ELLIPTICAL POLARIZATION OF THE RADIATION OF THE CRAB NEBULA

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Abstract

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E. G. MYGHELKIN, K. G. DZHAKUSHEVA

ELLIPTICAL POLARIZATION OF THE RADIATION OF THE CRAB NEBULA

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The continuous spectrum of the radiation of the Crab Nebula is caused, as is known, by the radiation of ultrarelativistic electrons in a magnetic field. The total radiation of an individual electron is elliptically polarized if the component of the particle momentum along the field is nonzero (see ([1])). In this case the relative circular polarization is equal to

$$(I_+ - I_-)/(I_+ + I_-) = \frac{\sqrt{3}}{4}\frac{cP_H}{E}\frac{mc^2}{E}, \tag{1}$$

where P_H is the projection of the momentum on the direction of the magnetic field; I_+ and I_- are the intensities of the components of, respectively, right-hand (the rotation of the electric vector forms a right-handed screw with the direction of propagation of the radiation) and left-hand circular polarization. It follows from (1): if the angle between the directions of the electron momenta and the corresponding magnetic fields is acute ($P_H > 0$), then the radiation is elliptically polarized with a right-handed screw, and vice versa. Obviously, for an anisotropic distribution of electrons with respect to momenta the total radiation is also elliptically polarized, and the degree of ellipticity will depend on the relative orientation of the corresponding magnetic fields and on the form of the distribution function. Limiting ourselves here to these remarks, we proceed to present the results and the method of the experimental investigation of the elliptical polarization of the radiation of the Crab Nebula (M1).

Photographic observations of the nebula M1 in the band 5300–6300 Å, according to the method described below, were carried out in October–December 1964 with the aid of D. A. Rozhkovskii' s automatic polarigraph ([2]), adapted to the 50-centimeter meniscus telescope of the Astrophysical Institute of the Academy of Sciences of the Kazakh SSR. Astronomical measurements of elliptical polarization had earlier been carried out by V. G. Fesenkov on the basis of the use of a $\lambda/4$-plate (see ([3])). We used a phase plate close in its properties to a quarter-wave plate ($\lambda/4' = 5340 \pm 40$ Å), and in the calculations the non-monochromaticity of the radiation was taken into account (see below).

Suppose that the polarization ellipse $\{a\cos\omega t,\ b\sin\omega t\}$, of right-handed rotation ( ), falls on the phase plate in such a way that the major axis of the ellipse a is oriented at an angle γ to one of the principal directions of the plate x. If the components of the beam, whose directions of oscillation are parallel to the principal axes y and x of the plate with optical path difference $\Delta = |n_o - n_e|d$, acquire in the plate a phase difference

$$\varphi = \varphi_y = 2\pi d(n_y/\lambda - n_x/\lambda) = \pm 2\pi\Delta/\lambda, \tag{2}$$

then at the exit from the plate the amplitudes acting are

$$E_x = x = A\sin(\omega t + \psi_1), \qquad E_y = y = B\sin(\omega t + \psi_2), \tag{3}$$

where $A = \sqrt{a_x^2 + b_x^2}$, $B = \sqrt{c^2 + d^2}$, $\operatorname{tg}\psi_1 = -a_x/b_x$, $\operatorname{tg}\psi_2 = d/c$, $c = b_y\cos\varphi - a_y\sin\varphi$, $d = b_y\sin\varphi + a_y\cos\varphi$. Hence there follows the relation for the angle θ between the axis of the plate x and the major axis of the ellipse (3)

$$\operatorname{tg}2\theta = \cos^{-1}2\gamma\,(2ab\sin\varphi/(a^2 - b^2) + \sin 2\gamma\cos\varphi). \tag{4}$$

An analogous calculation for the ellipse of left-hand rotation (L) gives a difference in sign at the first term in the parentheses. From (4) it follows:

$$\operatorname{tg}2\beta = 2\varepsilon/(1 - \varepsilon^2) = \sin^{-1}\varphi(\operatorname{tg}2\theta\cos 2\gamma - \sin 2\gamma\cos\varphi), \tag{5}$$

where $\varepsilon = b/a$ is the ratio of the amplitudes along the principal axes of the polarization ellipse. The sign of the elliptical polarization is determined from the condition

$$\operatorname{tg}2\beta > 0,\ \varepsilon > 0\text{—R}; \qquad \operatorname{tg}2\beta < 0,\ \varepsilon < 0\text{—L}. \tag{6}$$

