ON THE INFLUENCE OF THE BOUNDARY OF THE ACTIVE REGION ON THE SPECTRAL COMPOSITION OF INDUCED RADIATION

PHYSICS

1966

SovietRxiv

Abstract

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UDC 535.3

PHYSICS

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ON THE INFLUENCE OF THE BOUNDARY OF THE ACTIVE REGION ON THE SPECTRAL COMPOSITION OF INDUCED RADIATION

(Presented by Academician I. V. Obreimov on 23 II 1966)

In considering the spectral composition of induced radiation with allowance for the nonuniform spatial extraction of the inverted population by different modes, it is usually assumed ([1,2]) that the losses do not depend on the axial index of the mode. This assumption is valid if the entire resonator is filled with the active substance. If, however, there are dielectric boundaries inside the resonator, then an additional consideration is required.

Let the system consist of two plane-parallel mirrors with a large reflection coefficient, located at $z = 0$ (mirror 1) and $z = L$ (mirror 2). An active medium with refractive index μ_1 fills the region $0 \leq z < l$ (zone 1), and an inactive medium with refractive index μ_2 fills the region $l \leq z \leq L$ (zone 2).

The consideration will be carried out in the axial (with respect to z) approximation. In this one-dimensional approximation we assume that plane monochromatic waves propagate inside the resonator along the z axis. The boundary conditions at $z = 0$ and $z = L$ consist in the requirement that the electric vector of the wave be equal to zero; in addition, at $z = l$ the condition of continuity of the tangential component of the electric and magnetic fields must be satisfied. This leads to the equations:

$$\operatorname{tg} 2\pi\mu_1\nu_i l = -\frac{\mu_1}{\mu_2} \operatorname{tg} 2\pi\mu_1\nu_i l\delta,$$

$$\frac{a_i}{b_i} = \frac{\sin 2\pi\mu_1\nu_i l}{\sin 2\pi\mu_1\nu_i l\delta}, \qquad \delta = \frac{\mu_2}{\mu_1}\frac{L-l}{l}; \tag{1}$$

here ν_i is the oscillation frequency of the i-th mode; b_i, a_i are the amplitudes of the electric vector of the standing wave in zones 1 and 2.

The first equation (1) determines the frequency of the i-th axial mode. This transcendental equation cannot be solved for ν_i; therefore, for estimates, we replace the coefficient before the tangent on the right-hand side by -1; in this approximation

$$\nu_i = i\delta\nu, \qquad \delta\nu = \delta\nu_0/(1+\delta); \qquad \delta\nu_0 = 1/2l\mu_1 \qquad i = 1, 2, ... \tag{2}$$

Using equations (1) and (2), one can obtain the ratio of the volume energy densities in zones 2 and 1, or the ratio of N_{i2}, the number of photons of the i-th mode in zone 2, to N_{i1}, the number of photons in zone 1:

$$\frac{\mu_2^2 a_i^2}{\mu_1^2 b_i^2} = \frac{N_{i2}}{N_{i1}} \frac{l}{L-l} = \frac{\mu_1^2 + \mu_2^2}{2\mu_1^2} \left(1 + \frac{\mu_1^2 - \mu_2^2}{\mu_1^2 + \mu_2^2} \cos \frac{2\pi i}{1+\delta}\right). \tag{3}$$

As is seen, the ratio of the energy densities in zones 1 and 2 depends substantially on the axial index i of the mode. Since the number of photons appearing as a result of induced radiation is $\sim N_{i1}$, while the losses in zone 1 are $\sim N_{i1}$ and in zone 2 $\sim N_{i2}$, the losses, referred to one photon in the active zone, will be different for different i.

