PHOTOPRODUCTION OF MESONS ON NUCLEONS AND $SU(6)$ SYMMETRY

PHYSICS

1966

SovietRxiv

Abstract

Full Text

UDC 539.12

PHYSICS

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PHOTOPRODUCTION OF MESONS ON NUCLEONS AND $SU(6)$ SYMMETRY

In the present note, within the framework of $SU(6)$ symmetry, we investigate relations between the amplitudes for the production of mesons in a p-state in the interaction of magnetic-dipole γ-quanta with nucleons. In the case under consideration the photoproduction process is described by two partial amplitudes: one amplitude corresponds to the total angular momentum $I = \frac{1}{2}$ (M_{11}), the other to the total momentum $I = \frac{3}{2}$ (M_{13}). The spin structure of the amplitude for photoproduction of pseudoscalar mesons in the center-of-mass system, in terms of M_{11} and M_{13}, has the form ([1])

$$F_M = i(\vec{\sigma}\cdot\vec{e}\,\hat{\mathbf{k}}\cdot\hat{\mathbf{q}} - \vec{\sigma}\cdot\hat{\mathbf{k}}\,\vec{e}\cdot\hat{\mathbf{q}})(M_{13} - M_{11}) - \vec{e}\times\hat{\mathbf{k}}\cdot\hat{\mathbf{q}}(2M_{13} + M_{11}), \quad (1)$$

where $\vec{e}$ is the polarization vector of the γ-quantum; $\hat{\mathbf{k}}, \hat{\mathbf{q}}$ are unit vectors in the directions of the momenta of the γ-quantum and of the produced meson.

In $SU(6)$ symmetry a magnetic-dipole γ-quantum is described by a tensor of second rank ([2])

$$Q^A_{A'} \equiv Q^{ia}_{i'a'} = (\vec{\sigma}\cdot\vec{e}\times\hat{\mathbf{k}})^i_{i'}\, Q^a_{a'}, \qquad Q^a_{a'} = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (2)$$

Since the mesons are produced in a p-state, it is necessary to take into account the angular spurion ([3])

$$L^A_{A'} \equiv L^{ia}_{i'a'} = (\vec{\sigma}\cdot\mathbf{q})^i_{i'}\,\delta^a_{a'}, \quad (3)$$

where the Latin indices take the values $1, 2$, and the Greek indices $1, 2, 3$.

Then the number of independent parameters describing the process of meson photoproduction on nucleons upon absorption of a magnetic-dipole γ-quantum is determined by the number of common multiplets in the products 56×56 and $35 \times 35 \times 35$. Since

$$56 \times 56 = 1 + 35 + 405 + 2695,$$

from the product $35 \times 35 \times 35$ it is necessary to retain multiplets of dimensions $1, 35, 405, 2695$. For this purpose we note that

$$35 \times 35 \times 35 = (1 + 35_1 + 35_2 + 189 + 280 + 280^* + 405) \times 35.$$

Each of the products 189×35, 280×35, $280^* \times 35$, 405×35 contains the multiplet 35 once. Further, the products 280×35, $280^* \times 35$ contain the multiplet 405 once, and the product 405×35 contains the multiplet 405 twice. Thus, in the product $35 \times 35 \times 35$ there are contained 2 singlet multiplets, 9 multiplets 35, 6 multiplets 405, and one multiplet 2695. Therefore the photoproduction amplitude under consideration is determined by 18 parameters:

$$\begin{aligned} F_M = \bar{\psi}^{A'B'C'} \{ & \alpha \delta^A_{A'} \delta^B_{B'} (LQ)^D_D M^C_{C'} + \beta M^A_{A'} Q^B_{B'} L^C_{C'} \\ & + a_1 \delta^A_{A'} \delta^B_{B'} \delta^C_{C'} (LQ_d)^D_{D'} M^{D'}_D + a_2 \delta^B_{B'} \delta^C_{C'} [(LQ_d)^A_D M^D_{A'} + (LQ_d)^D_{A'} M^A_D] \\ & + a_3 \delta^B_{B'} \delta^C_{C'} [(LQ_d)^A_D M^D_{A'} - (LQ_d)^D_{A'} M^A_D] + a_4 \delta^C_{C'} (LQ_d)^A_{A'} M^B_{B'} + \end{aligned}$$

