ON DETERMINING THE DEPTH OF THE ZERO SURFACE

GEOPHYSICS

1966

SovietRxiv

Abstract

Full Text

UDC 551.465.555/261

GEOPHYSICS

V. F. KOZLOV

ON DETERMINING THE DEPTH OF THE ZERO SURFACE

(Presented by Academician V. V. Shuleikin on 13 I 1966)

In oceanology, the so-called dynamic method of calculating currents has become widely used; it is based on the assumption of the geostrophic character of the motion of waters. An indispensable element of the dynamic method is the concept of the zero surface, i.e., the surface on which the horizontal components of velocity vanish. There exist several ways of determining the position of the zero surface ([1,3]). All of them, to one degree or another, possess certain conventionalities admitted by the authors as sufficient criteria of the zero surface.

In the present communication a new method is proposed for determining the depth of the zero surface, using the necessary conditions for its existence within the framework of the geostrophic model.

The equations of geostrophic motion in an inhomogeneous ocean may be written in the form

$$u = -\frac{g}{\rho f}\left(\int_0^z \frac{\partial \rho}{\partial y}\, dz + \frac{\partial \eta}{\partial y}\right), \tag{1}$$

$$v = \frac{g}{\rho f}\left(\int_0^z \frac{\partial \rho}{\partial x}\, dz + \frac{\partial \eta}{\partial x}\right), \tag{2}$$

where the function η is determined by the shape of the free surface and by the distribution on it of the water density and atmospheric pressure. The coordinate axes x, y, and z are oriented respectively in the horizontal plane and vertically downward from the undisturbed ocean surface; u and v are the horizontal velocity components; ρ is density; g is the acceleration of gravity; f is the Coriolis parameter.

On the zero surface $z = h(x, y)$

$$\frac{\partial \eta}{\partial x} = -\int_0^h \frac{\partial \rho}{\partial x}\, dz, \qquad \frac{\partial \eta}{\partial y} = -\int_0^h \frac{\partial \rho}{\partial y}\, dz. \tag{3}$$

Fig. 1. Dependence of the conditional density on the zero surface on its depth

Figure 1: Fig. 1. Dependence of the conditional density on the zero surface on its depth

Eliminating the function η, we obtain

$$I(\rho, h)_{z=h} = 0, \tag{4}$$

where

$$I(\alpha, \beta) = \frac{\partial \alpha}{\partial x}\frac{\partial \beta}{\partial y} - \frac{\partial \alpha}{\partial y}\frac{\partial \beta}{\partial x}.$$

Let us denote $\rho_h = \rho(x, y, h)$; since

$$\frac{\partial \rho_h}{\partial x} = \left(\frac{\partial \rho}{\partial x}\right)_h + \left(\frac{\partial \rho}{\partial z}\right)_h \frac{\partial h}{\partial x}, \qquad \frac{\partial \rho_h}{\partial y} = \left(\frac{\partial \rho}{\partial y}\right)_h + \left(\frac{\partial \rho}{\partial z}\right)_h \frac{\partial h}{\partial y},$$

instead of (4) we find

$$I(\rho_h, h) = 0. \tag{5}$$

The last relation proves that between ρ_h and h, at the time under consideration, there is a dependence

$$\rho_h = F(h). \tag{6}$$

The function F is universal for each connected piece of the zero surface $z = h$. It follows from (6) that on every isobath of the zero surface the density retains a constant value, i.e., isopycnic po-

surfaces intersect the zero surface along lines located in horizontal planes. If the form of the function F is known from some considerations, the construction of a topographic map of the zero surface from the observed density distribution presents no fundamental difficulties. At each standard horizon z, the isoline $\rho = F(z)$ gives the corresponding isobath of the zero surface. Thus, the problem reduces to establishing the form of dependence (6) for various moments of time.

Fig. 1. Dependence of the conditional density on the zero surface on its depth

An idea of the character of function (6) is given by Fig. 1, on which the density values at individual points of the zero surface for the Pacific Ocean, constructed by the method of O. I. Mamaev [1], are marked. The density data were borrowed from [2]. The solid line shows the probable annual mean course of the curve $\rho_h = F(h)$.

Remaining within the limits of the geostrophic model, it is not difficult to obtain an equation for the direct determination of h. Let us integrate the continuity equation

$$\partial(\rho u)/\partial x + \partial(\rho v)/\partial y + \partial(\rho w)/\partial z = 0$$

vertically from the surface to the bottom $z = H(x, y)$, under the conditions $w = 0$ at $z = 0$ and $w = \partial H/\partial x + v\,\partial H/\partial y$ at $z = H$:

$$\rho_H \left(u_H \frac{\partial H}{\partial x} + v_H \frac{\partial H}{\partial y}\right) + \int_0^H \left[\frac{\partial(\rho u)}{\partial x} + \frac{\partial(\rho v)}{\partial y}\right] dz = 0.$$

Substituting expressions (1) and (2), taking into account (3) and (4), after transformations we finally obtain

$$\int_h^H I\left(\rho, \frac{H}{f}\right) dz - \int_0^H I\left(\rho, \frac{1}{f}\right) z\,dz = 0. \tag{7}$$

In practice there is no need to compute the depth of the zero surface by means of (7) over the entire ocean area; one may restrict oneself to a set of points with a sufficiently wide interval of variation in h, then determine from the data obtained the form of the function F, and carry out the further constructions by the method described above.

It is interesting to note that equation (7) remains meaningful directly at the equator. If the coordinate y is measured from the equator along the meridian, then (7), after multiplication by f^2 and passage to the limit as $y \to 0$, gives

$$H \int_h^H \frac{\partial \rho}{\partial x}\,dz - \int_0^H \frac{\partial \rho}{\partial x}\,z\,dz = 0, \qquad y = 0. \tag{8}$$

In conclusion, let us emphasize once again that relations (5) and (7) are only necessary conditions for the existence of a zero surface in geostrophic motions.

Far Eastern State University

Received
10 I 1966

CITED LITERATURE

1. O. I. Mamaev, *The Zero Dynamic Surface of the World Ocean*, Moscow, 1962.

2. A. M. Muromtsev, *Main Features of the Hydrology of the Pacific Ocean*, Leningrad, 1958.

3. L. M. Fomin, *Theoretical Foundations of the Dynamic Method and Its Application in Oceanology*, Moscow, 1961.

Note: Figure translations are in progress. See original paper for figures.