CHANGE IN THE SHAPE OF A LIGHT PULSE

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DURING NONLINEAR AMPLIFICATION

I. Introduction. Attempts have repeatedly been made to obtain a substantial increase in the power of a light pulse from an optical quantum generator with modulated Q by shortening the pulse duration during nonlinear amplification ([1,2]). Indeed, in a nonlinear amplifier, owing to the saturation effect, the excited particles of the medium should radiate energy on the leading edge of the pulse, and it seemed that this would lead to a shortening of the pulse duration ([3–5]). However, up to the present time it has not been possible experimentally to observe a substantial shortening of the pulse duration. As theoretical and experimental studies have shown ([6,7]), the pulse of an optical quantum generator with modulated Q, having an exponentially rising leading edge, when passing through an amplifying medium, does not substantially change its shape, but acquires an additional displacement forward. In this case the velocity of displacement of the pulse may prove to be considerably greater than the velocity of light in vacuum.

The present work is devoted to the study of the change in the shape of a light pulse during nonlinear amplification; the case considered is that in which the pulse duration is much greater than the transverse relaxation time of the medium T_2. In this case, representing the field in the form of amplitude and phase slowly varying over times $1/\omega$ and distances λ, $E(t,x) = \mathcal{E}(t,x)\times\cos[\omega t - kx + \varphi(t,x)]$, it is convenient to use the following equations for the radiation intensity $I(t,x) = \dfrac{1}{\hbar\omega}\dfrac{c}{8\pi}\mathcal{E}^2(t,x)$, the inverse population density $N(t,x)$, and the phase $\varphi(t,x)$ ([8,9]):

$$\frac{\partial I}{\partial t} + c\frac{\partial I}{\partial x} = c(\sigma N - \gamma)I,$$

$$\frac{\partial N}{\partial t} + \frac{1}{T_1}(N - N_0) = -2\sigma I N; \tag{1}$$

$$\frac{\partial\varphi}{\partial t} + c\frac{\partial\varphi}{\partial x} = (\omega_0 - \omega)\frac{T_2}{2}\, c\sigma N, \tag{2}$$

where ω_0 is the resonance frequency of the transition between levels; $\sigma = \sigma(\omega)$ is the cross section of the radiative transition at frequency ω; c is the speed of light in the medium in the absence of absorption; γ is the coefficient of linear radiation losses per unit length in the medium, for example, due to scattering by defects; T_1 is the lifetime of the atom in the upper level, which in what follows is assumed to be much greater than the time of pulse development.

II. Velocity of pulse propagation. Let us first consider the dependence of the velocity of pulse propagation on the shape of the initial pulse. To this end we rewrite equations (1) in the form*

$$\frac{\partial I}{\partial t} + c\frac{\partial I}{\partial x} = c\left[\sigma N_0 \exp\left(-2\sigma\int_{-\infty}^{t} I(t', x)\, dt'\right) - \gamma\right] I. \tag{3}$$

* The remaining equation for phase transport (2) describes the change in the phase and group velocity of light in the medium due to the active particles. It can be shown ([10]) that dispersion effects due to the active particles make a negligibly small contribution to the pulse-propagation velocity compared with the saturation effect.

The saturation boundary of the negative absorption of the medium moves with the same velocity as the leading front of the pulse. Therefore, to determine the propagation velocity of the pulse it is sufficient to follow the propagation in the medium of some definite saturation level

$$\delta = \frac{N(t,x)}{N_0} = \exp\left[-2\sigma\int_{-\infty}^{t} I(t', x)\, dt'\right].$$

The motion of the given saturation level δ is determined from the condition:

$$\int_{-\infty}^{t_s(x)} I(t', x)\, dx' = \mathcal{E}_s \ln\frac{1}{\delta} = \text{const}, \tag{4}$$

where $\mathcal{E}_s$ is the saturation energy. It is most convenient to trace the motion of a level of shallow saturation ($\delta \simeq 0.5 \div 0.8$), when the point $t_s(x)$ lies on the leading front of the pulse. In this case the leading front of the pulse, which has caused this saturation ($-\infty < t < t_s$), can be calculated by neglecting saturation in (3). As a result we find

$$I(t < t_s, x) = I_0(t - x/c) \times \exp[(\sigma N_0 - \gamma)x],$$

Figure 1

Figure 1: Figure 1

where $I_0(t-x/c)$ is the initial light pulse. Condition (4) is then reduced to the following:

$$\int_{-\infty}^{\tau_s(x)} I_0(\tau)\,d\tau = \mathcal{E}_s \ln\frac{1}{\delta}\,e^{-(\sigma N_0-\gamma)x}, \tag{5}$$

