ON THE STEADY WIND CIRCULATION IN A HOMOGENEOUS OCEAN

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A. I. FEL' ZENBAUM, N. B. SHAPIRO

ON THE STEADY WIND CIRCULATION IN A HOMOGENEOUS OCEAN

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Let us consider the problem of steady circulation excited in a rectangular ocean of constant depth by a system of zonal winds (Fig. 1). The problem reduces to integrating the equation

$$f_6^* \left(\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \right) + \frac{\partial f_5^*}{\partial y} \frac{\partial \psi}{\partial x} + \frac{d f_6^*}{dy} \frac{\partial \psi}{\partial y} = -\frac{H^2}{2A} \frac{d f_7 T_x}{dy} \tag{1}$$

with the boundary condition

$$(\psi)_{x=0,L} = (\psi)_{y=-b,b} = 0 \tag{2}$$

and the subsequent computation of the components of the current velocity u, v, w and of the level by known formulas ([1,2]). Thus, for the function ψ we have a Dirichlet boundary-value problem, whose solution in the case under consideration exists and is unique ([3]).

The problem is solved numerically using relaxation, splitting, and sweep methods ([4,5]). Instead of the elliptic equation (1), one considers the parabolic equation

$$\frac{\partial \Psi}{\partial \xi} = \frac{\partial^2 \Psi}{\partial {x'}^2} + \frac{\partial^2 \Psi}{\partial {y'}^2} + \chi(y) \frac{\partial \Psi}{\partial x'} + \delta(y) \frac{\partial \Psi}{\partial y'} + \gamma(y), \tag{3}$$

where $\Delta\xi$ is the relaxation parameter;

$$\Psi = \frac{\psi A}{H^2 L T_0}; \qquad \chi = \frac{1}{f_6^*} \frac{d f_5^*}{dy'}; \qquad \delta = \frac{1}{f_6^*} \frac{d f_6^*}{dy'};$$

$$\gamma = \frac{1}{2 f_6^*} \frac{d f_7 T_x' x}{dy'}; \qquad y' = \frac{y}{L}; \qquad x' = \frac{x}{L}; \qquad T_x = T_x' T_0.$$

Fig. 1

Figure 1: Fig. 1

On the interval $[\xi, \xi + \Delta\xi]$, equation (3) is split into a system of two equations

$$\frac{1}{2}\frac{\partial \Psi}{\partial \xi} = \frac{\partial^2 \Psi}{\partial {x'}^2} + \chi \frac{\partial \Psi}{\partial x'} + \alpha\gamma,$$

$$\frac{1}{2}\frac{\partial \Psi}{\partial \xi} = \frac{\partial^2 \Psi}{\partial {y'}^2} + \delta \frac{\partial \Psi}{\partial y'} + (1-\alpha)\gamma, \tag{4}$$

where $0 \leq \alpha \leq 1$, and on the first half-step $[\xi, \xi + \Delta\xi/2]$ the first equation is solved, while on the second—the second. Each of equations (4) is solved by the sweep method. The solution of equation (3), upon reaching the stationary regime, gives the solution of equation (1).

In equation (3) the coefficient $\chi \gg 1$, and therefore we have a problem with a boundary layer (western intensification of currents ([6,7])). In addition, one may expect certain peculiarities of the solution in the equatorial zone ([8]). Therefore, in concrete calculations it is advisable to take a nonuniform grid with refinements near the western coast and near the equator. The ocean cross-section L is divided into 10 equal parts, then the segment $[0, L/10]$ into 5 parts and, finally, the segment $[0, L/25]$ into 10 parts. Along the Y-axis, from

from the equator to 5° we proceed by 1°, and then by 5°. The calculations show that further refinement of the grid near the western coast has practically no effect on the result. At the same time, a calculation even on a relatively dense uniform grid $(\Delta x = L/22)$ leads to large errors in the western boundary layer.

