TURING MACHINES WORKING ON A PLANE

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Abstract

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In this note generalized Turing machines are considered. They differ from ordinary ones in only one respect: instead of a tape, these machines use for writing a plane divided into squares. Along this plane the machine can move both horizontally and vertically, passing at each step from the square being scanned to one of the four squares adjacent to it. Let us denote by $\xi(t), \eta(t)$ the coordinates of the square being scanned at time t, and by $\Delta_\alpha(t)$ the contents of the square α at time t.

We shall restrict ourselves here to nonerasing machines (see ([1])). This means that a partial order is given on the external alphabet and the machine is constructed so that, for all α (and independently of the initial configuration), one always has $\Delta_\alpha(t) \leqslant \Delta_\alpha(t+1)$. The machines considered here may have different external alphabets, ordered in different ways; however, we shall assume that the set of minimal (respectively maximal) symbols of the external alphabet is the same for all machines. Let us denote these two sets respectively by A and B; the machines under consideration will be called (A, B)-machines.

The capabilities of (A, B)-machines will be studied here as applied to the problem of computing partial recursive operators $Z(t) = T\{x(t)\}$ having the following property. If the values of the function $x(t)$ belong to the alphabet A, then the values of the function $Z(t)$ belong to the alphabet B. Such operators will be called (A, B)-operators. We shall assume that the "input" function $x(t)$ is everywhere defined. The computation of such an operator on an (A, B)-machine will be understood in the following sense. Initially, in the squares adjacent to the axis of abscissas, the values of the input function are entered, so that $X(\nu)$ is contained in the square with coordinates $(\nu, 0)$. In this same square, in the course of the machine' s operation, the value $Z(\nu)$ must appear, if the latter is defined. The remaining squares are used by the machine for intermediate computations.

This definition is a simple generalization of the definition of computability of operators considered in ([1]) for the case of ordinary nonerasing Turing machines. However, on machines with a "one-dimensional" tape it is impossible to compute without erasing all partial recursive operators; therefore in ([1]) the investigation was restricted to general recursive operators. At the same time, by modifying the construction of Hao Wang ([2]), one can show that on the "planar"Turing machines considered here all partial recursive operators are already computable without erasing. However, this construction does not make it possible to estimate what part of the plane will actually be used in the computation. Since the latter question is of independent interest, we shall introduce a more stringent concept of computability.

Let $\varphi(x)$ be a nondecreasing function. We shall say that the given machine φ **computes the given operator** if it computes it in the sense described above, and moreover in such a way that at every moment the inequality $\eta(t) \leqslant \varphi(\xi(t))$ holds. In other words, the computation is confined to the plane strip under the graph of $\varphi(x)$ (i.e., in fact, by a tape of pos-

of increasing width). We shall call a function φ a **universal bound** if every (A, B)-operator is φ-computable by means of a suitable (A, B)-machine. It can be shown that the function $\varphi(x) = x$ is a universal bound and that no constant is such. The purpose of the present note is to estimate how slowly φ can grow while still remaining a universal bound. Let us note at once that if φ is a universal bound, then $[\varphi/k] + 1$ also has this property, where k is any constant. Therefore it makes sense to estimate only the order of growth admissible for universal bounds. The following results have been obtained.

Theorem 1. *The function* $\log x$ *is a universal bound.*

Theorem 2. *If* φ *has a smaller order of growth than* $\log X / \log\log X$*, then* φ *is not a universal bound.*

In view of the remark made above, the choice of the base of the logarithm is immaterial.

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2. H. Wang, *J. Assoc. Comp. Mach.*, 4, No. 1 (1957).

Note: Figure translations are in progress. See original paper for figures.