THE INFLUENCE OF RADIATION DAMPING ON THE MOTION OF A RELATIVISTIC PARTICLE IN A HOMOGENEOUS MAGNETIC FIELD

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Abstract

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THE INFLUENCE OF RADIATION DAMPING ON THE MOTION OF A RELATIVISTIC PARTICLE IN A HOMOGENEOUS MAGNETIC FIELD

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When a charged particle moves in a magnetic field, owing to magnetic-bremsstrahlung (synchrotron) radiation, the energy of the particle decreases, which leads to a change in the particle trajectory.

In those cases in which the time spent by the particle in the field is comparable with the characteristic time of energy decrease

$$T \sim m^3c^5/e^4H^2$$

(as, for example, for electrons in cosmic fields), radiation damping must be taken into account in determining the trajectory.

The equation of motion of the particle has the form

$$\frac{d\mathbf{p}}{dt} = \frac{e}{c}[\mathbf{v}, \mathbf{H}] + \mathbf{f}_\tau, \tag{1}$$

where the damping force $\mathbf{f}_\tau$ is equal to [1]

$$\mathbf{f}_\tau = \frac{2e^4}{3m^2c^5}\left\{[\mathbf{H}[\mathbf{H}, \mathbf{v}]] - \frac{\mathbf{v}}{1 - v^2/c^2}\frac{1}{c^2}([\mathbf{v}, \mathbf{H}])^2\right\}. \tag{2}$$

Let us direct the z-axis of the Cartesian coordinate system along the field and denote

$$\omega = eH/mc, \qquad \delta = {}^2\!/_3 e^4H^2/m^3c^5. \tag{3}$$

We express the velocity v in units of c, and the energy E in units of mc^2:

$$u = v/c, \qquad w = E/mc^2 = 1/\sqrt{1-u^2}.$$

Then, passing to components and taking into account $\mathbf{p} = E\mathbf{v}/c^2$, we obtain

$$du_x w/dt = \omega u_y - \delta u_x (1-u_z^2) w^2,$$

$$du_y w/dt = -\omega u_x - \delta u_y (1-u_z^2) w^2, \tag{4}$$

$$du_z w/dt = -\delta u_z w^2 (u_x^2 + u_y^2).$$

It follows first of all from (4) that

$$u_z = \text{const}. \tag{5}$$

Taking (5) into account, we obtain from (4) the equation for the change of energy

$$dw/dt = -\delta\left[(w/w_\infty)^2 - 1\right], \tag{6}$$

where

$$w_\infty = 1/\sqrt{1-u_z^2} \tag{7}$$

is the limiting value of w as $t \to \infty$.

The integral of (6) is

$$w = w_\infty \operatorname{cth}(\delta t/w_\infty + C_0). \tag{8}$$

The constant C_0 is determined from the condition $w = w_0$ at $t = 0$.

It is expedient to express the quantity w_∞ in terms of the angle θ between the initial direction of the velocity and the direction of the field, and in terms of the initial value of the energy. Setting in (7) $u_z = u_0 \cos\theta$ and taking into account that $w_0 = 1/\sqrt{1-u_0^2}$, we have

$$w_\infty = \frac{w_0}{\sqrt{\cos^2\theta + w_0^2 \sin^2\theta}}. \tag{9}$$

Using (8), one can integrate the equations for the transverse components u_x and u_y,

$$u_x = u_\perp(0)e^{-\delta t}\sin(\omega\tau + \varphi_0),$$

$$u_y = u_\perp(0)e^{-\delta t}\cos(\omega\tau + \varphi_0), \tag{10}$$

where

$$u_\perp(0) = \sqrt{u_x^2(0) + u_y^2(0)};$$

φ_0 is the initial phase; τ is the proper time

$$\tau = \int_0^t \frac{dt'}{w} = \frac{1}{\delta}\ln\frac{\operatorname{ch}(\delta t/w_\infty + C_0)}{\operatorname{ch} C_0}. \tag{11}$$

It is seen from (10) that the transverse components of the velocity decay and tend to 0 as $t \to \infty$.

The proper time is a complicated function of t, in contrast to the motion without taking radiation friction into account, when $\tau = t/w_0$. As a result, the variation of u_x and u_y with time will no longer be harmonic, which will affect the radiation spectrum.

The expressions obtained are substantially simplified in passing to the extreme relativistic case, characterized by the relation $w_0 \gg 1$.

Outside a narrow cone around the field direction with angle $\theta_0 \sim 1/w_0^2$, from (9) we have $w_\infty \approx 1/\sin\theta$. Then (8) becomes

$$\frac{1}{w} = \frac{1}{w_0} + \sin\theta\ \operatorname{th}(\delta t\sin\theta). \tag{12}$$

Since $\operatorname{th} x$ practically reaches its limiting value when x is only slightly greater than unity, the main energy loss occurs over the time

$$t_0 \sim 1/\delta\sin\theta. \tag{13}$$

In the interval $0 \le t \le t_0$ it is sufficient to retain the first term in the expansion of $\operatorname{th} x$ in a series, so that

$$\frac{1}{w} = \frac{1}{w_0} + (\sin^2\theta)\delta t \qquad (t\delta\sin\theta \ll 1). \tag{14}$$

Hence it is seen that the energy decreases by half over a time $\sim 1/w_0\delta\sin^2\theta$. In the same approximation, for the proper time we have

$$\tau = \frac{t}{w_0} + t^2 \frac{\delta}{2} \sin^2 \theta. \tag{15}$$

Substituting (15) into (10), we obtain approximate expressions for the transverse components of the velocity.

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Note: Figure translations are in progress. See original paper for figures.

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