ON THE DIFFUSION OF PLASMA IN A TOROIDAL DISCHARGE

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Abstract

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ON THE DIFFUSION OF PLASMA IN A TOROIDAL DISCHARGE

As is known (^{1}), the plasma in experimental toroidal devices is subject to anomalous diffusion, which considerably exceeds classical diffusion due to pair collisions. This fact is in qualitative agreement also with theoretical considerations indicating the existence, in toroidal geometry, of mechanisms of plasma instability of a general type that may lead to enhanced diffusion. In discussing the experimental data, along with consideration of instabilities, one should not forget that in toroidal geometry even classical diffusion may lead to a particle loss considerably exceeding the loss from a straight discharge. This effect arises from the toroidal drift of charged particles, which, in the absence of rotational transform, would lead to charge separation and ejection of the plasma toward the outer wall of the torus (^{1}). In the presence of rotation of the field lines on magnetic toroidal surfaces, the charges are compensated by flow along the field lines (^{2}, ^{3}). However, because of the finite conductivity, complete compensation does not occur, and this leads to an effective increase in the mean plasma flux.

For stellarators in the absence of a vortex electric field, the corresponding effect was considered in works (^{4}, ^{6}) (an analogous effect of enhanced thermal conductivity was considered in (^{5})). In the present work a formula is obtained for the diffusion flux of plasma in a toroidal axisymmetric discharge in the presence of a vortex electric field.

Let us introduce a quasi-cylindrical coordinate system ρ, φ, ω with line-element square $dl^2 = d\rho^2 + r^2 d\varphi^2 + \rho^2 d\omega^2$, where $r = R(1 + \varepsilon \cos\omega)$; $\varepsilon = \rho/R$; R is the distance from the axis of rotation to the center of the cross section of a certain fixed magnetic surface. To the degree of accuracy sufficient for us, this cross section may be regarded as circular, $\rho =$ const. On the surface under consideration, by definition, the normal components of the current density and magnetic field are equal to zero, $j_\rho = B_\rho = 0$. The remaining quantities needed for the calculation have the form (^{3})

$$E_\varphi = \mathcal{E}_0(t,\rho)/2\pi r; \qquad B_\varphi = \frac{2I(\psi)}{cr} = \frac{B_{\varphi 0}(\rho)}{(1 + \varepsilon \cos\omega)}; \tag{1}$$

$$j_\varphi = 2\pi c \frac{dp(\psi)}{d\psi} r + \frac{1}{cr^2}\frac{dI^2(\psi)}{d\psi}; \tag{2}$$

$$B_\omega = B_{\omega 0}(\rho)\left[1 + \varepsilon\Lambda_1(\rho)\cos\omega + \varepsilon^2\Lambda_2(\rho)\cos 2\omega + \ldots\right] = \frac{1}{2\pi r}\frac{\partial\psi}{\partial\rho}; \tag{3}$$

$$j_\omega = j_\varphi \frac{B_\varphi}{B_\omega} - \frac{c}{B_\varphi}\frac{dp}{d\rho} = \frac{dI}{d\psi}B_\omega; \tag{4}$$

$$E_\omega = \left(\frac{j_\varphi}{\sigma_\parallel} - E_\varphi\right)\frac{B_\varphi}{B_\omega} + \frac{1}{\sigma_\parallel} j_\omega = E_{\omega 0}(t) - \frac{\partial\Phi}{\rho\,\partial\omega}; \tag{5}$$

$$v_\rho = \frac{c}{B_\omega}\left(E_\varphi - \frac{j_\varphi}{\sigma_\parallel}\right) - \frac{c^2}{B^2}\left(\frac{1}{\sigma_\perp} - \frac{1}{\sigma_\parallel}\right)\frac{dp}{d\rho}. \tag{6}$$

Here ψ is the transverse magnetic flux, so that $\psi(\rho,\omega) = \text{const}$ is the equation of a magnetic surface; $\mathcal{E}_0(t)$ is the voltage around the torus; $\sigma_\perp(\rho)$ and $\sigma_\parallel(\rho)$ are the transverse and longitudinal conductivities of the plas-

; φ is a single-valued potential; $E_{\omega 0}(t)$ is the mean value of the vortex azimuthal electric field.

The value of the normal velocity v_{eff}, averaged over the toroidal surface, is determined by the formula

$$v_{\text{eff}} = \left\langle v_\rho \frac{r}{R}\right\rangle = \langle v_\rho(1 + \varepsilon\cos\omega)\rangle, \tag{7}$$

where the angle brackets denote averaging over the azimuth ω.

