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A. G. ISTRATOV, V. B. LIBROVICH

HYDRODYNAMIC STABILITY OF A SPHERICAL FLAME

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A spherical flame, freely propagating in space in a combustible gas mixture from a point ignition source, is a convenient object for studying the stability of a laminar flame. Experiments on spherical flames are not complicated by the influence of certain secondary factors (pipe turbulence, heat removal, friction against the pipe walls, etc.), which can distort the phenomenon under investigation.

Experiments on spherical flames ([1−4]) have shown that the hydrodynamic instability of a flame appears at Reynolds numbers $\sim 10^4$—10^6. (The Reynolds number is formed from the flame radius, the normal propagation velocity, and the viscosity of the cold gas.) In ([1−4]) the results obtained were regarded as contradicting the theory of L. D. Landau ([5]), which predicts a rapid development of instability for a flame (the critical Reynolds number according to L. D. Landau cannot greatly exceed unity).

In the works mentioned, however, no account was taken of the special features of the onset of hydrodynamic instability for a spherical flame, connected with the fact that, in contrast to a plane flame, in a spherical one the wavelength of a disturbance on the burning surface continuously increases. In the present note it is shown that this circumstance leads to a considerable (by ~ 2 orders of magnitude) increase in the critical Reynolds number.

The quantities measured experimentally, by which one can judge the onset of instability, are the distortion in time of the shape of the spherical flame and the change in time of the flame propagation velocity. To detect instability it is therefore necessary that the disturbances on the flame surface be not only not too small in comparison with the flame radius, but also that their rate of growth exceed the rate of increase of the flame size. Otherwise the flame surface will be smoothed out and the relative magnitude of the recorded deviation from the undisturbed propagation velocity will decrease. In this connection, in the present note the criterion for the onset of instability of a spherical flame is taken to be the condition that the relative (referred to the flame radius) amplitude of the disturbance of the burning surface increase with time (cf. with a plane flame, for which an increase of the absolute value of the amplitude is sufficient).

The difference in the onset of instability for a spherical flame as compared with a plane one entails interesting consequences. As the solution of the stability problem in the assumptions of L. D. Landau' s theory shows, for large-scale disturbances of the flame surface (disturbances in the form of the first spherical harmonics) the amplitude of the disturbances grows more slowly than the flame size increases—the flame proves to be stable. For smaller disturbances, which, as is known from L. D. Landau' s theory, grow more rapidly, the flame is unstable. But for such disturbances (higher spherical harmonics) the basic assumption of L. D. Landau—that the burning velocity is constant when the curvature—

of the flame, and the solution of the problem should be carried out taking into account the change in the flame velocity. Such a solution was obtained, using a method proposed by J. Markstein, who related the change in flame velocity to the radius of curvature of the surface ([6,7]). J. Markstein introduced into the problem a certain undetermined constant of the dimension of length, taking into account the influence on stability of dissipative processes in the flame. (We note in passing that, long before work ([6]), the question of the influence on the diffusion-thermal stability of a flame of the relation between the coefficients of thermal conductivity and diffusion was discussed in the monograph ([8]).) The stabilization of small disturbances depends on the ratio of Markstein' s constant to the wavelength of the disturbance. Since, for a spherical flame, the wavelength of a disturbance corresponding to a given spherical harmonic grows proportionally to time, stabilization occurs only up to a certain moment, after which the harmonic becomes unstable. The calculations carried out showed that, among all harmonics, one can single out a characteristic one for which instability appears most rapidly. The time of onset of instability for this harmonic and the corresponding flame size may be regarded as determining the critical Reynolds number.

We shall now give some quantitative relations obtained in solving the problem.

For disturbances of the flame surface in the form of spherical harmonics, in the case where the problem was solved under the assumptions of L. D. Landau's theory, the following dependence of the relative amplitude of the flame disturbance f on time t was obtained:

$$f = \text{const} \cdot t^{\omega}, \tag{1}$$

where

$$\omega = -a/2 + \sqrt{a^2/4 - b} \tag{2}$$

(the second root of the characteristic equation does not lead to the onset of instability and is therefore omitted);

$$a = \frac{2\alpha n^2 + 4n + 3\alpha n + 3\alpha}{n + \alpha n + \alpha},$$

$$b = \frac{-\alpha(1-\alpha)n^3 + 2\alpha n^2 + 3n + 3\alpha n - \alpha^2 n + 2\alpha}{n + \alpha n + \alpha}. \tag{3}$$

Here α is the ratio of the densities of the hot and cold gas, and n is the number of the spherical harmonic.

Let us note that the appearance of the power-law dependence (1), instead of the exponential one for a plane flame, is connected with the fact that the wavelength of the disturbance on the flame sphere grows proportionally to time.

The stability of the solution is determined by the sign of ω, which depends on the sign of b in (2). The sign of b may be different depending on n and α: for a given α, at sufficiently small n, $b > 0$ and $\omega < 0$ (the flame is stable), while at larger n, $b < 0$, $\omega > 0$ (the flame is unstable). The stability boundary, corresponding to $b = 0$, is shown in Fig. 1 (curve 1); the values of n for which the flame is stable lie under the curve. It is seen that the stability region expands without bound as α decreases to zero or as α approaches unity. This is due to the fact that different degrees of thermal expansion α correspond to different rates of growth of disturbances.

