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Abstract

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ON THE STRUCTURE OF VECTOR AND AXIAL CURRENTS

IN THE BROKEN SYMMETRY $\widetilde{U}(12)$

(Presented by Academician N. N. Bogolyubov on June 4, 1965)

The theory of $SU(6)$ symmetry makes it possible to explain many experimental data. In a number of works ($^{1-10}$) it was shown that a possible relativistic generalization of the symmetry $SU(6)$ is the symmetry $SL(6)$ or $\widetilde{U}(12)$. It was also shown that the wave equations even for free particles break the symmetry $SL(6)$ or $\widetilde{U}(12)$, so that it is meaningless to construct an S-matrix invariant with respect to these groups. However, if the ordinary 4-momenta of particles are regarded as components of tensors* $P^{aa}_{b\dot{\beta}}$, $\dot{P}^{aa}_{b\beta}$, transforming as the corresponding spinors of the group $SL(6)$, then the wave equations are invariant with respect to the group $SL(6)$. On this basis Nguyen Van Hieu (^{7}) proposed a method for studying the symmetry $SL(6)$, which makes it possible to take into account the so-called internal breaking (^{4}) caused by the wave equations. This method is called in (^{7}) the spurion formalism and consists of the following. In constructing matrix elements of scattering processes, currents, or in studying wave equations, we first regard the 4-momenta p_μ of particles as components of tensors $P^{aa}_{b\dot{\beta}}$ and $P^{a\dot{a}}_{b\beta}$. The matrix elements of scattering processes or currents contain not only the wave functions of the initial and final particles, but also these 36-dimensional momenta. We require that, with these 36-dimensional momenta, the matrix elements or wave equations be invariant with respect to the group $SL(6)$, and then, in the invariant expressions obtained, we make the replacement $P^{aa}_{b\dot{\beta}} \to \delta^a_b p^\alpha_{\dot{\beta}}$, $P^{a\dot{a}}_{b\beta} \to \delta^a_b p^{\dot{\alpha}}_{\beta}$, i.e., set all superfluous components equal to zero. By this method, in the work of Nguyen Van Hieu and Smorodinsky (^{11}), the structure of currents in the symmetry $SL(6)$ was considered.

The purpose of the present work is to study the general structure of matrix elements of vector and axial currents for baryons in the symmetry $\widetilde{U}(12)$. Since the wave equations are invariant with respect to the group $\widetilde{U}(12)$ only if one introduces 143-component momenta, we shall require invariance of the matrix elements in this case and then, in the invariant expressions obtained, set all superfluous components equal to zero.

Let us consider the matrix element of the current $J^{a\alpha}_{b\beta}$, $a = 1, 2, 3, 4$, $\alpha = 1, 2, 3$, which is a 143-plet of the group $\widetilde{U}(12)$, between baryon states belonging to the 364-plet of the group $\widetilde{U}(12)$ (the 56-plet of the group $SU(6)$). We denote the momenta of the initial and final baryons by p and q, respectively, and first regard these momenta as components of 143-component tensors P^A_B and Q^A_B. Put $K^A_B = P^A_B - Q^A_B$, $L^A_B = P^A_B + Q^A_B$. These 143-dimensional momenta possess the following property

$$P^A_B Q^B_C + Q^A_B P^B_C = 2\delta^A_C (pq).$$

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* Here a, β are unitary indices, and α and b are spinor indices; for details see work ([7]).

Since the current J^A_B is self-adjoint,

$$(J^A_B)^+ = J^B_A,$$

its matrix element satisfies the condition

$$\langle Q|J^A_B|P\rangle^* = \langle P|J^B_A|Q\rangle. \tag{1}$$

From considerations of invariance and from the generalized Bargmann–Wigner equation for the given baryon multiplet it follows that the matrix element of the current J^A_B, satisfying condition (1), has the general form

$$\begin{aligned}\langle Q|J^A_B|P\rangle = {} & f_1(\chi)\,\bar{\psi}(Q)_{BCD}\psi(P)^{ACD} + \\ & + f_2(\chi)\left[\bar{\psi}(Q)_{DEF}(K/m)^D_B\psi(P)^{AEF} - \bar{\psi}(Q)_{BEF}(K/m)^A_D\psi(P)^{DEF}\right] + \\ & + f_3(\chi)\,\bar{\psi}(Q)_{DEF}(K/m)^D_B(K/m)^A_G\psi(P)^{GEF} + \\ & + f_4(\chi)(L/m)^A_B\bar{\psi}(Q)_{CDE}\psi(P)^{CDE},\end{aligned} \tag{2}$$

where m is the baryon mass, and $f_i(\chi)$ are functions of the invariant

$$\chi = {}^1\!/_{12}\,(K)^B_A(K)^A_B = k^2.$$

Substituting into the right-hand side of relation (2) the expression of the spinor ψ^{ABC} in terms of the physical wave functions of the particles, singling out the components transforming as octet vector and axial currents, and setting the extra components of the momenta equal to zero, we obtain

