_\xi^1 v(\eta)\,d\eta}. \tag{44}$$

Thus, v is a monotone function, and from (44) we obtain

$$\frac{d^2v}{d\xi^2} = \left(\frac{dv}{d\xi}\right)^2 - \gamma v\frac{dv}{d\xi};$$

putting $\frac{dv}{d\xi} = w$, we obtain

$$w'w = w^2 - \gamma vw$$

and

$$w = Ce^v + \gamma(v+1),$$

where $C = \lambda - \gamma(1+v_1)e^{-v_1}$, $v_1 = v(1)$.

It is now easy to see that, in order to determine the stationary solutions, it is enough to determine v_1 from the equation

$$\int_0^{v_1} \frac{dv}{\left[\frac{kQ}{\gamma}e^{\theta_0} - (1+v_1)e^{-v_1}\right]e^v + 1 + v} = \gamma. \tag{45}$$

The graph of the left-hand side of (45) shows that either there exist two stationary regimes, or none (uniqueness under the considered

values of the parameters and $kQe^{\theta_0} > 1$ occurs only if λ is a bifurcation point).

An analogous functional equation can be obtained for determining stationary solutions in the case of a first-order reaction (the reaction rate depends linearly on the amount of substance):

$$-\frac{d\theta}{d\xi} = Qke^{\theta}(1-x) - \gamma(\theta-\theta_x); \qquad -\frac{d\theta_x}{d\xi} = \gamma(\theta-\theta_x);$$

$$-\frac{dx}{d\xi} = k(1-x); \tag{46}$$

$$\theta_x\big|_{\xi=0} = \theta\big|_{\xi=0}; \qquad x\big|_{\xi=0} = 0; \qquad \theta_x\big|_{\xi=1} = \theta_0, \tag{47}$$

and in this case the substitution $\theta - \theta_x = v$ leads to the equation for v

$$\frac{dv}{d\xi} = k(Q-v)e^{\theta_0}e^{v+\gamma\int_\xi^1 v\,d\eta}. \tag{48}$$

It can be proved that the function v is a monotonically increasing function of ξ, $\frac{dv}{d\xi} > 0$, and therefore, taking the logarithm of (48) and differentiating the resulting equation, we arrive at the following equation for v:

$$(Q-v)v'' + v'^2 = (v' - \gamma v)v'(Q-v).$$

Putting $v' = w(v)$, we obtain

$$w' + \frac{w}{Q-v} - w = -\gamma v,$$

solving which we easily arrive at the functional equation for $v_1 = v(1)$, determining the stationary regime,

$$\int_0^{v_1} \frac{dv}{(Q-v)e^v\left[\frac{ke^{\theta_0}}{\gamma} + \int_{v_1}^{v} \frac{ye^{-y}}{y-Q}\,dy\right]} = \gamma. \tag{49}$$

Constructing graphs of the left-hand side of (49), we can verify that, for fixed Q, $\frac{ke^{\theta_0}}{\gamma}$, and different γ, there may be either one or several stationary regimes. It is easy to verify that at least one solution of problem (46), (47) always exists. The nonstationary mixed problem does not always have a solution defined for all t. This is easily seen by considering the case $\gamma = 0$, when any solution can be represented in the form

$$\theta = \begin{cases} \ln \dfrac{1}{-\dfrac{k}{\beta(1+\beta)}(t-\beta\xi) + e^{-\varphi\left(-\frac{t-\beta\xi}{\beta}\right)} - \dfrac{k}{1+\beta}(t+\xi)}, & -\beta < t-\beta\xi \leqslant 0, \\ \ln \dfrac{1}{\dfrac{k}{1+\beta}(t-\beta\xi) + e^{-\psi(t-\beta\xi)} - \dfrac{k}{1+\beta}(t+\xi)}, & 0 \leqslant t-\beta\xi \leqslant 1, \\ \ln \dfrac{1}{\dfrac{k}{1+\beta}(t-\beta\xi) + e^{-\theta_0} - \dfrac{k}{1+\beta}(t+\xi)}, & t-\beta\xi \geqslant 1, \end{cases}$$

$$\theta_x = \begin{cases} \psi(t+\xi), & 0 \leqslant t+\xi \leqslant 1, \\ \theta_0, & \xi + t \geqslant 1, \end{cases}$$

where $\theta|_{t=0} = \varphi(\xi)$; $\theta_x|_{t=0} = \psi(\xi)$.

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Note: Figure translations are in progress. See original paper for figures.