MOTION OF A NEUTRAL FERMION POSSESSING AN ANOMALOUS MAGNETIC MOMENT IN AN ELECTRIC FIELD

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Abstract

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MOTION OF A NEUTRAL FERMION POSSESSING AN ANOMALOUS MAGNETIC MOMENT IN AN ELECTRIC FIELD

(Presented by Academician N. N. Bogolyubov, 18 X 1965)

Let us consider the motion of a neutral Dirac particle with anomalous magnetic moment $\mu = -\mu_0$ ($\mu_0 > 0$) in a static electric field having a component along the z-axis of the Cartesian coordinate system, $\mathcal{E}(z)$. A solution of the Dirac equation (see [1])

$$i\hbar\frac{\partial}{\partial t}\Psi = \hat{\mathcal{H}}\Psi = \{c(\hat{\alpha}\mathbf{p}) + \rho_3 mc^2 + \mu_0\rho_2\sigma_3\mathcal{E}\}\Psi \tag{1}$$

will be an eigenfunction of the Hamiltonian operator $\hat{\mathcal{H}}$, as well as of the operators $\hat{p}_x$ and $\hat{p}_y$ that commute with it:

$$\hat{\mathcal{H}}\Psi = E\Psi; \qquad \hat{p}_x\Psi = \hbar k_1\Psi; \qquad \hat{p}_y\Psi = \hbar k_2\Psi. \tag{2}$$

To characterize the spin states it is expedient to introduce the polarization-tensor operator $\hat{\Lambda} = \rho_3[\hat{\sigma}\mathbf{p}]_3$, which also commutes with the Hamiltonian and characterizes the projection of the spin onto the direction perpendicular to the external field and to the velocity:

$$\hat{\Lambda}\Psi = \hbar\lambda\zeta\Psi \tag{3}$$

(for the transformation properties of this operator, see [2]). Taking (1), (2), and (3) into account, the wave function may be represented in the form

$$\Psi = e^{-icKt}\frac{e^{i(k_1x+k_2y)}}{\sqrt{L_1L_2}}\begin{pmatrix} f_1(z) \\ i\zeta e^{i\varphi}f_1(z) \\ if_2(z) \\ \zeta e^{i\varphi}f_2(z)\end{pmatrix}, \tag{4}$$

where $c\hbar K = E$ is the particle energy; $k_1 = k\cos\varphi$; $k_2 = k\sin\varphi$; $\lambda = k = \sqrt{k_1^2 + k_2^2}$; $\zeta = \pm 1$ corresponds to the two possible spin orientations. In this case the functions f_1 and f_2 must be determined from the system of equations

$$\left\{\frac{d}{dz} \pm \left(\frac{\mu_0}{e\hbar}\mathcal{E}(z) + \zeta K\right)\right\} f_{1,2} \mp (K - k_0) f_{2,1} = 0, \tag{5}$$

in which the upper and lower signs refer to the first and second component, respectively.

The fundamental possibility of the interaction of a neutral fermion with a static electric field of Coulomb type was noted in [3] (see also [4]). We consider here the case when the electric field $\mathcal{E}(z)$ has the form

$$\frac{\mu_0}{e\hbar}\mathcal{E}(z) = \gamma z + \gamma_0 \qquad (\gamma > 0). \tag{6}$$

Then the system of equations (5) admits the exact solution

$$f_1 = \frac{1}{2}\sqrt{1 + \frac{k_0}{K}}\, U_n(t), \qquad f_2 = \frac{1}{2}\sqrt{1 - \frac{k_0}{K}}\, U_{n+1}(t), \tag{7}$$

in which $U_n(t) = \sqrt[4]{\gamma/\pi}\,(2^n n!)^{-1/2} e^{-t^2/2} H_n(t)$ are Hermite functions, while the variable t is equal to

$$t = \sqrt{\gamma}\, z + (\xi k + \gamma_0)/\sqrt{\gamma}. \tag{8}$$

In this case the energy spectrum is completely discrete and depends only on a single quantum number $n = 0, 1, 2, \ldots$

$$K = \sqrt{k_0^2 + 2\gamma(n+1)}. \tag{9}$$

The solutions obtained physically correspond to the motion of the particle along a circle lying in a plane parallel to the z-axis and passing through the vector $\mathbf{p} = (p_1, p_2)$. The radius of this circle is

$$R^2 = 2\,\overline{(z - \bar{z})^2} = \frac{2}{\gamma}\left(n + 1 - \frac{k_0}{2K}\right) = \frac{1}{\gamma}\left(\frac{K^2 - k_0^2}{\gamma} - \frac{k_0}{K}\right), \tag{10}$$

and the position of the center of the orbit is characterized by the quantity

$$z_0 = \bar{z} = -(\xi k + \gamma_0)/\gamma. \tag{11}$$

For definite values of the momentum $k = \sqrt{k_1^2 + k_2^2}$, the spin projection ξ, and the quantity γ_0 ($\gamma_0 = -\xi k$), the center of the circle is at the origin of coordinates.

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CITED LITERATURE

1. W. Pauli, *Rev. Mod. Phys.*, **13**, 203 (1941).

2. A. A. Sokolov, I. M. Ternov, *DAN*, **153**, 1052 (1963).

3. E. L. Foldy, *Phys. Rev.*, **83**, 688 (1951).

4. I. A. Akhiezer, V. B. Berestetskii, *Quantum Electrodynamics*, Moscow, 1959, § 15.

Note: Figure translations are in progress. See original paper for figures.