Distribution of Surface Brightness in the Head of a Comet with Allowance for the Dispersion of Initial Velocities

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Abstract

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D. O. Mokhnach

Distribution of Surface Brightness in the Head of a Comet with Allowance for the Dispersion of Initial Velocities

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In constructing the simplest model of a comet's head, let us take a cometocentric rectangular coordinate system in which the axis Ox is directed toward the Sun, and the axis Oz is parallel to the line of sight. A particle is ejected at an angle $\pi/2-\alpha$ to the radius vector with initial velocity v and repulsive acceleration g, which may be regarded as constant for small changes in the heliocentric distance of the comet. The equations of motion in this case are

$$x = vt\sin\alpha - \frac{1}{2}gt^2; \qquad y = vt\cos\alpha\cos\varphi; \qquad z = vt\cos\alpha\sin\varphi \tag{1}$$

(φ is the azimuth of the particle).

Particles ejected in a certain direction within the solid angle $d\omega$ during the time dt will occupy the volume dV. If ρ is the number of particles per unit volume, and n is the emission coefficient, i.e., the number of particles ejected per unit solid angle in the given direction per unit time, then

$$\rho\, dV = n\, d\omega\, dt \tag{2}$$

or

$$\rho\, dV = n\cos\alpha\, d\alpha\, d\varphi\, dt, \tag{2'}$$

since $d\omega = \cos\alpha\, d\alpha\, d\varphi$.

Computing, from the equations of motion, the Jacobian $J = D(x,y,z)/D(\alpha,\varphi,t)$, we obtain $dV = J\, d\alpha\, d\varphi\, dt$ and $\rho(\alpha,\varphi,t) = n\cos\alpha/J$; whence

$$\rho(x,y,z) = \frac{n}{vt^2\left(v^2 - gx - \frac{1}{2}g^2t^2\right)},$$

since t is determined from the equation

$$\frac{1}{4}g^2t^4 - (v^2 - gx)t^2 + x^2 + y^2 + z^2 = 0.$$

For constant v and g, the envelope of all particle trajectories is a paraboloid of revolution with vertex $x_0 = v^2/2g$ and parameter $y_0 = v^2/g$.

The visible density $N(x, y)$—the number of particles in a cylinder with base area 1 sq. unit and axis coinciding with the line of sight, at the point (x, y) of the picture plane—can be computed from the formula

$$N(x, y) = \int_{z_1}^{z_2} \rho(x, y, z)\, dz \tag{3}$$

(z_1 and z_2 are the applicates of the points of intersection of the line of sight with the enveloping paraboloid).

It is often convenient to pass to the integration variable t. In this case

$$N(x, y) = \frac{4n}{vg} \int_{t_1}^{t_2} \frac{dt}{t\sqrt{(t_2^2 - t^2)(t^2 - t_1^2)}} =$$

$$= \frac{2n}{v\sqrt{x^2 + y^2}} \operatorname{arctg} \frac{t_2}{t_1} \sqrt{\frac{t^2 - t_1^2}{t_2^2 - t^2}}\Bigg|_{t_1}^{t_2} = \frac{n\pi}{v\sqrt{x^2 + y^2}}; \tag{4}$$

t_1 and t_2—the smallest and largest values of the time interval t needed for the particles to reach the point $(x, y, 0)$—can be computed from the formula

$$t_{1,2}^2 = 2\left(v^2 - gx \pm \sqrt{v^4 - 2v^2gx - g^2y^2}\right)/g^2.$$

In calculating nonstationary states it is sometimes necessary to take the upper limit $t < t_2$, while the emission coefficient is $n = n(t)$.

