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A. S. VOL' MIR, I. G. KIL' DIBEKOV

INVESTIGATION OF THE PROCESS OF BUCKLING OF RODS UNDER IMPACT

(Presented by Academician Yu. N. Rabotnov on 30 VI 1965)

In 1949 M. A. Lavrent' ev and A. Yu. Ishlinskii first showed ([1]) that, under the dynamic application of a parametric load, a rod or shell possessing initial imperfections undergoes preferential buckling in certain higher modes of static loss of stability. A number of works, set forth in the book ([2]), were devoted to establishing these modes and determining the nature of the behavior of the structure as a function of the loading program. In all these works it was assumed that the load, as it were, instantaneously "penetrates" the structure; the wave character of the propagation of forces along the structure was not taken into account. In the present article the unsteady process of buckling of a rod subjected to a longitudinal impact is studied, when the compression wave propagates along the length of the rod and is then reflected several times in succession from the ends. With a known approximation, the results of the study of this model may also be applied to the process of general loss of stability of a closed cylindrical shell under axial impact.

Let us assume that the rod is hinged at its ends and receives an impact from a rigid mass at one of the ends. We shall suppose that the rod has a certain initial curvature. We write the system of nonlinear equations describing the behavior of the rod:

$$\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2\right]-\frac{1}{c_1^2}\frac{\partial^2 u}{\partial t^2}=\frac{1}{2}\frac{\partial}{\partial x}\left(\frac{\partial v_0}{\partial x}\right)^2, \tag{1}$$

$$\frac{\partial^4 v}{\partial x^4}-\left(\frac{1}{c_1^2}+\frac{1}{c_2^2}\right)\frac{\partial^4 v}{\partial x^2\partial t^2}+\frac{1}{c_1^2c_2^2}\frac{\partial^4 v}{\partial t^4}-\frac{1}{i^2}\frac{\partial}{\partial x}\left\{\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial v}{\partial x}\right)^2-\frac{1}{2}\left(\frac{\partial v_0}{\partial x}\right)^2\right]\frac{\partial v}{\partial x}\right\}+\frac{1}{c_1^2 i^2}\frac{\partial^2 v}{\partial t^2}=\frac{\partial^4 v_0}{\partial x^4}. \tag{2}$$

Here u is the displacement of points of the axial line of the rod along its length; v and v_0 are the total and initial deflections; i is the radius of inertia of the section; x is the axial coordinate; t is time; c_1 and c_2 are the velocities of propagation of longitudinal and transverse waves in the material of the rod. The origin of coordinates is taken at the end of the rod subjected to impact. We take the boundary and initial conditions in the form:

$$\frac{G}{g}\frac{\partial^2 u}{\partial t^2} + P = 0 \quad \text{for } x = 0; \qquad u = 0 \quad \text{for } x = L; \tag{1}$$

$$v = 0, \qquad \partial^2 v/\partial x^2 = 0 \quad \text{for } x = 0,\ x = L; \tag{2}$$

$$u = 0 \quad \text{for } 0 \le x \le L; \qquad \partial u/\partial t = V_0 \quad \text{for } x = 0 \quad \text{at } t = 0; \tag{3}$$

$$\partial u/\partial t = 0 \quad \text{for } 0 < x \le L \quad \text{at } t = 0; \tag{3}$$

$$v = v_0, \qquad \partial v/\partial t = 0 \quad \text{at } t = 0, \tag{4}$$

where G/g is the magnitude of the impacting rigid mass; P is the compressive force in the sections of the rod; L is the length of the rod; V_0 is the impact velocity.

If the nonlinear terms are omitted in (1), (2), we obtain the well-known equations following from Timoshenko' s theory ([3,4]). Equations close to (1), (2) were contained in ([5]).

Fig. 1. Change of wave formation along the rod axis over time

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The solution of equations (1), (2), satisfying conditions (3), was obtained by means of a digital electronic computer. The finite-difference method was used; the step along the x axis was chosen from 1/16 to 1/60 of the rod length, and the step in normalized time from 1/5 to 1 relative to the dimensionless step along x.

Fig. 2. Critical half-wave lengths

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In Figs. 1–5 the results obtained are shown for certain values of the dimensionless parameters characterizing the stiffness of the rod and the impact velocity; the ratio G/G_1, where G_1 is the weight of the rod, is taken here to be infinitely large.

Figure 1 shows the elastic lines of the rod, corresponding to additional deflections, for a number of successive values of the time period measured from the moment of impact; the dashed lines indicate sections of the elastic lines corresponding to the times $t = 3$ and $t = 5$. The time is referred to the period of passage of the elastic wave along the length of the rod. After impact, a traveling bending wave is formed, whose nodal points are at first move to the right, toward the fixed end. After reflection of the compression wave from the right end, additional buckles arise here. Then, as the compressive force increases in different cross sections of the rod, the buckling pattern of the structure becomes increasingly symmetric with respect to the middle cross section.

Consider the process of change with time of the distances between neighboring nodal points, hereafter called half-wave lengths (Fig. 2). By $l_0, \dots, l_4$ we mean the half-wave lengths referred to the total length of the rod; they are counted successively, from the left end of the rod to the right. Let us follow, for example, the length of the half-wave l_2. At first it increases, and then, after reaching a maximum ($l_2 = 0.45$), begins to decrease. It was established that in all cases a critical length of each half-wave can be determined, which is the maximum over the entire history of the buckling of the rod. It turned out that the critical lengths $l_{1\text{cr}}, \dots, l_{4\text{cr}}$ depend almost not at all on the amplitude of the initial curvature; they prove to be the same, for example, for amplitudes equal to 0.01 and 0.001 of the radius of inertia of the cross section. Apparently, these results may also be extended to the ideal rod as a limiting case. Let us note that the peculiar "jerks" in the change of the lengths of the principal half-waves (see, for example, the curve l_1) correspond to snapping of the rod upon the formation or disappearance of secondary buckles. Subsequently the half-wave lengths become equalized. In Fig. 2, for the instant of time $t = 6.75$, the length of the "unified" half-wave is about $0.2L$. Beginning with this instant, the buckling may be regarded as established; to the subsequent process one may apply the approach to the problem proposed by M. A. Lavrent' ev and A. Yu. Ishlinskii.

Fig. 3. Process of dynamic buckling

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Figure 3 shows the change with time of the amplitude of one of the principal half-waves (l_1). The curve $A_1(t)$ is at first convex upward. The instant at which the second principal half-wave (l_2) reaches the "critical" value $l_{2\text{cr}}$ corresponds to the inflection point of the curve $A_1(t)$. Subsequently there is a sharp increase of the deflection amplitude with time. Apparently, the attainment by the principal half-waves of their "critical" values determines the onset of the violent buckling of the rod—a process that may conventionally be characterized as a dynamic loss of stability. Therefore the characteristic of the "packet" of critical half-waves, relating to the unsteady period of buckling of the system, is of practical interest.

Moscow Aviation Institute
named after S. Ordzhonikidze

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Note: Figure translations are in progress. See original paper for figures.