ON THE THEORY OF TEMPORAL PERTURBATIONS OF GRAVITATIONAL AND MAGNETIC FIELDS IN CONNECTION WITH CONTEMPORARY TECTONO-PHYSICAL PROCESSES IN THE EARTH

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Abstract

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Corresponding Member of the USSR Academy of Sciences E. E. FOTIADI, G. I. KARATAEV, V. I. SHCHEGLOV

ON THE THEORY OF TEMPORAL PERTURBATIONS OF GRAVITATIONAL AND MAGNETIC FIELDS IN CONNECTION WITH CONTEMPORARY TECTONO-PHYSICAL PROCESSES IN THE EARTH

1. At present, in a number of regions of the USSR, including Siberia, a network of stationary geophysical test sites is being developed, where at the same points repeated observations of geophysical fields, levelings, triangulations, etc. are periodically carried out—usually at intervals of 2–3 years (discrete observations). In addition, at some points (stations) continuous round-the-clock observations are being conducted of changes in time of the gravitational and magnetic fields and of inclinations of the Earth' s surface.

If discrete observations—especially with periods of tens of years—are of greater interest for the study of general theoretical questions of geophysics and geology, then continuous observations also have practical significance, providing information on rapidly occurring processes in the Earth' s crust and mantle. In particular, continuous observations with gravimeters and magnetometers—owing to substantial changes in the density, shape, and especially magnetization of perturbing masses when pressure changes at depth—as well as with tiltmeters, may be useful for studying the nature of stress redistributions in the focus of a volcano, the motion of magmatic melts before and during eruptions; they may be used to elucidate the pattern of energy accumulation before earthquakes and its dissipation after them, and ultimately used as essential data in developing a scheme for forecasting volcanic eruptions and earthquakes. Observations of temporal changes in the gravitational field may also be used at industrial oil and gas fields in order to establish changes in the reserves of these minerals during exploitation. The pattern of temporal perturbations of the gravitational field and of inclinations of the Earth' s surface, caused only by tidal forces, is substantially different for different geotectonic structures. The reflection in these perturbations of the influence of rapidly occurring deep processes will make it possible to judge the structure and composition of the Earth' s crust and mantle.

Despite the obvious geological value of continuous observations of variations in time of the gravitational and magnetic fields and of inclinations of the Earth' s surface, no substantial theoretical research in this direction is being carried out.

Below we give some results of studies of the relation between temporal gravitational and magnetic potentials and the time-varying shape, density, and magnetization of perturbing bodies.

2. By the direct problem of temporal perturbations of the gravitational (and equally of the magnetic) potential we shall understand the problem of determining an analytic expression in elementary functions, or the geometric image, of the gravitational potential as a function of coordinates and time for a given density (magnetization) and form of the surface of the continuous anomalous medium. The inverse problem, correspondingly, will be the problem of determining, from given variations of the gravitational (magnetic) potential, the temporal field of density (magnetization) and the form of the surface–

surface of the anomalous medium as a function of coordinates and time. Since, depending on the type of problems being solved, changes over time in the density (magnetization) and in the shape of the surface of the medium may be determined by the strain tensor, the stress tensor, and the components of external deforming forces, in a number of cases the variations of the potential fields also determine the external loads on the surface of the anomalous medium.

Let $\rho(\xi_i, t)$, $i = 1, 2, 3$ (where $\xi_1 = \xi$, $\xi_2 = \eta$, $\xi_3 = \zeta$ are spatial coordinates, t is time), be a time-varying density field of an anomalous continuous medium bounded by a regular surface $S(t)$, changing in time without violation of the continuity of the medium, and let $T(t)$ be the region of space in which the anomalous mass m is contained. For such a medium, at any instant of time t, the gravitational potential* may be represented in the form

$$V(x_i, t) = f \int_{T(t)} \frac{\rho(\xi_i, t)}{r} \, d\tau, \tag{1}$$

where f is the gravitational constant; $r^2 = (\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2$; $x_1 = x$, $x_2 = y$, $x_3 = z$ are the coordinates of the points at which the potential is determined.

Differentiating (1) with respect to t, we find an expression for the material rate of change of the gravitational potential $V'(x_i, t)$. However, since the region of integration $T(t)$ is moving in time, it is more convenient to determine the material velocity $V'(x_i, t)$ from the relation

$$V' dt = f \int_{T(t+dt)} \varphi(\xi_i + c_i dt, x_i, t + dt) \, d\tau_1 - f \int_{T(t)} \varphi(\xi_i, x_i, t) \, d\tau, \tag{2}$$

where $\varphi(\xi_i, t) = \rho(\xi_i, t)/r(\xi_i, x_i)$; c_i are the components of the velocity vector of displacement of the points of the body.

