ON THE THEORY OF INERTIAL DEPOSITION OF PARTICLES FROM A GAS OR LIQUID

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Abstract

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HYDROMECHANICS

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ON THE THEORY OF INERTIAL DEPOSITION OF PARTICLES FROM A GAS OR LIQUID*

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Let us consider the most idealized stationary motion of aerosol particles, described by the equation

$$k\frac{d\mathbf{V}}{dt} + \mathbf{V} = \mathbf{W} + \mathbf{F}, \quad (1)$$

where k is the Stokes number; $\mathbf{V}$ and $\mathbf{W}$ are the velocity vectors of the motion of the aerosol particles and of the air carrying them, referred to the value of the air velocity at infinity u_∞.

The quantity t is the ratio of time to the time of passage, with velocity u_∞, of the characteristic length l, which is the size of the obstacle. By $\mathbf{F}$ is also meant, in dimensionless form, the sum of the forces of gravity and other forces, for example electrostatic forces ([1]). The number concentration of aerosol particles n is considered as the density of a pseudo-liquid, and the condition of conservation of aerosol particles can be written in the form of the continuity equation

$$\operatorname{div} n\mathbf{V} = 0. \quad (2)$$

If the right-hand side of equation (1) has a potential

$$\mathbf{W} = \operatorname{grad}\varphi, \qquad \mathbf{F} = -\operatorname{grad} U, \quad (3)$$

then, according to A. M. Yaglom and A. Robinson ([1,2]),

$$\operatorname{rot}\mathbf{V} = 0, \quad (4)$$

and equation (1) has the first integral

$$\frac{k}{2}\left[\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial y}\right)^2+\left(\frac{\partial\Phi}{\partial z}\right)^2\right]+\Phi=\varphi-U \qquad (\operatorname{grad}\Phi=\mathbf{V}). \tag{5}$$

We shall try to construct an analytical method for determining the capture coefficient on the basis of equations (2) and (5). From equation (2) there follows the existence, for plane and spatial axisymmetric flows, of a stream function Ψ. For plane flows of the pseudo-liquid, equations (2), (4) in the functions Φ, Ψ can be written in the form

$$n\frac{\partial\Phi}{\partial x}=\frac{\partial\Psi}{\partial y}, \qquad n\frac{\partial\Phi}{\partial y}=-\frac{\partial\Psi}{\partial x}. \tag{6}$$

In the case of a spatial axisymmetric flow the system

$$rn\frac{\partial\Phi}{\partial x}=\frac{\partial\Psi}{\partial r}, \qquad rn\frac{\partial\Phi}{\partial r}=-\frac{\partial\Psi}{\partial x}. \tag{7}$$

is valid.

Let at infinity there be an undisturbed flow parallel to the x-axis and let $\mathbf{F}=0$, $\mathbf{V}_\infty=\mathbf{W}_\infty=1$. In plane motion, for the portion of the body captured between the streamlines $\Psi_1=n_\infty y_{\infty 1}$ and $\Psi_2=n_\infty y_{\infty 2}$, we have for the capture coefficient E

$$E=\frac{y_{\infty 1}-y_{\infty 2}}{y_1-y_2}=\frac{\Psi_1(x_1,y_1)-\Psi_2(x_2,y_2)}{n_\infty(y_1-y_2)}, \tag{8}$$

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where (x_1,y_1), (x_2,y_2) are the coordinates of the points of intersection of the streamlines Ψ_1 and Ψ_2 with the collecting body. When calculating the total coefficient E in formula (8), (x_1,y_1), (x_2,y_2) are understood to be the coordinates of the upper and lower points of the collecting body. In the case of a flow symmetric with respect to the x-axis, formula (8) has the form $E=\Psi(x,y)/n_\infty y$.