Allowance for the dispersion of initial velocities. The generally accepted view is that the luminous molecules forming the head of a comet cannot be emitted directly from the cometary nucleus, but are the product of the decay of more complex parent molecules. The latter, having weak bonds and consequently a short lifetime with respect to dissociation, decay near the nucleus in regions of high density. Therefore collisions of daughter molecules during their formation and the establishment of a Maxwellian distribution of initial velocities seem quite natural. The emission coefficient is $n = n(v)$, and the visible density must be calculated from the formula

$$N(x,y) = \frac{16n}{v_0 g\sqrt{\pi}} \int_{v_1}^{\infty} \int_{t_1}^{t_2} \frac{e^{-(v/v_0)^2}(v/v_0)^2\,dv\,dt}{v\sqrt{(t_2^2 - t^2)(t^2 - t_1^2)}},$$

where v_0 is the most probable velocity, and v_1 is the smallest velocity at which a particle reaches the point (x, y) of the picture plane. For the stationary model

$$N(x,y) = \frac{2n\sqrt{\pi}}{v_0\sqrt{x^2 + y^2}} e^{-(v_1/v_0)^2} \tag{5}$$

(n is the emission coefficient for molecules with velocities $v \geq v_1$). If $v_1 \ll v_0$, $N(x,y) \sim 1/R$.

Distribution laws of visible density for different values of the most probable velocity. At present there are data on the distribution of surface brightness in CN and C_2 emissions for some comets. It is known, for example, that in the carbon coma the $1/R$ law is not obeyed for circular isophotes. To determine whether dispersion of initial velocities is the cause of the deviation from the inverse-distance law, $N(x,y)$ was computed for various values of v_0 and g. The smallest velocity v_1 was calculated from the formulas $v_1^2 = gy$ for $x = 0$ and $v_1^2 = 2gx$ for $y = 0$. Table 1 gives the data for C_2 at $g = 0.36$ cm/sec^2.

Table 1

	0.05	0.10	0.15	0.2	0.4	0.6	0.8	1.0	$1.6 \cdot 10^{10}$ cm	
$N(0,y)$	17.2	7.4	4.1	2.6	0.62	0.18	0.06	0.02	0.001	$v_0 = 0.4 \cdot 10^5$ cm/sec
$N(0,y)$	18.2	8.3	5.1	3.35	1.05	0.43	0.20	0.10	0.001	$v_0 = 0.5 \cdot 10^5$
$N(0,y)$	18.8	8.8	5.5	3.8	1.4	0.68	0.38	0.21	0.046	$v_0 = 0.6 \cdot 10^5$
$N(0,y)$	19.8	9.8	6.3	4.6	2.2	1.4	0.94	0.70	0.36	$v_0 = 1.2 \cdot 10^5$

On the basis of Table 1 one may suppose that the law of distribution of visible density along the Oy axis, discovered by F. Miller ([1]), is explained by the dispersion of initial velocities for the value $v_0 = 0.5 \cdot 10^5$ cm/sec. The inverse-distance law with deviations smaller than the usual errors of measurement is obtained for $v_0 = 3 \cdot 10^5$ cm/sec.

It is necessary to note the appreciable asymmetry of the isophotes with respect to the photometric center at small values of v_0.

F. Miller speaks of circular isophotes in comet 1959 k. Table 2 shows that only for a value of the most probable velocity $v_0 = 3 \cdot 10^5$ cm/sec (or larger) can circular isophotes be expected, but in this case the velocity dispersion will not change the $1/R$ law obtained in the pred-

the assumption of a single initial velocity of the molecules in the simplest stationary model. If one tries to explain the $1/R^2$ law along the Oy axis by a Maxwellian velocity distribution, one must note a sharp deviation from the circular form of the isophotes already at a distance of 20,000–30,000 km, which is not confirmed by observations.

Table 2

	x	$1 \cdot 10^{10}$	0.5	0.3	0.2	-0.5	-1.0	-5.0	
$N(x, O)$		0.007	0.19	0.86	2.1	2.0	1.0	0.2	$v_0 = 0.5 \cdot 10^5$
$N(x, O)$		0.5	1.4			2.0	1.0		$v_0 = 1.2 \cdot 10^5$
$N(x, O)$		0.9	1.9			2.0	1.0		$v_0 = 3 \cdot 10^5$

Analogous calculations were carried out for different values of the most probable velocities v_0 at $g = 0.04$, which corresponds to the acceleration of CN molecules at a distance of 1.2 AU. At this distance from the Sun, B. A. Vorontsov-Vel' yaminov ([2]) observed the comet of 1942 and found that the $1/R$ distribution law is obeyed in the direction toward the Sun at distances greater than 10^6 km. With a Maxwellian velocity distribution, even along the Oy axis, for $v_0 \leq 1.2 \cdot 10^5$ cm/sec, deviations from the $1/R$ law are noticeable at a distance of 500,000 km (an error of $\sim 30\%$), and in the direction toward the Sun the deviations are still larger. For the inverse-distance law to be obeyed up to $R = 10^6$ km, in this case as well a value $v_0 \sim 3 \cdot 10^5$ cm/sec is necessary.