Figure 2 gives the dependence on α of the growth rate of disturbances for a plane flame front, which at the initial instant of time (the instant at which the disturbances are imposed) coincides with the dependence on α of the characteristic frequency. (In the figure the dimensionless frequency $\Omega = -i\Omega_{\mathrm{L}}\alpha/ku_n$ is plotted; Ω_{L} is the frequency in L. D. Landau' s theory ([5]), u_n is the normal flame velocity, and k is the wave number.) It is precisely this quantity that is substan—

at the onset of instability in a spherical flame.) It is evident that Ω vanishes at $\alpha = 0$ and $\alpha = 1$, while the maximum value $\Omega_m = \sqrt{5} - 2$ is attained at $\alpha_m = \sqrt{5} - 2$. In accordance with Fig. 2, curve 1 in Fig. 1 gives the minimum values of n for α close to α_m. For small perturbations of the flame,

Fig. 1
Fig. 1

Fig. 2
Fig. 2

having the form of higher spherical harmonics, the solution of the problem, carried out with allowance for the change in the flame velocity on curved portions of the front, leads, instead of (1), to the following dependence $f(t)$:

$$f = \mathrm{const} \cdot \left(\frac{v_2 t}{\mu}\right)^{\omega} \exp\left[\frac{\omega c + d}{2\omega + a - 1}\frac{\mu}{v_2 t}\right], \tag{4}$$

where

$$v_2 = u_n/\alpha, \qquad c = \frac{\alpha n(n+1)(2n+1)}{n+n\alpha+\alpha}, \qquad d = c(n+1),$$

μ is the constant in the dependence of the perturbed flame velocity u on the radius of curvature of the distortion Λ (for stabilization of the flame it is necessary that $\mu > 0$):

$$u = u_n(1+\mu/\Lambda). \tag{5}$$

For $\omega > 0$, i.e., in cases where the solution of the problem under L. D. Landau' s assumptions indicates instability, dependence (4) has a minimum at

$$v_2 t/\mu = c(\omega+n+1)/\omega(2\omega+a-1). \tag{6}$$

According to (6), for different n the minimum value of the relative amplitude is reached at different times. The value n_* for which the minimum occurs earliest can be found by examining dependence (6) for an extremum with respect to n.

In Fig. 3, in the coordinates $\lg f$—$\lg v_2 t/\mu$ for $\alpha = 0.2$, curves are shown for the dependence of the relative perturbation amplitude on time for various n. The straight lines in the figure correspond to the solution of the problem under L. D. Landau' s assumptions; the positions of the minima of the curves are marked by a dashed line. For the given α, $n_* = 12$. The dependence $n_*(\alpha)$ is plotted in Fig. 1 (curve 2). This curve lies entirely within the region bounded by curve 1.

As the moment of onset of instability of the spherical flame, as was already stated, it is expedient to take the time $\tau_* = v_2 t_*/\mu$ at which the spherical harmonic with number n_* passes through its minimum. The dimensionless time τ_*, as is easy to verify, is proportional to the Reynolds number Re, constructed using the flame size corresponding to this time. Namely,

$$\tau_* = \frac{\mathrm{Pr}}{l}\mathrm{Re}, \tag{7}$$

where Pr is the Prandtl number for the cold gas, l is the proportionality coefficient between the constant μ and the flame-front width; the condition of proportionality is evident from dimensional considerations. From general

considerations, l may depend on α, on the Lewis and Prandtl numbers, and on the dimensionless activation energy. In papers ([9]), in which l was calculated under various assumptions concerning the flame-stabilizing effects, it was shown that, because of the large activation energy, the value of l may be 10–20. The dependence $\tau_*(\alpha)$, constructed with the aid of relation (6), is given in Fig. 4. For most flames for which

$$\alpha = 0.05 \div 0.2,$$

the value is

$$\tau_* = 50 \div 60.$$

Fig. 3

Let us give a numerical example showing what critical Reynolds numbers correspond to the results obtained. Take, for example, a flame with $\alpha = 0.2$, $l = 10$, $\mathrm{Pr} = 1$. According to Fig. 4, for such a flame $\tau_* = 60$, and therefore $\mathrm{Re} = 600$, i.e., two orders of magnitude greater than unity.

Fig. 4

The value of the critical Reynolds number given above, nevertheless, is insufficient to explain the experimental data ([1–4]). Apparently, there are two reasons for this. First, in experiment it is difficult to determine the critical Reynolds number from the passage of one of the perturbation harmonics through a minimum; a flame with already rather large distortions of the surface is taken to be unstable. Second, nonlinear effects play an essential role in delaying the onset of instability. One such effect is considered in paper ([10]).

The qualitative picture of the appearance of instability in a spherical flame confirms the conclusions of the theory. In experiments, for example, it has been observed that the initial large-scale perturbations arising from the ignition source are smoothed out, while instability arises because smaller perturbations appear on the expanding flame.

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Institute of Chemical Physics
Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.