ON THE RELATION BETWEEN THE HYDRODYNAMIC AND ELECTRICAL CHARACTERISTICS OF A DISCHARGE IN A LIQUID

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Abstract

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ON THE RELATION BETWEEN THE HYDRODYNAMIC AND ELECTRICAL CHARACTERISTICS OF A DISCHARGE IN A LIQUID

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For discharges with moderate currents, when magnetic forces may be neglected and, correspondingly, when the rates of channel expansion are small in comparison with the speed of sound, one may write the approximate equation of energy balance per unit length l of the channel ([1])

$$p_{\mathrm{k}} \frac{ds}{dt} + \frac{1}{\gamma - 1} \frac{d}{dt} p_{\mathrm{k}} s = N(t), \tag{1}$$

where p_{k} is the pressure in the channel; $s = \pi R^2$; R is the channel radius; $N(t)$ is the power released in the channel per unit length; $\gamma = 1.20$ is the effective adiabatic exponent of the plasma ([1,2]).

The pressure p_{k} can be expressed in terms of R from the solution of the hydrodynamic problem of the expansion of a cylinder in a liquid. In our case this problem can be solved in the acoustic approximation. If we restrict consideration to discharges with channels short in comparison with the length of the emitted compression pulse, but with $l > R$, then, using formulas (4), (5), (6) of ([1]), for $\theta = \ln(l/R)$ one may find

$$p_{\mathrm{k}} \approx \rho_0 \frac{\ddot{s}}{2\pi} \ln \frac{l}{R} - \frac{1}{2} \rho_0 \frac{\dot{s}^2}{4\pi^2 R^2}. \tag{2}$$

Substituting (2) into (1) and introducing the dimensionless variables

$$x = t/T, \quad f(x) = F(x)/F(T); \quad F(x) = \int_0^x N(x)\, dx, \quad y = R/R_0,$$

where it is convenient to choose as T the duration of the discharge, determined, for example, from the current duration (Fig. 1), we obtain approximately*

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$\frac{d}{dx}\,y\dot{y} = \frac{\gamma - 1}{\rho_0 \pi} \frac{T^2 F(T)}{R_0^4} \frac{f(x)}{y^2 \ln(l/R_0 y)}. \tag{3}$$

Choosing R_0 from the condition

$$R_0^4 = (\gamma - 1) T^2 F(T)/\rho_0 \pi, \tag{4}$$

we obtain the system of equations:

$$\frac{dz}{dx} = \frac{f(x)}{y^2 \ln(l/R_0 y)}, \qquad \frac{dy}{dx} = \frac{z}{y}, \qquad \text{where } z = y\dot{y}, \tag{5}$$

with the initial conditions $x = 0,\ z = 0,\ y = y_0$, where y_0 may be any quantity satisfying the requirement $y_0 \ll 1$, owing to the insensitivity of the calculation results to the value of y_0 when this requirement is fulfilled.

The system (5) describes the process of channel expansion, if $f(x)$ is known—the time dependence, normalized to unity, of the energy released per unit length of the channel. If the solution of the system (5) is known, which in practice is most conveniently found by numerical integration, then the velocity of expansion of the channel and the pressure in it

Fig. 1. Oscillograms of the current ($1'$ and $2'$) and voltage (1 and 2) for discharges No. 3 (1 and $1'$) and No. 6 (2 and $2'$)

Fig. 2. Function $f(x)$ for discharges No. 3 (curve), No. 6 (a), No. 1 (b), No. 2 (v), No. 4 (g), No. 5 (d)

can be found from the formulas

$$\frac{dR}{dt} = \frac{R_0}{T} \frac{z}{y}, \qquad p_k = \rho_0 \frac{R_0^2}{T^2} \left[\frac{f(x)}{y^2} - \frac{z}{2y^2} \right]. \tag{6}$$

Considering the discharge channel as a set of point sources which create, at a distance r from the middle of the channel in the Fraunhofer zone, the velocity potential

$$\varphi = -\frac{1}{4\pi r}\int_{-l/2}^{l/2} \dot{s}\left(t - \frac{r - \xi\cos\theta}{c}\right) d\xi, \tag{7}$$

where θ is the angle between the channel axis and the direction of observation, $-l/2 \leq \xi \leq l/2$, and restricting ourselves to the first two terms of the expansion of the integrand in $l/cT < 1$, and taking into account that $p = -\rho\varphi$, we obtain, in dimensionless variables, the pressure in the compression pulse

$$p \approx \rho_0 \frac{l}{4\pi r}\frac{\ddot{s}(x - r/cT)}{T^2} + \rho_0 \frac{l^2\cos\theta}{8\pi^2 rcT^2}\dddot{s}\left(x - \frac{r}{cT}\right). \tag{8}$$