Averaging (5) over ω, we find the relation between the distributions $p(\psi)$ and $I(\psi)$ and $\mathcal{E}_0$:

$$\mathcal{E}_0 = \frac{\pi c\left(\langle j_\omega\rangle/\sigma_\parallel - \langle E_\omega\rangle\right)}{I\langle 1/r^2 B_\omega\rangle} + \frac{4\pi I'}{c\sigma_\parallel} + \frac{4\pi^2 cp'}{\sigma_\parallel}\frac{\langle 1/B_\omega\rangle}{\langle 1/r^2 B_\omega\rangle}. \tag{8}$$

Substituting the expressions for $\mathcal{E}_0$, j_φ, B_φ, j_ω into (6) and averaging (7), we obtain an expression for the mean expansion velocity of the plasma cord

$$v_{\text{eff}} = -\frac{c^2}{\sigma_\perp B^2}\left(1 + \frac{\sigma_\perp}{\sigma_\parallel}\frac{B_{\omega 0}^2}{B_{\varphi 0}^2}\right)\frac{dp}{d\rho} + \frac{c j_{\varphi 0} B_{\omega 0}}{\sigma_\parallel B_{\varphi 0}^2} - c\frac{E_{\omega 0}}{B_{\varphi 0}} - \frac{2c^2\rho^2}{\sigma_\parallel B_{\omega 0}^2 R^2}\frac{dp}{d\rho}. \tag{9}$$

The first term in this expression represents the diffusion velocity. If by $\sigma_\perp$ one understands a certain effective conductivity with allowance for instabilities, then this term also describes anomalous diffusion. The second term in (9) is the velocity of plasma contraction by the magnetic field of the current (the pinch effect in the longitudinal magnetic field), and the third is the drift in the vortex azimuthal field associated with the change of the longitudinal field; for $B_{\varphi 0} \gg B_{\omega 0}$

$$E_{\omega 0} = \frac{\rho}{2c} \frac{\dot{B}_{\varphi 0}}{B_{\varphi 0}}. \tag{10}$$

The diffusion enhanced by toroidicity is described by the last term of expression (9). This term is the drift, averaged over the toroidal surface, of charges in crossed longitudinal magnetic field B_φ and electric field of charge separation. Indeed, the component normal to the magnetic surface of the velocity of such a drift is equal to

$$v_{\mathrm{dr}} = -c\frac{E_\omega}{B_\varphi} = -c\frac{E_\omega}{B_{\varphi 0}}(1 + \varepsilon \cos\omega). \tag{11}$$

The electric field $\mathbf{E} = -\nabla\varphi$, associated with charge separation, is determined from the projection of Ohm' s law onto the magnetic field, $-\mathbf{B}\nabla\varphi = j_\parallel B/\sigma_\parallel$, where $j_\parallel = h\mathbf{B}$ is the density of the longitudinal current that removes the charge separation caused by the toroidal drift. From the condition $\operatorname{div}\mathbf{j} = 0$, for h one obtains the equation $\mathbf{B}\nabla h = \mathbf{j}_\perp \nabla B^2/B^2$. For simplicity we restrict ourselves to the case of a strong longitudinal field $B_\varphi^2 \gg B_\omega^2$; then

$$\mathbf{j}_\perp \nabla B^2 \approx \frac{j_{\perp 0}\, \partial B_\varphi^2}{\rho\, \partial\omega} = 2\varepsilon j_{\perp 0} \sin\omega.$$

Consequently, $h = -2\varepsilon j_{\perp 0} \cos\omega / B_{\omega 0}$, and from Ohm' s law we obtain

$$E_\omega = -2\varepsilon \frac{j_{\perp 0} B_{\varphi 0}^2}{\sigma_\parallel B_\omega^2} \cos\omega = \frac{2c\varepsilon}{\sigma_\parallel} \frac{B_{\varphi 0}}{B_\omega^2} \frac{dp}{d\rho} \cos\omega. \tag{12}$$

Substituting these values of the electric field into the expression for the velocity v_{dr} and averaging (7) over the magnetic surface, we obtain

$$v_{\mathrm{dr}} = -\frac{2c^2\varepsilon^2}{B_\omega^2 \sigma_\parallel} \frac{dp}{d\rho}, \tag{13}$$

in agreement with the last term in formula (9). For $B_\varphi^2 \gg B_\omega^2$, the total effective diffusion coefficient D_{eff} is expressed in terms of the coeffithe diffusion coefficient in cylindrical geometry $D_\perp$ in the form:

$$D_{\text{eff}} = D_\perp \left(1 + \frac{2\sigma_\perp}{\sigma_\parallel} q^2\right). \tag{14}$$

The parameter $q = \rho B_{\varphi 0}/R B_{\omega 0}$ (the "safety factor" with respect to helical perturbations) in experiments on a tokamak is usually several units ($q \sim 5$).

Let us note that this parameter is related to the angle of twist of the magnetic-field lines ι by the relation $q = 2\pi/\iota$. Substitution of this value of q into (14) leads to an expression for D_{eff} that coincides with the result of Pfirsch and Schlüter obtained for stellarators:

$$D_{\text{eff}} = D_\perp \left(1 + \frac{8\pi^2 \sigma_\perp}{\iota^2 \sigma_\parallel}\right). \tag{15}$$

It can be shown that this formula is applicable to any toroidal systems with a circular magnetic axis and with a cross section of the magnetic surfaces close to circular.

In the presence of vortex electric fields, the plasma expansion velocity can in principle be made equal to zero. With a constant longitudinal magnetic field ($E_{\omega 0} = 0$), compensation of diffusion is possible, as follows from formula (9), only at comparatively low plasma pressure $p \sim B_{\omega 0}^2/8\pi q^2$.

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Note: Figure translations are in progress. See original paper for figures.