LOSS OF STABILITY OF A STRICTLY CONVEX SHELL UNDER CONCENTRATED LOADING

![Fig. 1](#)

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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LOSS OF STABILITY OF A STRICTLY CONVEX SHELL UNDER CONCENTRATED LOADING

Let us consider a strictly convex shell, rigidly fixed along its edge, reinforced by a rigid element H (Fig. 1). Let a concentrated force Q, normal to the surface of the shell, act on the element H. If this force is sufficiently large, it may cause loss of stability of the shell with buckling over some region G containing the element H. We shall find the least force Q determined by this condition. It is called the **upper critical** force.

As shown in the author' s paper (^{1}) (Chap. II, § 2), the magnitude of the upper critical load is determined from the stationarity condition of a certain functional $W = U - A$, defined on infinitesimal bendings of the middle surface of the shell with discontinuities at the boundary of buckling. The discontinuity of the bending field on the boundary of buckling γ must be directed along the binormal ν. In this connection, we first consider the character of the bending fields in the present problem.

Fig. 1

Let τ be the bending field inside the buckling region G, and τ' the bending field outside this region, i.e., on the remaining part of the shell. In view of the rigid fastening of the edge and the strict convexity of the shell, the field τ' is identically equal to zero. For the same reason, owing to the rigid attachment of the element H to the shell, the field τ in the region G is trivial, i.e., it reduces to the field of velocities of motion of the region G as a whole. With this character of the bending fields τ and τ', the above-mentioned matching of them on the boundary of buckling γ is possible only if the curve γ is plane and the field τ is the field of velocities of rotation of the region G about an axis lying in the plane of the curve γ; in particular, if it reduces to a simple displacement in a direction perpendicular to this plane. Hence it follows that the buckling region G, if of sufficiently small dimensions, must have the form of an ellipse similar and similarly situated to the indicatrix of curvature at the center of buckling.

In the paper (¹) (Chap. II, § 1) it is shown that the deformation energy of the shell associated with buckling over an elliptical region similar to the indicatrix of curvature is determined by the formula

$$U = 4\pi E\delta^2 h\lambda^2/\sqrt{12(1-\mu^2)}\sqrt{R_1R_2},$$

where R_1 and R_2 are the principal radii of curvature at the center of buckling; h is the deflection at the center of buckling; λ is a parameter characterizing the size of the buckling region; E and μ are the elastic constants of the shell material (modulus of elasticity and Poisson' s ratio). The work done by the force Q during buckling is

$$A = Qh.$$

From the stationarity condition of the functional W with respect to the deformation parameter h, we find the magnitude of the force Q sustained by the shell during buckling:

$$Q = 4\pi E\delta^2\lambda^2/\sqrt{12(1-\mu^2)}\sqrt{R_1R_2}.$$

If the tangent plane at the center of buckling is taken as the xy-plane, then the boundary of the buckling region is given by the equations

$$x = \lambda\sqrt{R_1}\cos t, \qquad y = \lambda\sqrt{R_2}\sin t.$$

The area of the buckling region is

$$S = \pi\lambda^2\sqrt{R_1R_2}.$$

Hence

$$Q = 2E\delta^2 S/\sqrt{3}(1-\mu^2)R_1R_2.$$

The smallest value of the load Q capable of causing loss of stability of the shell corresponds to the minimum of S. This minimum is the area of an ellipse similar to and similarly situated with respect to the indicatrix of curvature described about the rigid element H. If this area is denoted by $S(H)$, then the least force Q_e causing loss of stability of the shell is determined by the formula

$$Q_e = 2\pi E\rho^2\delta^2/\sqrt{3}(1-\mu^2)R^2.$$

Consider the example of a spherical shell of radius R. Let the rigid element H, through which the action of the force Q is transmitted, have the shape of a

Fig. 2

Figure 2: Fig. 2

circle of radius ρ. In this case $S(H) = \pi\rho^2$, and, consequently, the critical force is

$$Q_e = 2\pi E\rho^2\delta^2/\sqrt{3}(1-\mu^2)R^2.$$

Fig. 2

Let now a moment M, acting in a plane normal to the surface of the shell, be applied to the rigid element H reinforcing the shell (Fig. 2). Under buckling, the action of the moment M is equivalent to a pair of forces Q, one of which is applied to H at the center of the buckling region and the other at its boundary. The deformation energy of the shell is

$$U = 4\pi E\delta^2 h\lambda^2/\sqrt{12}(1-\mu^2)\sqrt{R_1R_2}.$$

The work performed by the moment M during deformation is

$$A = Qh = \frac{M}{d}\,h,$$

where d is the arm of the pair of forces Q. From the stationarity condition for the functional $W = U - A$, for the moment M causing buckling, one obtains

$$M = 2\pi E\delta^2\lambda^2 d/\sqrt{3}(1-\mu^2)\sqrt{R_1R_2}. \tag{*}$$

The quantity $2d$ has a simple geometrical meaning: it is the diameter of the buckling region in the section by the plane of the moment M.

The smallest value of M, determined by formula $(*)$, is obtained when the buckling region is minimal. In this case it is an ellipse similar to and similarly situated with respect to the indicatrix of curvature described about the element H. Let $S(H)$ be the area of this ellipse, and $2d(H)$ its diameter in the plane of the moment M. Then the least moment acting on the shell through the support H and capable of causing loss of stability is determined by the formula

$$M_e = \frac{2E\delta^2 S(H)}{\sqrt{3}(1-\mu^2)\sqrt{R_1R_2}}\,d(H).$$

For a spherical shell of radius R with a support H in the form of a circle of radius ρ,

$$S(H) = \pi\rho^2, \qquad d(H) = \rho.$$

Accordingly, the critical moment is

$$M_e = 2\pi E\rho^3\delta^2/\sqrt{3}(1-\mu^2)R^2.$$

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1. A. V. Pogorelov, *Geometrical Theory of Stability of Shells*, “Nauka,” 1966.

Note: Figure translations are in progress. See original paper for figures.