$$\langle q|j_\mu^V|p\rangle = \frac{l_\mu}{2m}\left(f_1 - \frac{k^2}{m^2}f_3 + \frac{k^2}{m^2}f_2\right)\bar{N}NF +$$
$$+ \left(f_1 - \frac{k^2}{m^2}f_3 + 4f_2\right)\left(\bar{N}\frac{r_\mu}{4m^2}N\right)_{D+{}^2/{}_3F} +$$
$$+ \frac{3l^2}{4m^2}\left[\left(f_1 - \frac{k^2}{m^2}f_3 + 4f_2\right)\bar{D}_\lambda\gamma_\mu D_\lambda - \frac{2l_\mu}{m}f_2\bar{D}_\lambda D_\lambda\right] + \qquad (3)$$
$$+ \frac{3k_\lambda k_\sigma}{2m^2}\left[\left(f_1 - \frac{k^2}{m^2}f_3 + 4f_2\right)\bar{D}_\lambda\gamma_\mu D_\sigma - \frac{2l_\mu}{m}f_2\bar{D}_\lambda D_\sigma\right] +$$
$$+ \frac{1}{m^2}\left(f_1 - \frac{k^2}{m^2}f_3 + 4f_2\right)\left(\varepsilon_{\mu\nu\rho\lambda}l_\rho k_\lambda\bar{D}_\nu N + \text{h.c.}\right),$$

$$\langle q|j_\mu^A|p\rangle = \left(f_1 + \frac{k^2}{m^2}f_3\right)\left[\frac{l^2}{4m^2}(\bar{N}\gamma_\mu\gamma_5 N)_{D+{}^2/{}_3F} + \frac{3l^2}{4m^2}\bar{D}_\lambda\gamma_\mu\gamma_5 D_\lambda + \frac{3k_\lambda k_\nu}{2m^2}\bar{D}_\lambda\gamma_\mu\gamma_5 D_\nu\right]$$
$$+ \frac{2k_\mu}{m}(2f_3 - f_2)\left[\frac{l^2}{4m^2}(\bar{N}\gamma_5 N)_{D+{}^2/{}_3F} + \frac{3l^2}{4m^2}\bar{D}_\lambda\gamma_5 D_\lambda + \frac{3k_\lambda k_\nu}{2m^2}\bar{D}_\lambda\gamma_5 D_\nu\right]$$
$$- 2\left(f_1 + \frac{k^2}{m^2}f_3\right)\frac{l^2}{4m^2}(\bar{D}_\mu N + \text{h.c.}) +$$
$$+ \left[\left(f_1 + \frac{k^2}{m^2}f_3\right)\frac{p_\lambda}{m}\frac{q_\mu}{m} - (f_2 - 2f_3)\frac{p_\lambda}{m}\frac{k_\mu}{m}\right](\bar{D}_\lambda N + \text{h.c.}). \qquad (4)$$

Here N and D_μ denote, respectively, the physical wave functions of the baryon octet and decuplet, while the indices F and D indicate the type of coupling. We note that f_4 gives no contribution. Thus, in the $\widetilde{U}(12)$ symmetry theory the matrix elements of the octet vector and axial currents for particles in the 56-plet of the group $SU(6)$ are expressed in terms of three independent functions $f_i(\chi)$, $i = 1, 2, 3$.

Let us now consider the consequences of several additional assumptions of a special character. If one assumes that the main contribution to the vector form factors is given by pole diagrams with exchange of a vector meson, then

$$f_3 = 0, \qquad f_2 = \frac{m}{2\mu}f_1,$$

where μ is the meson mass. In this case we obtain the results of Refs. ([9,10]). In particular, the magnetic moment of the proton is equal to $1 + 2m/\mu$. Within the framework of the model of composite particles with the relativistic equations of Bogoliubov et al. ([12]) we have

$$f_3 = 0, \qquad f_2 = \frac{1}{2}f_1.$$

Let us now discuss some possibilities for an experimental test of the results obtained. From expression (3) it is seen that annihilations of the type

$$\bar{p} + p \to e^+ + e^-, \qquad p + p \to \mu^+ + \mu^-$$

do not occur for an antiproton at rest. Further, all experimental form factors are expressed through $f_1 - \frac{k^2}{m^2} f_3$ and f_2, while the latter can be expressed through the electric and magnetic form factors of the proton. Thus, knowing the proton form factors, one can obtain the cross sections of the processes

$$e + p \to e + \Delta^+, \qquad e + n \to e + \Delta^0,$$

as well as the probabilities of radiative decays of baryon resonances. Experimental study of weak-interaction processes involving leptons—leptonic decays of baryons and of the Ω^- hyperon, production of baryons and baryon resonances in a neutrino experiment, etc.—also makes it possible to test the relations obtained.

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Note: Figure translations are in progress. See original paper for figures.