Using (8), we obtain the energy of the compression pulse

$$W_{\text{ak}} = 4\pi\int_0^{\pi/2}\int_0^T \frac{p^2}{\rho c} r^2 \sin\theta\, d\theta\, dt =$$

$$= \frac{\pi\rho_0 l^2 R_0^4}{cT^3}\left\{\int_0^1 \left[\frac{f(x)}{y^2\ln(l/R_0 y)}\right]^2 dx + \frac{l^2}{12c^2T^2}\int_0^1 \left[\frac{d}{dx}\frac{f(x)}{y^2\ln(l/R_0 y)}\right]^2 dx\right\}. \tag{9}$$

In the experimental investigation of the discharges, the discharge current and the voltage across the interelectrode gap, the channel radius (from photographs taken on an SFR-1) and the pressure in the compression pulse were measured. Dischar-

in water were initiated by thin (0.04 mm) tungsten wires. The parameters of the investigated discharges are given in Table 1.

Figure 1 presents oscillograms of current and voltage.

The functions $f(x)$, calculated from these and analogous oscillograms for 5 discharges, are shown in Fig. 2. The solid curve shows $f(x)$ for discharge No. 3, which was chosen as the approximating curve when integrating system (5).

Table 1

Discharge No.	U, kV	C, μF	l, cm	L, μH	T, μsec	$F(T)$, J/cm	R_0, cm
1	6	150	5	1.5	100	215	1.10
2	6	150	5	1.5	80	450	1.17
3	6	150	6	1.5	75	377	1.08
4	6	150	7	1.5	95	314	1.17
5	6	150	12	1.5	160	195	1.35
6	6	150	3	0.07	40	830	1.00

Fig. 3 and Fig. 4

Figure 3: Fig. 3 and Fig. 4

The results of the calculation are compared with the experimental data in Fig. 3, where the dependences of the dimensionless radius on time are given (the solid curve is the result of the calculation using the approximating curve), and in Fig. 4, where curves 1 and 2 show the theoretical, and curves 1′ and 2′ the experimental, shape of the compression pulse at a distance of 100 cm from the discharge at $\theta = \pi/2$ for discharges No. 3 and No. 6, respectively.

Fig. 3. Dependence of the dimensionless channel radius on time. Theory (curve) and experiments for discharges No. 1 (*a*) and No. 2 (*b*), No. 3 (*c*), No. 4 (*g*), No. 5 (*d*), and No. 6 (*e*).

Fig. 4. Pulse profile at a distance of 100 cm from the discharge. Theory –curves 1 and 2; experiment–curves 1′ and 2′, 1 and 1′ for discharge No. 3 ($C = 150\ \mu$F, $U = 6$ kV, $l = 7$ cm); 2 and 2′ for discharge No. 6 ($C = 150\ \mu$F, $U = 6$ kV, $l = 3$ cm).

In the latter case, corresponding to comparatively high velocities of expansion of the channel, the theoretical curve differs noticeably from the experimental one, which has a steep leading front caused by the influence of nonlinear effects not described by our equations. Another reason for the discrepancy between curves 2 and 2′ is that the shape of the channel of discharge No. 6, unlike the other discharges, does not substantially correspond to the cylindrical model adopted in the calculation, but rather resembles a dumbbell with gradually merging spherical near-electrode regions enveloping the electrodes. We note that the values of the channel radius of discharge No. 6 shown in Fig. 3 were measured near the electrode.

Let us note that, according to (5) and (7), for unchanged $f(x)$ the magnitude of the pressure in the compression pulse is proportional to $\sqrt{F(T)/T^2}$, while the total energy of the pulse is proportional to the ratio $F(T)/T$, which characterizes the magnitude of the derivative, with respect to time, of the energy released in the channel. Accordingly, the electroacoustic efficiency of the discharge $\eta = W_{\rm ac}/F(T)l$ is proportional to the rate at which energy is introduced into the discharge channel. For example, for discharge No. 3, η, calculated by means of (9), is about 15% (the corresponding experimental value is 12%), while for discharge No. 6 $\eta \simeq 24\%$ (the corresponding experimental value, obtained from pressure measurements at a distance of 1 m from the discharge, where wave absorption already had a noticeable effect, is 18%). This conclusion is also consistent with the results of (3), which indicate an increase of η with increasing rate of energy release in the discharge channel.

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Note: Figure translations are in progress. See original paper for figures.