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Abstract

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ON THE VIOLATION OF THE RECIPROCITY PRINCIPLE IN THE PROPAGATION OF VERY-LONG RADIO WAVES AROUND THE EARTH IN THE DAYTIME

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1. Experiments [(1–4)] have established that the attenuation of very-long waves (VLF) on paths running from east to west is greater than in the reverse direction. This phenomenon, called below the gyrotropic effect, is especially large on daytime paths close to the geomagnetic equator. It is caused by the anisotropy of the ionosphere [(5–8)] and is analogous to the gyrotropic effect in microwave waveguides with ferrite magnetized by a transverse magnetic field [(9)]. In the waveguide theory of VLF [(10–12)] it has not been taken into account. However, this can be done in a natural way if, in calculating the ionospheric impedances by the method of [(13)], which enter the equations for the wave numbers ν of the normal waves, one takes into account not only the vertical component of the Earth' s magnetic field H_r, but also the horizontal components: H_θ (along the path) and H_φ (across it). The scanty information on the gyrotropic effect in VLF does not permit the use of the method of matching the data on the field and the medium [(10–12)]. Therefore, as a first approximation we shall take the profiles of the electron concentration $N_e(h)$ and collision frequency $\nu_{\text{eff}}(h)$, obtained without allowance for the gyrotropic effect and corrected for the equatorial zone by aeronomical calculations [(14)], introducing them, together with H_θ, H_φ, and H_r, into the dielectric-constant tensor of the ionosphere (ε). The components of the tensor ε_{jk}: $(i, j, k$ are unit vectors along θ, φ, and $r)$

$$\varepsilon_{ij} = \pm\frac{i\nu U_k}{Z^2 - U^2} + \frac{\nu U_{iU} j}{Z(Z^2 - U^2)}\ (i \neq j); \qquad \varepsilon_{ii} = 1 - \frac{\nu}{Z} - \frac{\nu U_{jk}^2}{Z(Z^2 - U^2)}\ (i = j), \tag{1}$$

where

$$\nu = 4\pi N_e(h)e^2/m\omega^2; \qquad Z = 1 + i\nu_{\text{eff}}(h)/\omega; \qquad U_j = -eH_j/mc\omega,$$

j is the unit vector θ (φ or r), $U = U_i\mathbf{i} + U_j\mathbf{j} + U_k\mathbf{k}$; U_{jk} is the projection of $\mathbf{U}$ on the plane with unit vectors j, k; the plus sign is taken when i, j in ε_{ij} stand in the order θ, φ, r; H_θ, H_φ, and H_r are the components of $\mathbf{H}_0$ in the coordinate system connected with the wave path: its origin lies on the polar axis.

2. For a path running along the geomagnetic equator, only H_φ differs from 0, and the system of Maxwell equations for the ionosphere (4)(13) splits into purely TH- and purely TE-waves. For the TH-waves of interest to us, the impedance equation (9)(13) assumes the simple form:

$$\left(\frac{\varepsilon_{\theta\theta}}{\Delta}u\right)'_r + \frac{\varepsilon_{\theta\theta}}{\Delta}u^2 + k^2\left[1 - \frac{\varepsilon_{\theta\theta}}{\Delta}\left(\frac{\nu}{kr}\right)^2 \pm \frac{ir}{k}\left(\frac{\varepsilon_{\theta r}}{r\Delta}\right)'_r\left(\frac{\nu}{kr}\right)\right] = 0, \tag{2}$$

where $\Delta = \varepsilon_{\theta\theta}^2 + \varepsilon_{\theta r}^2$. The last term in (2) is responsible for the violation of the reciprocity principle and for the gyrotropic effect associated with it. Integration of (2) with respect to r from $r_\infty = c + h_\infty$, where u assumes the adiabatic value

$$\bar{u}^e(r_\infty) = ik\sqrt{\varepsilon_{\theta\theta} + \frac{\varepsilon_{\theta r}^2}{\varepsilon_{\theta\theta}} - \left(\frac{\nu}{kr_\infty}\right)^2}, \tag{3}$$

to the point $r = c$, where $u(c)$ gives the impedance $Z_y^e = iu(c)/k$, entering the equation

$$\overline{D_\nu(a', c')} - iZ_y^e\overline{D_\nu(a', c)} = 0, \tag{4}$$

whose roots are the wave numbers of normal waves of the TH type. It was obtained from (14) (13) under the condition that $X^e = 0$. The excitation coefficients are determined by formula (13) (12).