Fig. 1. Isolines of the function ψ $(10^6\ \mathrm{m}^3 \cdot \mathrm{s}^{-1})$: on the left—for a uniform wind $(T_x = -1\ \mathrm{dyn} \cdot \mathrm{cm}^{-2};\ T_y = 0)$; on the right—for a nonuniform wind $(T_x = T_x(y),\ T_y = 0)$

Equations (4) are solved by an implicit scheme. Consider, for example, the second of these equations and write it in difference form

$$\frac{\psi_j^{k+1} - \psi_j^{k+1/2}}{\Delta\xi} = \frac{2}{(\Delta y')_j + (\Delta y')_{j+1}} \left[\frac{\psi_{j+1} - \psi_j}{(\Delta y')_{j+1}} - \frac{\psi_j - \psi_{j-1}}{(\Delta y')_j}\right]^{k+1} +$$

$$+\delta_j \left[\frac{\psi_j - \psi_{j-1}}{(\Delta y')_j}\alpha^* + \frac{\psi_{j+1} - \psi_j}{(\Delta y')_{j+1}}(1-\alpha^*)\right]^{k+1} + (1-\alpha)\gamma, \tag{5}$$

where $(\Delta y')_j = y'_j - y'_{j-1}$, $\Delta\xi = \xi_k - \xi_{k-1} = \mathrm{const}$, and $0 \leq \alpha^* \leq 1$.

Equation (5) reduces to a three-point equation

Fig. 2. Hodographs of the velocity of the current caused by a uniform wind at several characteristic points: u at $x = L/2,\ y = 0$ (curve 1), v at $x = 20$ km, $y = 5^\circ$ (curve 2)

Figure 2: Fig. 2. Hodographs of the velocity of the current caused by a uniform wind at several characteristic points: u at $x = L/2,\ y = 0$ (curve 1), v at $x = 20$ km, $y = 5^\circ$ (curve 2)

$$a_j \Psi_{j+1} - b_j \Psi_j + c_j \Psi_{j-1} + f_j = 0, \tag{6}$$

which is solved by the sweep method. It can be shown that, for the adopted values $\alpha^* = 0$ ($\delta_j \geq 0$) and $\alpha^* = 1$ ($\delta_j < 0$), the scheme is absolutely stable.

Specific calculations were performed for $L = 5000$ km, $b = 6660$ km (60° north and south of the equator), $H = 200$ m, $\Omega = 2\omega \sin\varphi$ ($\omega = 7.29 \cdot 10^{-5}$ sec^{-1}), $A = 50$ g $\cdot$ cm$^{-1} \cdot$ sec^{-1}, $T_0 = 1$ dyne $\cdot$ cm^{-2}. As a result of a series of preliminary calculations, the optimum value $\Delta\xi = 0.001$ and the value $a = 1/2$ were selected.

First of all, let us note that integral circulation in the ocean arises not only under a nonuniform wind, as follows from works ([6,7]), but also under a uniform wind. This is the result of a "more refined" accounting of bottom friction in combination with the latitudinal variation of the Coriolis parameter in equation (1), as compared with the equations of Stommel and Munk. The isolines of the integral stream function ψ in this case, for $T_x = -T_0$, are shown in Fig. 1 on the left. It is seen that intensification of the total transports occurs near the western coast and at the equator. The maximum values of the vertically averaged velocity are reached near the western coast, $v_{\text{av}} = 50$ cm/sec, and at the equator, $u_{\text{av}} = -15$ cm/sec. Despite the fact that the total transport at the equator is directed westward, at depths $z > H/3$ a substantial countercurrent is observed. Its velocity reaches 25 cm/sec, whereas at the surface $u \approx -120$ cm/sec. Figure 2 shows hodographs of the velocity of the current caused by a uniform wind at several characteristic points. The behavior of sea level in the meridional direction has an interesting feature (a depression at the equator and a local rise at approximately 2° northern and southern latitude).