The conditions for a mode to enter oscillation depend substantially on ψ_i—the photon losses per unit time in the i-th mode. In the case under consideration

$$\psi_i = (\gamma_0^I + \gamma_3^I)\frac{N_{i1}}{l} + (\gamma_0^{\text{II}} + \gamma_3^{\text{II}})\frac{N_{i2}}{L-l};$$

here γ_3^I and γ_3^{II} are associated with losses upon reflection from mirrors 1 and 2; γ_0^I and γ_0^{II} take into account all the remaining losses. Using (3), we obtain

$$\psi_i = \gamma_i N_{i1} = \gamma^0 \left(1 + \theta \cos \frac{2\pi i}{1+\delta}\right) N_{i1},$$

$$\gamma^0 = \frac{1}{l}\left(\gamma_0^I + \gamma_3^I + \gamma_0^{\text{II}} + \gamma_3^{\text{II}}\right)\left(1 - \frac{\mu_1^2 - \mu_2^2}{2\mu_1^2}\chi\right),$$

$$\chi = \frac{\gamma_0^{\text{II}} + \gamma_3^{\text{II}}}{\gamma_0^I + \gamma_3^I + \gamma_0^{\text{II}} + \gamma_3^{\text{II}}}, \qquad \theta = \frac{\chi(\mu_1^2 - \mu_2^2)}{2\mu_1^2 - \chi(\mu_1^2 - \mu_2^2)}. \tag{4}$$

Thus, for $\mu_1 = \mu_2$ the relative losses γ_i do not depend on i; for $\mu_1 \neq \mu_2$, the relative losses depend on i the more strongly, the larger θ is, i.e., the larger the fraction χ of the losses that falls on zone 2. The quantity θ is not difficult to estimate: let $\mu_1^2 \simeq 3$ (ruby), $\mu_2^2 = 1$; if all losses are associated with zone 2, then $\chi = 1$, and hence the maximum value of θ will be $\theta_{\max} = 0.5$; if the losses in zone 2 amount to 0.03 of the total losses, then $\theta = 0.01$.

In the case considered, the equations determining the number of photons in the different modes will be ([1]):

$$\frac{dn}{dt} = \frac{n_0 - n}{\tau_1} - 2nD \sum_k g_{kN_{k1}} \Phi_k, \tag{5}$$

$$\left[1 + \frac{L-l}{l} \frac{\mu_1^2 + \mu_2^2}{2\mu_1^2} \left(1 + \frac{\mu_1^2 - \mu_2^2}{\mu_1^2 + \mu_2^2} \cos \frac{2\pi i}{1+\delta}\right)\right] \frac{dN_{i1}}{dt} =$$

$$= \left[-\gamma^0 \left(1 + \theta \cos \frac{2\pi i}{1+\delta}\right) + Dg_i \int_0^l n\Phi_i \, dz\right] N_{i1};$$

here n is the density (per unit length z) of the inverted population of active centers responsible for the working transition; n_0 is the density of the inverted population at the given pumping in the absence of induced radiation; τ_1 is the quantity (2), close to the spontaneous decay time of the active center if the pump energy is close to threshold; $Dg_i\Phi_i$ is the probability (per unit time) of spontaneous emission of a photon into the i-th mode by an excited active center located at the point z; $\Phi_i = 1 - \cos \frac{2\pi i}{1+\delta} \frac{z}{l}$ is a quantity characterizing the energy distribution of the standing wave; the summation is carried out over the modes that have entered oscillation.

Solving system (5) for the nonstationary case is difficult. To estimate the influence of losses on the spectral composition of the induced radiation, let us consider stationary stable solutions of (5), i.e., solutions for which $\dot{n} = 0$, $\dot{N}_{i1} = 0$ for modes that have entered oscillation, and $\dot{N}_{i1} < 0$ for the remaining modes. Stationary solutions of equations (5) without taking into account the dependence of the losses on the mode number were obtained in ([1,2]); it turns out that in this case, as the pump is increased, the axial modes enter oscillation one after another, beginning with the mode located closest to the center of the luminescence line.