The local capture coefficient E_l in the neighborhood of the axis of symmetry is equal to

$$E_l=\frac{1}{n_\infty}\left(\frac{\partial\Psi}{\partial y}\right)_{y=0}.$$

For spatial axisymmetric flows,

$$E = \frac{r_\infty^2}{r^2} = \frac{2\Psi}{n_\infty r^2}. \tag{9}$$

Equation (5) can be represented in the form

$$k\frac{\partial\Phi'}{\partial x} + \Phi' - \varphi' = -\frac{k}{2}\left[\left(\frac{\partial\Phi'}{\partial x}\right)^2 + \left(\frac{\partial\Phi'}{\partial y}\right)^2 + \left(\frac{\partial\Phi'}{\partial z}\right)^2\right], \tag{10}$$

where

$$\Phi' = \Phi - x, \qquad \varphi' = \varphi - x - k/2 \tag{11}$$

are the perturbation potentials. For large values of the Stokes number k, $\partial\Phi'/\partial x$, $\partial\Phi'/\partial y$, $\partial\Phi'/\partial z \ll 1$, and then it is natural, as a first approximation $\Phi' = \Phi_1'$, to solve the equation

$$k\,\partial\Phi_1'/\partial x + \Phi_1' - \varphi' = 0. \tag{12}$$

Hence

$$\Phi_1' = k^{-1}\exp\left(-\frac{x}{k}\right)\int_{-\infty}^{x}\exp\left(\frac{x}{k}\right)\varphi'(x,y,z)\,dx. \tag{13}$$

As $|x|+|y|+|z| \to \infty$, derivatives of all orders of Φ_1' tend to zero. At sufficiently small velocities of air motion, φ satisfies Laplace' s equation. It is not difficult to verify that then Φ_1' also satisfies Laplace' s equation. Approximation (12) corresponds to a constant numerical concentration n. Indeed, from equation (2) in approximation (12) it follows that $\operatorname{grad} n \cdot \operatorname{grad}\Phi_1 = 0$ ($\Phi_1 = x + \Phi_1'$), i.e., n is constant along each streamline and, consequently, taking into account that at infinity $n = n_\infty = \text{const}$, it is constant throughout the entire region of the pseudoliquid flow. Conversely, from the admissibility of $n = \text{const}$ follows the negligibility of the right-hand side of equation (10), which does not satisfy Laplace' s equation, since the left-hand side for $n = \text{const}$ satisfies Laplace' s equation. In the approximation $\Phi' = \Phi_1'$, i.e. $n = n_\infty = \text{const}$, from equations (6), (7), (12) it follows that

$$k\,\partial\Psi_1'/\partial x + \Psi_1' - n_\infty\psi' = 0, \tag{14}$$

$$\Psi_1' = \frac{n_\infty}{k}\exp\left(-\frac{x}{k}\right)\int_{-\infty}^{x}\exp\left(\frac{x}{k}\right)\psi'\,dx, \tag{15}$$

where $\psi' = \psi - n_\infty y$, $\Psi_1' = \Psi_1 - n_\infty y$ for plane motion, $\psi' = \psi - \frac{1}{2}r^2$, $\Psi_1' = \Psi_1 - \frac{n_\infty}{2}r^2$ for spatial axisymmetric motion; ψ is the stream function of the air; Ψ_1 is the stream function of the pseudoliquid for $n =$ const. For plane flows, the functions ψ' and Ψ_1' are harmonically conjugate to the functions φ' and Φ_1', respectively, and the solution can be represented in complex form as

$$W_1'(z) = k^{-1} \exp\left(-\frac{z}{k}\right) \int_{-\infty}^{z} \exp\left(\frac{z}{k}\right) w'(z)\, dz, \tag{16}$$

$$W_1'(z) = \Phi_1' + \frac{i}{n_\infty}\Psi_1', \qquad w'(z) = \varphi' + i\psi', \qquad z = x + iy.$$

The lower limit is understood in the sense $x = -\infty$, $y =$ const. For a cylinder $\psi' = -y/(x^2 + y^2)$ and, according to formulas (8), (15), in the first approximation E_1 the total capture coefficient is equal to

$$E_1 = 1 - \frac{1}{k}\int_0^\infty \frac{\exp(-s/k)}{1+s^2}\, ds. \tag{17}$$

Below, the values of E_1 are compared with the data of work ([3]), where, for the calculation of E, a system of ordinary differential equations was used.