The bow-shaped form of the head of the 1942 comet, which B. A. Vorontsov-Vel' yaminov noted in photographs obtained in integral light, can apparently be explained by the superposition on the image of the cyanogen coma of CO^+ emission, concentrated mainly in the part of the head opposite the direction toward the Sun.

On the combined influence of velocity dispersion and molecular dissociation on the distribution of visible density. An attempt to explain

F. Miller' s distribution law by dissociation of C_2 molecules leads to the following conclusions: 1) if $v_0\tau \approx 10^{10}$ cm (τ is the lifetime before dissociation, v_0 the initial velocity), then along the Oy axis the distribution of visible density coincides with the observed one; 2) for $v_0 < 3 \cdot 10^5$ cm/sec, an asymmetry of the isophotes along the direction toward the Sun is noticeable; for $v_0 \geq 3 \cdot 10^5$ cm/sec, the isophotes are practically circular and symmetric with respect to the photometric center; 3) for $v_0\tau < 10^{10}$, the visible density decreases faster than observed, and for $v_0\tau > 10^{10}$, more slowly. If one assumes a Maxwellian velocity distribution and takes the most probable velocity to be $3 \cdot 10^5$ cm/sec, then calculations show that the distribution law of the visible density does not differ from that obtained under the assumption of a single velocity. This has a simple explanation. Let us consider molecules with velocities $v_1 = 2 \cdot 10^5$ cm/sec and $v_2 = 4 \cdot 10^5$ cm/sec (the values v_1 and v_2 are symmetric with respect to v_0). The number of molecules with such velocities is practically the same, i.e. $n(v_1) = n(v_2)$. For $v = v_1$ and $v = v_2$, taken separately, distribution laws are obtained that differ from the specified one, but the combined influence of these two groups of molecules is the same as for molecules with velocities $v = v_0$, i.e., to within a constant factor,

$$\frac{1}{2}(N(v_1) + N(v_2)) \approx N(v_0).$$

This was to be expected, since it was shown above that, in the stationary model and for $v_0 = 3 \cdot 10^5$ cm/sec, the distribution law $N = 1/R$ is preserved; that is, at this value of the most probable velocity the velocity dispersion may be neglected.

For $v_0 < 3 \cdot 10^5$ cm/sec, the velocity dispersion enhances the effect of isophote asymmetry obtained in photodissociation.

As shown by the author ([3,4]), the short lifetime of gas halos is explained by the fact that $N(R)$ in the brightest part of the halo decr–

is a function decreasing with time. If in this case the dispersion of initial velocities is taken into account, then the decrease of $N(R)$ will occur more rapidly than under the assumption of a single initial velocity.

Yu. N. Gnedin and A. Z. Dolginov showed theoretically [5] that the distribution of the apparent density in a neutral coma, similar to that observed along the axis Oy in comet 1959 k, can be explained by dissociation under the conditions $v_0\tau = 10^{10}$ cm and $gR/v_0^2 \ll 1$. The velocity dispersion in the Maxwell distribution is essential for explaining the change of the apparent density $N(R)$ at the periphery of the comet' s head.

As is evident from the calculations given above, the isophotes can be circular if $gR/v_0^2 \sim 0.05$ or less. From this one can find the smallest value of the most probable velocity $v_0 = 3 \cdot 10^5$ cm/sec, corresponding to a thermal velocity at $T = 13\,000^\circ$ K.

In conclusion it should be noted that the lack of observational material is hindering the development of the theory of cometary forms. Series of observations in monochromatic light are needed, which would make it possible to study cometary forms in their development.

Leningrad Agricultural
Institute

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.