Fig. 1. Change in the shape of a Gaussian pulse under nonlinear amplification. $\sigma N_0/\gamma = 6$

where $\tau = t - x/c,\ \tau_s(x) = t_s(x) - x/c$. Since the velocity of motion v of the pulse or of the saturation boundary is determined by the relation

$$(v-c)/c = -c\,\partial t_s(x)/\partial x, \tag{6}$$

then, with the aid of (5), we find the final expression for the pulse velocity:

$$v/c = 1 + (\sigma N_0 - \gamma)c\int_{-\infty}^{\tau_s} I_0(\tau)\,d\tau \big/ I_0(\tau_s). \tag{7}$$

In the particular case of an initial pulse with an exponential leading front $I_0(\tau) \sim \exp(\tau/\tau_0)$, expression (7) gives $v/c = 1 + c(\sigma N_0 - \gamma)\tau_0$, which coincides with the expression found earlier. Formula (7), however, describes a more general situation, when the velocity of motion of the pulse changes as the pulse maximum shifts along the leading front.

III. **Change in pulse shape.** Let us now consider the change in the shape of the pulse during propagation as a function of the shape of the initial pulse. This can be done the basis of expression (7) for the velocity of motion of the pulse.

Motion of the pulse with velocity $v > c$ prevents a shortening of its duration. However, if v tends to c under nonlinear amplification, then the pulse undergoes a shortening of duration and tends toward a solution

equation (3) in the form of a δ-function $I(t,x) = \mathcal{E}_m\delta(t-x/c)^*$, where $\mathcal{E}_m$ is the stationary energy of the pulse ([5,12]). It follows from (7) that the propagation velocity of the light pulse v tends to the velocity of light in the medium c, if

Fig. 2. Change in the shape of an exponential pulse under nonlinear amplification. $\sigma N_0/\gamma = 6$

the leading front of the initial pulse satisfies the condition

$$\lim_{\tau \to -\infty} \frac{1}{I_0(\tau)} \int_{-\infty}^{\tau} I_0(\tau')\, d\tau' = 0. \tag{8}$$

Obviously, condition (8) is satisfied by any pulse with a cut-off leading front, when $I_0(\tau) = 0$ for $\tau \leq \tau'$. It turns out that condition (8) is satisfied by a pulse of Gaussian form $I_0(\tau) \sim \exp(-\tau^2/\tau_0^2)$. Despite the infinite extent of the leading front, a Gaussian pulse contracts when propagating in a medium with negative absorption. This is confirmed by numerical integration of the nonstationary equation (3). In Fig. 1 are shown the results of solving equation (3) on an electronic computer for a Gaussian initial pulse—the dependences $I(t, x)$ after traversing a distance x_0 in a medium with negative absorption are depicted.

Pulses for which

$$\lim_{\tau \to -\infty} \frac{1}{I_0(\tau)} \int_{-\infty}^{\tau} I_0(\tau')\, d\tau' = \text{const} > 0,$$

tend to the stationary form $I(t - x/v)$ without contraction of their duration. Stationary solutions $I(t - x/v)$ of equation (3) were investigated in (7). In particular, the indicated condition is satisfied by pulses with an exponential rise of the leading front. Numerical solutions of the nonstationary equation (3) for a pulse with an exponential leading front are given in Fig. 2, where one can see the approach of the pulse to a stationary state.

Fig. 3. Change in the shape of a pulse with a stepwise rise of the leading front under nonlinear amplification. $\sigma N_0/\gamma = 6$

Finally, light pulses satisfying the condition

$$\lim_{\tau \to -\infty} \frac{1}{I_0(\tau)} \int_{-\infty}^{\tau} I_0(\tau')\, d\tau' = \infty$$

undergo an infinite broadening of their duration. Po—

* In the real case the minimum duration is, of course, of the order of T_2 (5,8,9,11).

Since the total energy of the pulse, irrespective of its shape, is limited, the intensity of such pulses tends to zero. The class of "expanding" pulses includes initial pulses with a power-law rise of the leading edge $I_0(\tau) \sim |\tau_0/\tau|^n,\ n > 1$. Figure 3 presents the results of a numerical integration of equation (3) for an initial pulse with a leading edge of the form $|\tau_0/\tau|^8$.

Thus, from the form of the leading edge of the initial light pulse,* one can unambiguously determine the change in its shape under nonlinear amplification.

We note that, under nonlinear amplification of short light pulses in media whose refractive index changes under the action of a strong light field, an additional change (shortening, broadening) of the shape of the light pulse will occur.

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* We note that the results obtained are applicable not only to the case of propagation of a light pulse. For example, the propagation of an ultrasound pulse in a two-level quantum amplifier of phonons [13] in the region of gain saturation is described by analogous equations, and in this case phenomena similar to those considered above may be expected.