To estimate the order in which the modes enter oscillation, we shall restrict ourselves to the approximation proposed in ([1]) and applicable only for weak pumping. Let the luminescence line responsible for the working transition possess-

has a homogeneous half-width Γ cm^{-1} and has a Lorentzian shape; then, assuming that the mode with axial index i_0 falls at the center of the luminescence line, using (2) we obtain:

$$g_i = g_{i_0}(1 + \beta\Delta i^2)^{-1}, \qquad \beta = (2\delta\nu/\Gamma)^2 = [l\mu_1\Gamma(1+\delta)]^{-2}, \qquad \Delta i = i - i_0.$$

In this case equations (5) lead to the system

$$\sum_k Q_k \left(1 + \frac{\varphi_{i-k}}{2}\right) = 1 - \frac{1}{\alpha}\left(1 + \theta \cos\frac{2\pi i}{1+\delta}\right)(1 + \beta\Delta i^2), \qquad (6)$$

where

$$Q_k = 2Dg_k\tau_1 N_{k1}, \qquad d = \frac{Dg_{i_0} n_0 l}{\gamma^0}, \qquad \varphi_{i-k} = \frac{\sin 2\pi(i-k)/(1+\delta)}{2\pi(i-k)/(1+\delta)}.$$

Using system (6), let us determine which modes, as the pumping is increased, will first begin oscillating. The order in which the modes begin oscillating will depend on the value of $\cos 2\pi i_0/(1+\delta)$. In order not to complicate the discussion, we shall examine two cases: a) $\cos 2\pi i_0/(1+\delta) = 1$, i.e., the central mode i_0 is under unfavorable conditions with respect to losses, and b) $\cos 2\pi i_0/(1+\delta) = -1$, i.e., the central mode has minimum losses.

Since equations (6) are even with respect to a change of sign of Δi, in case a), depending on the ratio of the parameters δ, β, θ, either one mode with $\Delta i = 0$ will first begin oscillating, or two modes $\pm\Delta i_m$ at once, separated by the frequency interval $2\Delta i_m \delta\nu$. The value Δi_m is determined by the condition that, at $\Delta i = \Delta i_m$, the expression $(1 + \theta\cos 2\pi\Delta i/(1+\delta))(1+\beta\Delta i^2)$ should have the smallest value. In case b), the central mode always begins oscillating first ($\Delta i = 0$); the next to appear may be either the neighboring modes ($\Delta i_2 = \pm 1$), or modes lying far from the central one. The value Δi_2 is determined by the condition that the expression

$$\left[2\theta\sin^2\frac{\pi\Delta i}{1+\delta} + \beta\Delta i^2\left(1 - \theta\cos\frac{2\pi\Delta i}{1+\delta}\right)\right](1-\varphi_{\Delta i})^{-1}$$

should have a minimum at $\Delta i = \Delta i_2$.

Let us consider several specific examples (assuming $\theta \ll 1$).

1) Let $\delta \ll 1$, i.e., mirror 2 is located close to the end face. Then in case a) $\Delta i_m = 0$ for $\chi < \dfrac{3}{2\pi^2}\dfrac{\beta}{\delta^2}$; in case b) $\Delta i_2 = \pm 1$ for $\chi < \dfrac{3}{2\pi^2}\dfrac{\beta}{\delta^4}$, and $\Delta i_2 = \pm\dfrac{1}{\delta}$ for $\chi > \dfrac{3}{2\pi^2}\dfrac{\beta}{\delta^4}$.
2) Let $1+\delta = 2(1+1/k)^{-1}$. Then in case a) (k even): $\Delta i_m = 0$ for $\chi < {}^3/_2\beta$; $\Delta i_m = \pm 1$ for ${}^3/_2\beta < \chi < \dfrac{6k^4}{\pi^2}\beta$; $\Delta i_m = \pm k$ for $\chi > \dfrac{6k^4}{\pi^2}\beta$; in case b) ($k > 5$, k odd): $\Delta i_2 = \pm 1$ for $\chi < {}^9/_2\beta$; $\Delta i_2 = \pm 2$ for ${}^9/_2\beta < \chi < \dfrac{3k^4}{2\pi^2}\beta$; $\Delta i_2 = \pm k$ for $\chi > \dfrac{3k^4}{2\pi^2}\beta$.
3) Let $l = 12$ cm, $L = 50$ cm, $\mu_1 = 1.76$ (ruby), $\mu_2 = 1$, $\beta = 3\cdot 10^6$, $1+\delta = 3(1+{}^1/_{14})^{-1}$. In case a): $\Delta i_m = 0$ for $\chi < 6\cdot 10^{-6}$; $\Delta i_m = \pm 1$ for