Fig. 2. Hodographs of the velocity of the current caused by a uniform wind at several characteristic points: u at $x = L/2,\ y = 0$ (curve 1), v at $x = 20$ km, $y = 5^\circ$ (curve 2)

With the appearance of transverse nonuniformity of the zonal wind, the integral circulation, as a rule, intensifies. Figure 1 on the right depicts such a circulation, caused by the wind shown in the same figure and close to that adopted in work ([7]). In general, the obtained pattern of the distribution of total transports is analogous to Munk' s pattern. An interesting difference from Munk' s results is the weakening of western intensification to the north and south of the equator. The maximum values of the vertically averaged velocity are still

Fig. 3. Vertically averaged current velocities and current velocities at individual horizons for $y = 25°$N (top) and $x = L/2$ (bottom)

Figure 3: Fig. 3. Vertically averaged current velocities and current velocities at individual horizons for $y = 25°$N (top) and $x = L/2$ (bottom)

reached near the western coast and reach $v_{\text{av}} = 85$ cm/sec. At the equator, under the wind system considered, a slight total westward transport is observed ($u_{\text{av}} \approx -1$ cm/sec). It is important to note that, as a rule, the velocities of the current on individual horizons, both in direction and in magnitude, differ significantly from the vertically averaged velocities of the current. This applies especially to the ocean surface and to the equatorial region as a whole.

Figure 3 shows the distribution of current velocity on several sections. With respect to the vertical component of the current velocity w, it may be noted that it does not change sign with depth and along the parallel. In the equatorial region there is an intense upward motion of waters. Further, with motion to the north and south, zones of upward and downward motion of waters alternate.

Calculations for a nonuniform wind were also carried out using a refined distribution of the tangential wind stress $T_x(y)$, taking into account a local calm zone in the equatorial region ([8,9]). The weakening of the wind here leads to a sharp strengthening of the eastern total transport at the equator, as a result of which a powerful and more isolated from the trade-wind countercurrent deep countercurrent of the type of currents ⋯

Cromwell, Lomonosov currents ([10,11]). It is interesting to note that an increase in the transverse size of the ocean, other conditions being equal, leads to a weakening of the western intensification of currents and, at the same time, to an amplification of zonal flows. Possibly this is precisely what explains the fact that in the Pacific Ocean ($L = 10000$ km) the Kuroshio is weaker, while the intertrade countercurrent and the Cromwell current are stronger than the Gulf Stream, the intertrade countercurrent, and the Lomonosov current in the Atlantic Ocean ($L = 5000$ km).

Fig. 3. Vertically averaged current velocities and current velocities at individual horizons for $y = 25°$N (top) and $x = L/2$ (bottom)

The computations were carried out on an electronic computer at the Computing Center of the Siberian Branch of the USSR Academy of Sciences. The authors express their gratitude to G. I. Marchuk and L. N. Gutman for indicating the method of numerical solution of the problem and for a number of interesting remarks during discussion of the results of the computations.

Marine Hydrophysical Institute
Academy of Sciences of the Ukrainian SSR

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REFERENCES

1. A. I. Fel' zenbaum, *Theoretical Foundations and Methods for Calculating Steady Marine Currents*, Moscow, 1960.
2. A. I. Fel' zenbaum, DAN, 167, No. 4 (1966).
3. R. Courant, *Partial Differential Equations*, Moscow, 1963.
4. G. I. Marchuk, DAN, 155, No. 5 (1964).
5. G. I. Marchuk, *Meteorology and Hydrology*, No. 5 (1964).
6. H. Stommel, Trans. Am. Geophys. Union, 29, No. 2 (1948).
7. V. M. Malkus, *Meteorology*, 7, No. 2 (1950).
8. N. B. Shapiro, DAN, 164, No. 2 (1965).
9. K. Hidaka, Records of Oceanogr. Works in Japan, 7, No. 1 (1963).
10. H. Knaus, Deep-Sea Res., 3, No. 5 (1960).
11. G. P. Ponomarenko, DAN, 149, No. 5 (1963).

Note: Figure translations are in progress. See original paper for figures.