k	100	50	20	10	5	3	2	1	0.5
E	0.98	0.97	0.94	0.89	0.78	0.67	0.57	0.37	0.17
E_1	0.98	0.97	0.93	0.87	0.77	0.67	0.57	0.38	0.21

For a plate with separationless flow past a neighborhood of the axis of symmetry, we obtain the following formula for the local E_{1l}:

$$E_{1l} = \frac{1}{k}\int_0^\infty \frac{s\exp(-s/k)}{\sqrt{1+s^2}}\, ds, \tag{18}$$

which can be computed with the aid of tables of functions ([4,5]). Below, for several values of k, (18) is compared with E_l from ([1]).

k	10	2.5	1	0.5
E_l	0.91	0.71	0.51	0.27
E_{1l}	0.91	0.74	0.54	0.37

The comparisons presented indicate that the linear approximation gives satisfactory accuracy for E down to rather small values of k.

In order to refine the linear theory and compute n for small $k > k_{\text{cr}}$, where k_{cr} is the critical Stokes number, let us note that from the continuity equation or the system of equations (6), (7) it follows that

$$\ln \frac{n}{n_\infty} = -\int_{-\infty}^{x} \left[\frac{\Delta\Phi}{\Phi_x}\right]_{\Psi=\text{const}} dx. \tag{19}$$

In the spatial axisymmetric case

$$\Delta\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial r^2} + \frac{1}{r}\frac{\partial\Phi}{\partial r}.$$

From the systems of equations (6), (7) we also have

$$\operatorname{grad}\Phi \cdot \operatorname{grad}\Psi = 0, \tag{20}$$

and, if Φ and Ψ are represented in the form

$$\Phi = \Phi_1 + \Phi', \qquad \Psi = \Psi_1 + \Psi_2', \tag{21}$$

where $\Phi_1 = x + \Phi_1'$, $\Psi_1 = n_\infty y + \Psi_1'$, or $\Psi_1 = n_\infty r^2/2 + \Psi_1'$, and the unknown Φ' and Ψ' supplement Φ_1 and Ψ_1 to the exact solution, then from (20) we obtain

$$\Psi' = -\int_{-\infty}^{x} \left[\frac{\operatorname{grad}\Phi' \cdot \operatorname{grad}\Psi_1}{\Phi_x}\right]_{\Psi=\text{const}} dx. \tag{22}$$

Where the linear theory is the main part of the solution, the integrals (19), (22) can be calculated approximately by setting $\Phi_x \approx \Phi_{2x}$, $\Phi' \approx \Phi_2' - \Phi_1'$, where Φ_2' is the solution of equation (10) in the second approximation

$$k\frac{\partial\Phi_2'}{\partial x} + \Phi_2' - \varphi' = -\frac{k}{2}\left[\left(\frac{\partial\Phi_1'}{\partial x}\right)^2 + \left(\frac{\partial\Phi_1'}{\partial y}\right)^2 + \left(\frac{\partial\Phi_1'}{\partial z}\right)^2\right],$$

and integrate (19) and (22) with respect to x along $\Psi_1 = \text{const}$. The calculation is still simpler in the hydraulic approximation along $\Psi_1 \approx n_\infty y \Phi_{1x}(x,0) \approx \text{const}$, or

$$\Psi_1 \approx \frac{n_\infty r^2}{2}\Phi_{1x}(x,0) \approx \text{const}$$

for a jet of a definite width containing the axis of symmetry. The first rough estimates confirm a small variation of n within the limits of satisfactory accuracy of the linear approximation. Thus, for $k = 1$ on the axis of symmetry of the plate, $n/n_\infty \approx 1.1$. For k close to k_{cr}, n varies more strongly; however, this variation occurs near the capture body, and this circumstance can be used to simplify the calculation of the integrals (19), (22). For example, the variation of n along the axis of symmetry at different distances from the plate for $k = 0.5$ will be as follows:

x	0	−0.25	−0.5	−0.75	−1
n/n_∞	1.26	1.10	1.06	1.02	1.01

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Note: Figure translations are in progress. See original paper for figures.