$$\begin{aligned} & + b_1 \delta^A_{A'} \delta^B_{B'} \delta^C_{C'} (LQ_f)^D_{D'} M^{D'}_D + b_2 \delta^B_{B'} \delta^C_{C'} \left[(LQ_f)^A_D M^D_{A'} + (LQ_f)^D_{A'} M^A_D \right] + \\ & + b_3 \delta^B_{B'} \delta^C_{C'} \left[(LQ_f)^A_D M^D_{A'} - (LQ_f)^D_{A'} M^A_D \right] + b_4 \delta^C_{C'} (LQ_f)^A_{A'} M^B_{B'} + \\ & + \delta^B_{B'} \delta^C_{C'} \left(c_1 R^{[AD]}_{[A'D']} M^{D'}_D + c_2 R^{AD}_{[A'D']} M^{D'}_D + c_3 R^{[AD]}_{A'D'} M^{D'}_D + c_4 R^{AD}_{A'D'} M^{D'}_D \right) + \\ & + d_1 \delta^C_{C'} \left(R^{AB}_{[A'D]} M^D_{B'} + R^{AB}_{[B'D]} M^D_{A'} \right) + d_2 \delta^C_{C'} \left(R^{[AD]}_{A'B'} M^B_D + R^{[BD]}_{A'B'} M^A_D \right) + \\ & + d_3 \delta^C_{C'} \left(R^{AB}_{A'D} M^D_{B'} + R^{AB}_{B'D} M^D_{A'} \right) + d_4 \delta^C_{C'} \left(R^{AD}_{A'B'} M^B_D + R^{BD}_{A'B'} M^A_D \right) \Big\} \psi_{ABC}, \end{aligned} \tag{4}$$

where ψ_{ABC} is the tensor describing baryons, $M^A_{A'}$ is the tensor describing mesons,

$$(LQ_d)^A_{A'} = L^A_B Q^B_{A'} + L^B_{A'} Q^A_B, \qquad (LQ)^A_A = L^A_B Q^B_A,$$

$$(LQ_f)^A_{A'} = L^A_B Q^B_{A'} - L^B_{A'} Q^A_B,$$

$$R^{[AB]}_{[A'B']} = L^A_{A'} Q^B_{B'} + L^B_{B'} Q^A_{A'} - L^B_{A'} Q^A_{B'} - L^A_{B'} Q^B_{A'},$$

$$R^{[AB]}_{A'B'} = L^A_{A'} Q^B_{B'} - L^B_{B'} Q^A_{A'} - L^B_{A'} Q^A_{B'} + L^A_{B'} Q^B_{A'},$$

$$R^{AB}_{[A'B']} = L^A_{A'} Q^B_{B'} - L^B_{B'} Q^A_{A'} + L^B_{A'} Q^A_{B'} - L^A_{B'} Q^B_{A'},$$

$$R^{AB}_{A'B'} = L^A_{A'} Q^B_{B'} + L^B_{B'} Q^A_{A'} + L^B_{A'} Q^A_{B'} + L^A_{B'} Q^B_{A'}.$$

In (4) the invariant proportional to α arises from a term of the type $1(LQ) \times 35(M)$; the invariant proportional to β corresponds to the extraction of the multiplet 2695 from the product of three multiplets 35. Further, the invariants a_i and b_i ($i = 1, 2, 3, 4$) arise as a result of multiplying $35_1(LQ) \times 35(M)$ and $35_2(LQ) \times 35(M)$; the invariants c_i correspond to the extraction of the multiplet 35 from the products

$$189(LQ) \times 35, \qquad 280(LQ) \times 35, \qquad 280^*(LQ) \times 35, \qquad 405(LQ) \times 35.$$

and, finally, d_i correspond to the extraction of the multiplet 405 from the products

$$280(LQ) \times 35, \qquad 280^*(LQ) \times 35, \qquad 405(LQ) \times 35.$$

The symbol (LQ) after the dimension of a multiplet means that the multiplet under consideration arises in the multiplication of the corresponding multiplets 35.

We note that from the explicit form of the tensors L and Q it follows that

$$(LQ)^A_A = 0.$$

In addition, the invariant in (4) proportional to b_1 gives no contribution to the photoproduction of pseudoscalar mesons. Thus, the amplitude for meson production in a p-state upon absorption of a magnetic-dipole γ-quantum is determined by 16 parameters, with 12 of them, as follows from (4), contributing to the combination of amplitudes $f_1 = M_{13} - M_{11}$, and 13 parameters contributing to the combination $f_2 = 2M_{13} + M_{11}$. After examination of the invariants (4), it turns out that in fact the amplitudes f_1 are determined by only 5 parameters. Thus, for the photoproduction amplitudes on the proton we have:

$$
\begin{aligned}
f_1(\gamma p \to p\eta) &= 4\sqrt{6}\,x_1, \\
f_1(\gamma p \to p\pi^0) &= 10\sqrt{2}\,x_1 \qquad\quad + 2\sqrt{2}\,x_3, \\
f_1(\gamma p \to n\pi^+) &= 5x_1 \qquad + 5x_2 \qquad + 4x_3, \\
f_1(\gamma p \to \Lambda K^+) &= -\frac{9}{\sqrt{6}}x_1 - \frac{9}{\sqrt{6}}x_2 - \sqrt{6}\,x_3 + \sqrt{6}\,x_5, \\
f_1(\gamma p \to \Sigma^0 K^+) &= \frac{1}{\sqrt{2}}x_1 + \frac{1}{\sqrt{2}}x_2 + \sqrt{2}\,x_3 + 3\sqrt{2}\,x_5, \\
f_1(\gamma p \to \Sigma^+ K^0) &= -2x_1 \qquad\quad + 2x_3;
\end{aligned} \tag{5a}
$$