Elliptically polarized radiation is equivalent to partially (linearly) polarized radiation when the instrument does not respond to phases (for example, a polaroid). The angles γ and θ, measured by such an instrument, determine the preferential directions of polarization, coinciding with the directions of the major axes of the ellipses. The presence of a component of natural light I_e introduces no changes into relation (5). Thus, two polarimetric measurements (with and without the plate) make it possible, according to (5) and (6), to judge the magnitude and sign of the ellipticity of partially (elliptically) polarized radiation. Further, if expression (3) is transformed to the canonical form $nx'^2 + my'^2 = \text{const}$, then the measured degree of polarization is determined as follows: $P = (m-n)/(m+n+2I_e)$. Hence, using the known invariants of a second-order curve, we find

$$k = P_2/P_1 = \sqrt{\cos^2 2\gamma + (\sin 2\gamma \cos\varphi \pm \mathrm{tg}^2\,\beta \sin\varphi)^2}, \tag{7}$$

where P_1 is the degree of polarization found in measurements without the plate; P_2 is the degree of polarization of the radiation transformed by the plate. The double sign under the root is specified as follows: (+)–R, (−)–L. In the particular case when the plate is a quarter-wave plate ($\varphi = \pm\pi/2$) and is oriented along the axes of the original ellipse ($\gamma = 0$), from (7) there follows the well-known result (see (3)): $P_2/P_1 = (a^2+b^2)/(a^2-b^2)$. The quantities P_1 and P_2, of course, differ from the true degree of polarization P, as it is customarily defined from the Stokes parameters (see ([4],[5])). Namely: $P = I_{\mathrm{el}}/I = (Q^2+U^2+V^2)^{1/2}/I$, whereas $P_1 = (Q^2+U^2)^{1/2}/I = P\cos 2\beta$, where $I = I_{\mathrm{el}} + I_e$, $Q = I_{\mathrm{el}}\cos 2\beta \cos 2\gamma$, $U = I_{\mathrm{el}}\cos 2\beta \sin 2\gamma$, $V = I_{\mathrm{el}}\sin 2\beta$.

Analogously to formula (5), from (7) we have

$$\mathrm{tg}\,2\beta = \sin^{-1}\varphi \left(\pm\sqrt{k^2 - \cos^2 2\gamma} - \sin 2\gamma \cos\varphi\right). \tag{8}$$

Here the sign before the root corresponds to the sign of the expression $\mathrm{tg}\,2\theta \cos 2\gamma$.

Formulas (5) and (8) are easily generalized to nonmonochromatic radiation, if in the spectral interval under consideration the dependence of the polarization on wavelength may be neglected, i.e. $\varepsilon(\lambda) = \mathrm{const}$, $\gamma(\lambda) = \mathrm{const}$. The phase plate gives dispersion: for each λ the transformation of the original polarization ellipse obeys dependences (4) and (7), while φ varies according to (2). Let us consider, as applied to our problem, V. G. Fesenkov's method ([6]) of determining polarization. In this case the directly measured quantities are the intensities at three fixed positions of the polaroid, $I_{1,2,3}(\varphi)$, so that, owing to the dispersive properties of the plate, the question is of measuring certain mean quantities:

$$\overline{I_{1,2,3}(\varphi)} = \int_{\varphi_1}^{\varphi_2} V(\varphi) I_{1,2,3}(\varphi)\, d\varphi \Big/ \int_{\varphi_1}^{\varphi_2} V(\varphi)\, d\varphi. \tag{9}$$

Here $\varphi_{1,2} = \pm 2\pi\Delta/\lambda_{1,2}$ for the transmission band ($\lambda_1 \div \lambda_2$); $V(\varphi)$ is a weighting function, determined in our observations by the throughput of the telescope, filters, polaroids, and plate, the sensitivity of Kodak Oa D photographic plates, and the transparency of the atmosphere. A simple calculation for the transformed ellipse (3) shows that the required intensities $I_{1,2,3}(\varphi)$ are linear functions of $\sin\varphi$ and $\cos\varphi$. Thus, the averaging (9) reduces to calculating the mean values of $\sin\varphi$ and $\cos\varphi$ (in our case it was found that $\overline{\sin\varphi} = -0.997$, $\overline{\cos\varphi} = 0.115$).