$6 \cdot 10^{-6} < \chi < 5 \cdot 10^{-4}$; $\Delta i_m = \pm 4$ for $5 \cdot 10^{-4} < \chi < 3 \cdot 10^{-3}$; $\Delta i_m = \pm 7$ for $\chi > 3 \cdot 10^{-3}$; in case b): $\Delta i_2 = \pm 1$ for $\chi < 3 \cdot 10^{-5}$; $\Delta i_2 = \pm 3$ for $3 \cdot 10^{-5} < \chi < 1.7 \cdot 10^{-2}$; $\Delta i_2 = \pm 14$ for $\chi > 1.7 \cdot 10^{-2}$.

4) Let $1 + \delta = n \gg 1$ (n an integer). Then in case a): $\Delta i_m = 0$ for $\chi < \frac{3}{2\pi^2}(\mu_1 l \Gamma)^{-2}$; $\Delta i_m = \pm n/2$ for $\chi > {}^3/_8(\mu_1 l \Gamma)^{-2}$; in case b): $\Delta i_2 = \pm 1$ for $\chi < (\mu_1 l \Gamma)^{-2}$; $\Delta i_2 = \pm n$ for $\chi > (\mu_1 l \Gamma)^{-2}$. Thus, despite the fact that in case 4) $L \gg l$ and the spacing between the frequencies of neighboring modes is $\delta\nu$, a frequency period $\delta\nu_0$ may be observed in the spectrum of the induced radiation, associated not with the resonator length L, but with the length of the active zone l.

As we see, depending on the ratio of the parameters δ, β, χ, modes whose axial indices differ greatly may successively begin oscillating.

Although the sequence $\{\Delta i\}$ for the transition of axial modes into generation at high pump powers has not been calculated in the present work, one may expect that, to a rough approximation, $\{\Delta i\}$ is determined by the fact that in this case the sequence $\{\gamma_i / g_i\}$ increases monotonically, starting from the smallest value. For example, for case 3b) this sequence ($\theta = 0, 1$) will be $\{\Delta i\} = 0, 14, 28, 3, 11, 17, 25, 42, 31, ...$

Because formula (2) is approximate, all the results presented are rather in the nature of estimates, and in a numerical solution of equations (1) (an analytic solution is possible only for a few values of δ) the values of $\Delta i_m, \Delta i_r$ given above may change somewhat; however, the character of the spectrum will remain the same.

If, during the generation process, the active substance is heated, then, as a result of changes in l, μ_1, ν_i, the losses of the i-th mode will also change.

If the active zone does not adjoin the mirror and moves inside the resonator in the z direction with velocity v, then the frequencies and losses of the i-th mode will vary periodically with frequency $f_i = 2v/\lambda_i$; with this same frequency the intensity of the induced radiation will be modulated (for $v = 50$ cm/sec, $f_i \simeq 1.4$ MHz for ruby).

Inhomogeneity of the active zone or nonparallelism of the boundaries at different points of the cross section will lead to different relative losses across the section, and this may complicate the transverse structure of the mode. In the case of spherical mirrors and plane boundaries of the active zone, one should expect that the effect considered above will complicate the transverse structure of the mode the more, the greater the angular divergence of the beam.

The dispersive properties of a resonator with a dielectric boundary considered above can be suppressed by antireflection coating of the end faces of the active zone.

The author considers it a pleasant duty to express gratitude to Academician I.

V. Obreimov for his attention to and interest in the work, and to B. L. Livshits and V. N. Tsikunov for discussion.

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Received
21 January 1966

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.