for the photoproduction amplitudes on the neutron we have:

$$
\begin{aligned}
f_1(\gamma n \to n\eta) &= -\sqrt{6}\,x_1 + 2\sqrt{6}\,x_3, \\
f_1(\gamma n \to n\pi^0) &= 5\sqrt{2}\,x_1 - 2\sqrt{2}\,x_3, \\
f_1(\gamma n \to p\pi^-) &= 5x_1 - 5x_2 + 4x_3, \\
f_1(\gamma n \to \Lambda K^0) &= 3\sqrt{6}\,x_1 - \frac{15}{\sqrt{6}}x_3 - \frac{1}{\sqrt{6}}x_4 + \sqrt{6}\,x_5, \\
f_1(\gamma n \to \Sigma^0 K^0) &= \sqrt{2}\,x_1 + \frac{1}{\sqrt{2}}x_3 - \frac{1}{\sqrt{2}}x_4 + 3\sqrt{2}\,x_5, \\
f_1(\gamma n \to \Sigma^- K^+) &= x_1 + x_2 + x_3 + x_4,
\end{aligned} \tag{5b}
$$

where

$$
x_1 = 18(b_1 + c_2 - c_3), \qquad x_2 = 6(b_2 + c_1 - c_4), \qquad x_3 = b_4,
$$

$$
x_4 = \frac{1}{9}a, \qquad x_5 = \frac{1}{3}(d_1 + d_2 - d_3 - d_4).
$$

From (5a) and (5b) the following relations between photoproduction amplitudes follow:

$$
f_1(\gamma p \to n\pi^+) + f_1(\gamma n \to p\pi^-) = \sqrt{2}\,[f_1(\gamma p \to p\pi^0) - f_1(\gamma n \to n\pi^0)], \tag{6a}
$$

$$
f_1(\gamma p \to \Sigma^+ K^0) + f_1(\gamma n \to \Sigma^- K^+) = \sqrt{2}\,[f_1(\gamma p \to \Sigma^0 K^+) - f_1(\gamma n \to \Sigma^0 K^0)],
$$

$$
\sqrt{2}\,f_1\left(\gamma p \to p\,\frac{\pi^0 - \sqrt{3}\eta}{2}\right) = f_1(\gamma p \to \Sigma^+ K^0),
$$

$$
\sqrt{2}\,f_1\left(\gamma p \to \frac{\Sigma^0 - \sqrt{3}\Lambda}{2}\,K^+\right) = f_1(\gamma p \to n\pi^+), \tag{6b}
$$

$$
f_1\left(\gamma n \to n\,\frac{\pi^0 - \sqrt{3}\eta}{2}\right) = -f_1\left(\gamma n \to \frac{\Sigma^0 - \sqrt{3}\Lambda}{2}\,K^0\right),
$$

$$
f_1(\gamma p \to p\eta) - f_1(\gamma n \to n\eta) = \sqrt{3}\,f_1(\gamma n \to n\pi^0), \tag{6c}
$$

$$f_1(\gamma p \to p\pi^0) + f_1(\gamma n \to n\pi^0) = \frac{\sqrt{75}}{4} f_1(\gamma p \to p\eta).$$

Let us note that the relations (6a) are a consequence of the known transformation properties of the Hamiltonian of electromagnetic interactions with respect to isotopic rotations; the relations (6b) are satisfied in $SU(3)$-symmetry ([4]); and, finally, the relations (6c) are valid only in $SU(6)$-symmetry.

For an experimental test, the last relation in (6c) is of special interest. In the impulse approximation, the expression $f_1(\gamma p \to p\pi^0) + f_1(\gamma n \to n\pi^0)$ is the amplitude for photoproduction of π^0-mesons on the deuteron. Therefore this relation between amplitudes is equivalent to a relation between the cross sections of π^0-photoproduction on the deuteron and η-meson production on the proton, if absorption of magnetic dipole γ-quanta takes place. Such a situation occurs in the region of the first resonance of pion photoproduction on nucleons.

As for the combination of amplitudes $f_2 = 2M_{13} - M_{11}$, the relations (6a) and (6b) are satisfied for them. No new relations, valid only in $SU(6)$-symmetry, arise.

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Received
6 IX 1965

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Note: Figure translations are in progress. See original paper for figures.