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Abstract

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DETERMINATION OF THE JUMP AT THE FERMI SURFACE OF THE ELECTRON MOMENTUM DISTRIBUTION FUNCTION BY MEANS OF THE TWO-QUANTUM ANNIHILATION EFFECT OF POSITRONS

(Presented by Academician Ya. B. Zel̆ dovich on 29 XI 1965)

As is known, the distribution function in momentum space for a real degenerate electron gas has a jump at the Fermi surface ([1–4]). This jump, generally speaking, can be observed by studying the angular correlation of pairs of γ-quanta produced in the annihilation of positrons in metal single crystals. If one disregards the interaction of the positron with the surrounding electron gas, then the pattern of angular correlation of pairs of γ-quanta directly determines the electron momentum distribution function (see, for example, ([5])). In the present work it is shown that the interaction of the positron with the electron medium leads to an additional contribution to the jump that should be observed for pairs of γ-quanta at momenta equal to p_F. This effect was calculated in the approximation of a compressed electron gas.

The probability of an annihilation event of a positron with emission of a pair of γ-quanta possessing total momentum $\mathbf{p}$ is given by the expression ([5], [6]):

$$W(\mathbf{p}) = -C \lim_{t-t'\to 0} \int \exp[-i\mathbf{p}(\mathbf{x}-\mathbf{x}')]\, G_{ep}(\mathbf{x}t, \mathbf{x}t; \mathbf{x}'t', \mathbf{x}'t')\, d^3x\, d^3x', \qquad (1)$$

where $G_{ep}(xx;\, x'x')$ is the two-particle electron-positron Green function ([6]), $C = 7.47 \cdot 10^{-15}$ cm$^3 \cdot$ s^{-1}. We shall express the two-particle Green function in expression (1) in terms of one-particle Green functions, using the known equation:

$$G_{ep}(xx;\, x'x') = G_e(x-x')G_p(x-x') + i \int G_e(x-x_1)G_p(x-x_2)\times$$

$$\times\Gamma(x_1x_2;\, x_3x_4)G_e(x_3-x')G_p(x_4-x')\, d^4x_1d^4x_2d^4x_3d^4x_4. \qquad (2)$$

For a compressed electron gas

$$G_e(x-x')G_p(x-x') \simeq G_e^0(x-x')G_p^0(x-x')+$$

$$+G_p^0(x-x')\delta G_e(x-x') + G_e^0(x-x')\delta G_p(x-x'). \qquad (3)$$

Substituting expressions (3) and (2) into (1), we find that $W(\mathbf{p})$ is equal to the sum of four terms having the following physical meaning.

1. The term containing $G_e^0(x-x')G_p^0(x-x')$ determines the annihilation probability without taking into account the interaction between the particles, i.e.

$$W_1(\mathbf{p}) = Cn_0(\mathbf{p}),$$

where $n_0(\mathbf{p})$ is the distribution function of an ideal degenerate Fermi gas.

2. The term containing $G_p^0(x-x')\delta G_e(x-x')$ arises as a consequence of the interaction of the electrons with one another. Therefore

$$W_2(\mathbf{p}) = C\delta n(\mathbf{p}),$$

where $\delta n(\mathbf{p})$ is a quantity related to the complete electron momentum distribution function $n(\mathbf{p})$ in the following way ([4]):

$$n(\mathbf{p}) = n_0(\mathbf{p}) + \delta n(\mathbf{p}).$$

3. The third term, containing $G_p^0(x-x')\delta G_p(x-x')$, arises because of the interaction of the positron with the surrounding medium.

4. The last term, containing the vertex part $\Gamma(x_1x_2;\, x_3x_4)$, takes into account the interaction of the two annihilating particles.

Let us consider the expression for $W_3(\mathbf{p})$. It can be shown that the quantity $\delta G_p(x-x')$ is calculated according to the same rules as the correction to the ordinary one-electron Green' s function for the given electron gas, i.e.,

$$\delta G_p(x-x') = i\int G_p^0(x-x_1)M_p(x_1-x_2)G_p^0(x_2-x')\, d^4x_1d^4x_2,$$

where

$$M_p(x_1 - x_2) = G_p^0(x_1 - x_2)[u(x_1 - x_2) - v(|\mathbf{x}_1 - \mathbf{x}_2|)];$$

$u(x_1 - x_2)$ and $v(|\mathbf{x}_1 - \mathbf{x}_2|)$ are the effective and Coulomb interaction potentials.

The general expression for $W_3(\mathbf{p})$ has the form:

$$W_3(\mathbf{p}) = -C\frac{1}{(2\pi)^4}i\int[u(k,\varepsilon) - v(k)]\times$$

$$\times\left\{\frac{\theta(p_F - |\mathbf{p}|)\theta(k)}{(\varepsilon + k^2 - i\delta)^2} - \frac{\theta(p_F - |\mathbf{p} - \mathbf{k}|)\theta(k)}{(\varepsilon - k^2 + i\delta)^2}\right\} d^3k\, d\varepsilon; \quad (4)$$

$$\theta(x) = 0, \quad x \leq 0,$$

$$\theta(x) = 1, \quad x > 0.$$

In expression (4), the first term in the braces makes an additional contribution to the discontinuity of the quantity $W(\mathbf{p})$ at $|\mathbf{p}| = p_F$. Numerical integration of this term shows that, for electron-gas densities corresponding to real metals $(1.8 \leq r_s \leq 5.5)$,

$$\frac{W_3(p_F + 0) - W_3(p_F - 0)}{W(0)} \sim 0.1.$$

In this case the effect increases in absolute magnitude with increasing r_s (r_s is the radius of a unit electron sphere, expressed in units of the Bohr-orbit radius).

Thus, the interaction of the positron with the electron medium surrounding it leads to the fact that the experimentally observed discontinuity in the momentum distribution of γ-ray pairs must be smaller than the value corresponding to the discontinuity of the function $n(\mathbf{p})$.

Note added in proof. After this article had been sent to press, it became known to us that the existence of the effect discussed here had been noted in the work of Carbotte and Kahana ([7]).

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Note: Figure translations are in progress. See original paper for figures.