3. We shall show that the calculation of the parameters of TH-waves for any daytime paths on the Earth can be carried out with sufficient accuracy by the formulas of Sec. 2. For this purpose we divide the interval of integration (r_∞, c) of equations (9) (13) into two parts: (r_∞, d), where the adiabatic approximation is valid, and (d, c), where the buildup of radio-wave reflections occurs. The point $r = d$ can be determined by comparing the hodograph $u(r)$ of equation (9) (13) with the hodograph of the adiabatic approximation $\bar{u}(r)$ of equation (10) (13); $r = d$ depends on ν, H_0, and the profiles $N_e(h)$ and $\nu_{\text{eff}}(h)$. We shall call the profile $N_e(h)$ a daytime profile if, for ν close to ka and any $|H_0| \leq 0.5$ Oe, the condition $Z(d) > U$ is satisfied. Owing to the adiabatic nature of (r_∞, d), we transfer the beginning of integration from r_∞ to d. Equation (10) (13), which determines $\bar{u}(d)$, when the daytime-profile condition is fulfilled, is approximated

Fig. 1. Hodographs $u(h)$ and electron-concentration profiles $N_e(h)$

Figure 1: Fig. 1. Hodographs $u(h)$ and electron-concentration profiles $N_e(h)$

Fig. 2 and Fig. 3

Figure 2: Fig. 2 and Fig. 3

by a biquadratic equation. One of its roots coincides with u^e, determined from (3) at $r = d$. Since the profile ν_{eff} has the form

Fig. 1. Hodographs $u(h)$ and electron-concentration profiles $N_e(h)$

$$\nu_{\text{eff}} = s_0 10^5 \exp[-0.148(h - 89)], \tag{5}$$

where $s_0 = 5 \div 10$, $h = r - d$ in km, then for $Z(d) > U$ this inequality is also fulfilled for all $r < d$. Consequently, equation (9) (13) for the impedances Z_y of TH-waves for any H_θ, H_φ, and $H_r \leq 0.5$ Oe is approximated by (2). Confirmation of this important result, simplifying the correction of the theory (10–12), is given in Fig. 1, where the hodographs $u(r)$ obtained by integrating (9) (13) for $f = 15$ kHz, $\nu = 2000$, for the profile $N_e(h)$ of middle latitudes (curve I in Fig. 1) are shown. Replacing $N_e(h)$ and $\nu_{\text{eff}}(h)$ by stepwise profiles with jumps at the points $h_0, h_1, \dots, h_l, h_{l+1}, h_d$, the hodograph $u(r)$ can be represented as a piecewise-continuous curve undergoing a jump at each step h_l, ensuring the continuity of Z_y^e according to equation (11) (13).

$$Z_e(h_l) = \frac{1}{\Delta}\left[\frac{\varepsilon_{\theta\theta}}{ik} u^e(h_l) \mp \varepsilon_{\theta r}\frac{\nu}{kr_l}\right], \qquad r_l = a + h_l. \tag{6}$$

Between the steps the hodograph moves along a circle whose radius depends on the accumulated reflection coefficient in the overlying layers. Hodograph **A** is constructed for $H_\theta = H_\varphi = 0$ and $H_r = 0.445$ Oe, **B** for $H_\theta = H_\varphi = H_r = 0$. In accordance with what has been said, their final values $u(c)$ coincide, despite the great difference of the initial ones; accumulation of reflection takes place only due to the first term in (6). In hodographs Γ and , $H_\theta = H_r = 0$, and $H_\varphi = 0.445$ Oe, and the second term in (6) introduces a different effect of accumulation of reflections depending on the choice of wave direction

Fig. 2. Parameters $\beta(f)$ and $\Delta\alpha(f)$ of the TH_1 wave for the profile $N_e(h)$ of middle latitudes I at
$S_0 = 5$

Fig. 3. Parameters $\beta(f)$ and $\Delta\alpha(f)$ of the TH_1 wave for profile I, $S_0 = 10$ (dashed curves) and profile II, $S_0 = 5$ (solid curves). Values of H_φ, equal to 0.445 and 0, are indicated on the curves

$v > 0$ (W $\to$ E) or $v < 0$ (E $\to$ W). **B** is a case similar to **A**, but for the ordinary wave.