Taking into account that the element dS of the surface $S(t)$ during an infinitely small interval dt is displaced by the amount $c_i dt$ in the direction ν_i, instead of (2) one may write:

$$V'(x_i,t) = f\int_{T(t)} \varphi'(\xi_i,x_i,t)\,d\tau + f\int_{S(t)} \varphi(\xi_i,x_i,t)c_i\nu_i\,dS, \tag{3}$$

whence, by the Ostrogradsky-Gauss theorem, we have

$$V'(x_i,t) = f\int_{T(t)} \left[\varphi'(\xi_i,x_i,t) + \varphi(\xi_i,x_i,t)\frac{\partial c_i}{\partial \xi_i}\right] d\tau, \tag{4}$$

where $\varphi' = \partial\varphi/\partial t + c_i\partial\varphi/\partial\xi_i$ (this expression is not difficult to obtain by expanding $\varphi' dt = \varphi(\xi_i + c_i dt, t+dt) - \varphi(\xi_i,t)$ in a Taylor series up to first-order terms).

On the other hand, one may also obtain the following expression for the time-dependent gravitational potential:

$$V(x_i,t) = V(x_i,t_0) + \int_{t_0}^{t} \delta V(x_i, t-t')\,dt', \qquad t_0 < t, \tag{5}$$

where $\delta V(x_i, t-t')$, for very small intervals $t_{j+1} - t_j$, is the envelope of the values of the quantities

$$\delta V(x_i, t_{j+1} - t_j) = f\int_{T(t)} J(j+1,j)\left[\varphi(\xi_i', x_i, t_{j+1}) - \varphi(\xi_i, x_i, t_j)\right] d\tau, \tag{6}$$

where J is the modulus of the Jacobian of the transformation from the coordinate system for the instant t_{j+1} to the coordinate system at the instant t_j for small displacements; in this case $J = 1 + \operatorname{div}\mathbf{u}$, $\xi_i' = \xi_i + c_i dt$.

* Analogously also for the magnetic potential; in what follows, formulas are given only for the gravitational potential.

One may give the following examples of solutions of the direct problem for the temporal gravitational potential (and, equally, the magnetic potential).

A. Let some force $p(t)$, distributed over a circle of radius R, be applied to the plane boundary of an elastic half-space. (For example, the weight of a vertical cylindrical column acts on the Mohorovičić surface, or a tidal force.)

In the theory of elasticity it is shown that, in the case of a uniform distribution of loads, the displacement of the elastic half-space will be equal to

$$u_3(r,t) = u_z(r,t) = \frac{4(1-\sigma^2)}{\pi^2 R^2 E}\, p(t) \int_0^{\pi/2} \sqrt{R^2 - r^2 \sin^2\alpha}\, d\alpha, \tag{7}$$

where σ is Poisson' s ratio, and E is Young' s modulus.

Suppose that at $t = t_0$ one has $V_z(x_i, t_0) = C$; then the equation for temporal perturbations of the gravitational field will have the form

$$V_z(r', \alpha', z, t) = C + f \int_0^{2\pi} \int_0^R \int_z^{z+u_3(t)} \rho(r, \alpha, \zeta)\, \frac{\zeta r\, dr\, d\alpha\, d\zeta}{(r^2 + \zeta^2)^{3/2}}. \tag{8}$$

If, in addition, $\rho = \text{const}$, then, taking into account that $u_3 \ll z$, one can approximately obtain

$$V_z(r', \alpha', z, t) = C + f\rho \int_0^{2\pi} \int_0^R \frac{zr\, u_3(t, r, \alpha)\, dr\, d\alpha}{(r^2 + z^2)^{3/2}}. \tag{9}$$

B. Let, beginning at some moment t_0, from a depth H_0 along a vertical hollow cylinder of radius R, according to a prescribed law $H_0 \geq H(t) \geq 0,\ t_0 \leq t \leq t_1$, some mass of anomalous density move. (For example, the motion of magmatic matter in a volcanic conduit or along fissures, the motion of oil in a borehole, etc.) In this case the equation for the temporal potential of the gravitational field may be written as

$$V_z(x_i, t) = V_z(x_i, t_0) - 2\pi f\rho \int_{H_0}^{H(t)} \left(1 - \frac{\xi}{\sqrt{R^2 + \xi^2}}\right) d\xi, \tag{10}$$

or, solving (10),

$$V_z(x_i, t) = V_z(x_i, t_0) + 2\pi f\rho \left[H_0 - H(t) - \sqrt{R^2 + H_0^2} + \sqrt{R^2 + H^2(t)}\right]. \tag{11}$$

A number of other examples may be proposed for the equation of temporal perturbations of the gravitational field, both in the quasistatic and in the dynamic cases, for various rheological bodies.

It seems to us that the primary directions of further work in the study of temporal perturbations of the gravitational and magnetic potentials should be: 1) the isolation, from observed temporal variations of the gravitational and magnetic fields, of components due to various tectonophysical processes in the Earth; 2) the construction of a model of a rheological body describing the behavior of rocks under conditions of high pressures and temperatures; 3) the solution of the direct problem for bodies under various rheological conditions; 4) the construction of a linear solution with respect to external loads and the displacement vector.

Institute of Geology and Geophysics
Siberian Branch of the Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.