Fig. 1. Diagram of the elliptical polarization of the radiation of the Crab Nebula

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By direct substitution of the intensities $I_{1,2,3}(\varphi)$, averaged according to (9), into the formulae of Fesenko' s method (see (6)), after transformations we find two expressions for determining ε:

$$\operatorname{tg} 2\beta = \operatorname{tg} 2\beta' = (\overline{\sin\varphi})^{-1}(\pm\sqrt{k^2 - \cos^2 2\gamma} - \sin 2\gamma\,\overline{\cos\varphi}), \tag{10}$$

$$\operatorname{tg} 2\beta = \operatorname{tg} 2\beta'' = (\overline{\sin\varphi})^{-1}(\operatorname{tg} 2\theta \cos 2\gamma - \sin 2\gamma\,\overline{\cos\varphi}), \tag{11}$$

where $\theta = \alpha_2 + \delta$; δ is the angle between the axis x of the plate and the first position of the polaroid, from which the position angles α_2 (measurements with the plate) and α_1 (measurements without the plate) are reckoned; $\operatorname{tg} 2\beta' = 2\varepsilon'/(1-\varepsilon'^2)$, $\operatorname{tg} 2\beta'' = 2\varepsilon''/(1-\varepsilon''^2)$. It is convenient to use formulae (10) and (11) jointly: errors in the measurements may make one or the other numerical result more preferable.

On the basis of the method considered, two groups of observations were carried out (the first group without the plate, the second group with the plate), with 5 series of exposures in each group. Measurements of 30 exposures were made at 54 points of the object on an MF-4 microphotometer with a circular diaphragm $d = 0'.254$. Processing of the exposures of the first group (determination of P_1 and $\alpha_1 = \gamma - \delta$) showed quite satisfactory agreement with the observations of Walter (7) and Walraven (8). The observations of the second group differ in principle only by the introduction of the phase plate, so that the comparison mentioned made it possible to judge the reliability of the determination of P_2

and θ. As a result, data were obtained on the elliptical polarization of the radiation of the Crab Nebula. The very fact of the presence of ellipticity is beyond doubt: the plate is close to a quarter-wave plate, and in the absence of ellipticity there would have been a systematic deviation of the angles α_2 toward the directions of the plate axes, with an equally systematic decrease in the quantities P_2 as compared with P_1 at every point of the object; this was not observed.

Part of the final results is given in Fig. 1.

The quantity $\varepsilon_{\min}$–the smaller (in absolute value) value of the ellipticity factor from ε' and ε''–is plotted in the direction of the minor axes of the polarization ellipses. Along the major axes, i.e., in the directions α_1, the corresponding minimum values of the degree of elliptical polarization at each point are plotted: $P_{\min} = P_1/(\cos 2\beta)_{\max}$. Averaged over the nebula, the following was found:

$\overline{P}_1 = 17\%$, $\overline{P}_{\min} = 19\%$, $\overline{P}_{\max} = 25\%$. Thus the degree of elliptical polarization P exceeds the value of the "degree of polarization" P_1, obtained under the assumption that the radiation is partially (linearly) polarized, by an average of $(5 \pm 3)\%$. At 5 measured points (Nos. 1, 2, 9, 13, 14) $\varepsilon > 0$ (right-handed screw); at the remaining points $\varepsilon < 0$ (left-handed screw). It follows from this that the ultrarelativistic electrons in the M1 nebula move predominantly in directions opposite to the magnetic field.

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Siberian Physicotechnical Institute
named after V. D. Kuznetsov

Astrophysical Institute
of the Academy of Sciences of the Kazakh SSR

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Note: Figure translations are in progress. See original paper for figures.