4. Figures 2 and 3 give computer calculations of the attenuation coefficients $\beta(TH_1)$ and of the differences of angular wave numbers $\Delta\alpha(TH_1) = \alpha(H_\varphi) - \alpha(0)$ as functions of the frequency f and H_φ for two types of profiles: I—middle latitudes in summer and II—the equatorial zone (Fig. 1), for $s_0 = 5$ and $s_0 = 10$.* On the curves $\beta(f)$ and $\Delta\alpha(f)$ the quantities H_φ, s_0, and the profile type are indicated. On the right ordinate the attenuations in decibels per 1000 km are given. The calculations were carried out by the formulas of Sec. 2, but, by virtue of the assertion of Sec. 3, they are applicable to any daytime paths. Profile I is applicable for latitudes $|\varphi| > 40°$, and II for $|\varphi| < 20°$. In the intermediate zone $20° < |\varphi| < 40°$, β and $\Delta\alpha$ are estimated by mean values. The values $\alpha(0)$ may be taken from the corresponding graphs $\alpha(f)$ in works ([10-12]). If $H_r = \sqrt{H_\varphi^2 + H_\theta^2}$ is the total horizontal component of the field $\mathbf{H}_0$ at a given point of the path, and ψ is the angle between H_r and the path, then β for it is determined approximately by the formula $\beta = \beta(H_r)\sin\psi$, where $\beta(H_r)$ is obtained from Figs. 2 or 3 by linear interpolation. It is seen from Figs. 2 and 3 that in the equatorial zone the valve effect reaches the largest values. Thus, for example, for $f = 10$ kHz on the Panama Canal path

* The Earth is considered ideally conducting; therefore, in (9), $\bar{D}_\nu$ is replaced by D_ν.

channel—the Hawaiian Islands, of length 1.33 rad, where the mean value $H_\varphi = 0.3$ oersted; according to Fig. 3 (profile II) it follows that $\beta(\mathrm{W,E}) = 1.2$ neper/rad, and $\beta(\mathrm{E,W}) = 2.9$ neper/rad, whence the ratio of the field strengths $P = \exp[\beta(\mathrm{E,W}) - \beta(\mathrm{W,E})]\theta \simeq 10$, which is close to the experimental value 12.3 obtained in [3]. In middle latitudes the valve effect is weakened owing to the appearance of the C layer, created by cosmic rays [11,12,14], and to the decrease of H_φ. On summer transatlantic paths, where $H_\varphi \simeq 0.15$ oersted, for $f = 15$ kHz $\beta(\mathrm{E,W}) = 1.75$ neper/rad, $\beta(\mathrm{W,E}) = 1.4$ neper/rad, which gives $P = 1.4$ for $\theta = 1$ rad. In winter daytime the calculation gives $\beta(\mathrm{E,W}) = 2.8$, $\beta(\mathrm{W,E}) = 2.1$, and for $\theta = 1$ rad we obtain $P = 2.5$. These results are close to the experimental data of the round-the-world expedition of 1922-1923 [4]. The valve effect is weakened with increasing atmospheric pressure because of an increase in ν_{eff} (see the graphs of Figs. 2–3 for $s_0 = 5; s_0 = 10$) and with increasing frequency f. The results of the calculations for profiles I and II are good confirmation that the C layer in the equatorial zone is practically absent, as is required by the aeronomic theory of the origin of the C layer through primary cosmic rays. If the C layer existed in the equatorial zone and the profile N_e were close to profile I, then the calculated P would be equal to 5, which is considerably below the experimental value 12. At night, in order to take account of the valve effect, all three components of the field H_θ, H_φ and H_r must be considered.

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Note: Figure translations are in progress. See original paper for figures.