A NUMERICAL SCHEME FOR SHORT-RANGE FORECASTING OF STRATIFORM CLOUDINESS

GEOPHYSICS

1966

SovietRxiv

Abstract

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UDC 551.511

GEOPHYSICS

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A NUMERICAL SCHEME FOR SHORT-RANGE FORECASTING OF STRATIFORM CLOUDINESS

(Presented by Academician E. K. Fedorov on July 8, 1965)

In recent years, considerable attention has been devoted to developing the theoretical foundations for the formation and forecasting of clouds, which exert a determining influence on the thermal and radiation regime of the atmosphere and the Earth' s surface. The first numerical (hydrodynamic) schemes for cloudiness prediction were developed by M. E. Shvets and his students (1, 2). However, these schemes do not take into account the influence of turbulent exchange, heat of condensation, and other factors. The theory of cloud formation developed by one of the authors (3, 4) is based on the idea (supported by quantitative estimates) of a sufficiently complete enlargement of cloud elements by air particles (moles) participating in turbulent exchange. If the concept of the specific moisture content of air is introduced,

$$s = q + \delta, \tag{1}$$

then the equation can be written

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} + v\frac{\partial s}{\partial y} + w\frac{\partial s}{\partial z} = \frac{\partial}{\partial z}k\frac{\partial s}{\partial z}, \tag{2}$$

where u, v, w are the projections of the velocity of motion on the coordinate axes x, y, z, respectively (the z-axis is directed vertically upward), k is the coefficient of turbulence, q is the specific humidity of the air, δ is the specific water content of the cloud (the mass of water droplets and ice crystals in 1 g of air), and t is time.

We obtain another equation of the problem if we add the left- and right-hand sides of the heat- and water-vapor-transfer equations in a turbulent atmosphere,

$$\frac{\partial \Pi}{\partial t} + u\frac{\partial \Pi}{\partial x} + v\frac{\partial \Pi}{\partial y} + w\frac{\partial \Pi}{\partial z} = \frac{\partial}{\partial z}k\frac{\partial \Pi}{\partial z}, \tag{3}$$

where

$$\Pi = \theta + (L/c_p)q, \tag{4}$$

$\theta = T(1000/p)^{0.286} \approx T + \gamma_a z$, θ and T are the potential and kinetic temperatures of the air, p is the air pressure, L is the latent heat of vaporization, c_p is the specific heat capacity of air at $p =$ const, and γ_a is the dry-adiabatic lapse rate. Equations (2)–(3) retain the same form for both unsaturated and saturated (cloudy) air.

In the scheme whose development is the subject of the present note, precipitation fallout from the cloud and radiative heat inputs are not taken into account. Ways of accounting for these effects are outlined in papers (5, 6).

For a cloud in which the water vapor is in the saturated state, system (1)–(3) is supplemented by the relation

$$q = q_m = 0.622\, E(T)/p, \tag{5}$$

where $E(T)$ is the maximum vapor pressure of water vapor at temperature T.

In constructing a cloudiness-forecasting scheme, system (1)–(4) must be supplemented by the equations of motion and continuity, from which out-

three components of the velocity of motion are involved. By the present time, quasi-geostrophic and adiabatic schemes for forecasting the fields of motion and the heights of isobaric surfaces have been developed most fully and implemented on computers. Since, for forecasting cloudiness, information on the fields of motion at as large a number of levels as possible is needed, we used a five-level scheme for forecasting the heights of isobaric surfaces, the algorithm and program of which were developed by G. I. Marchuk, G. P. Kurbatkin, and others ([7]). In a number of studies (see, for example, ([8], [9])) it has been shown that the heat influx not taken into account in scheme ([7]) gives, when calculating the components of the velocity of motion, corrections of the order of 10–15%.

Introducing a new coordinate system $x_p = x,\ y_p = y,\ \eta = p/p_0$ and $t_p = t$, we reduce equations (2) and (3), in the quasistatic and geostrophic approximations, to the form

$$\partial s/\partial t = a\, \partial^2 s/\partial \eta^2 + (b - \tau/p_0)\, \partial s/\partial \eta + (g_0/2\omega_z)(s, \Phi), \tag{6}$$

$$\partial \Pi/\partial t = a\, \partial^2 \Pi/\partial \eta^2 + (b - \tau/p_0)\, \partial \Pi/\partial \eta + (g_0/2\omega_z)(\Pi, \Phi), \tag{7}$$

where $\tau = dp/dt \approx -gpw$ is the analog of the vertical velocity in the coordinate system $x_p,\ y_p,\ \eta$; $a = k(g\eta/RT)^2$; $b = 2k(g\eta/RT)^2[1/\eta - (1/T)(\partial T/\partial \eta)]$; $p_0 =$ 1000 mb; $2\omega_z$ is the Coriolis parameter; Φ is the geopotential height of the isobaric surface $p = \text{const}$; g is the acceleration of gravity; R is the gas constant of air; $(A, B) = (\partial A/\partial x)(\partial B/\partial y) - (\partial A/\partial y)(\partial B/\partial x)$.

The coefficients a and b may, in the first approximation, be considered functions only of η. Their values for $k = 10$ m^2/sec are as follows:

η	1	0.85	0.7	0.5	0.3
$10^4 a$, hr^{-1}	6.12	4.7	3.4	1.9	0.76
$10^4 b$, hr^{-1}	13.3	13.0	11.5	8.6	5.4

Let us briefly discuss the initial and boundary conditions satisfied by the unknown functions Π and s:

1) At $t = 0$, the values of these functions are assumed known throughout the entire space,

$$\Pi(x, y, \eta, 0) = \Pi^0(x, y, \eta), \qquad s(x, y, \eta, 0) = s^0(x, y, \eta). \tag{8}$$

We take temperature and specific humidity from direct-measurement data; for calculating the water content of the cloud at the initial instant, since mass measurements of it are lacking, we use the formula

$$\delta = 0.201 \cdot 10^{-6}(T/\eta)\exp[17.86(1 - 258/T)].$$

Such a dependence of cloud water content on temperature was established on the basis of an analysis of numerous measurement data obtained in various regions of the country.

2) On the lateral boundary Ω of the domain enclosed between 20° W and 75° E and 35° and 75° N, the functions Π and s are assumed constant and equal to their values at the initial instant:

$$\Pi(\Omega, t) = \Pi(\Omega, 0), \qquad s(\Omega, t) = s(\Omega, 0).$$

3) The tropopause was adopted as the upper boundary of the domain; in performing the calculations it was identified with the 200-mb isobaric surface. According to observational data, the vertical gradients of temperature and specific humidity above the tropopause are close to zero, and cloudiness is absent. Using (1) and (4), we find

$$\text{at } \eta = \eta_{\text{tr}} = 0.2 \qquad \partial\Pi/\partial\eta = -0.286\, T_{0.2}/\eta_{\text{tr}}^{1.286}, \qquad \partial s/\partial\eta = 0, \tag{9}$$

where $T_{0.2}$ is the temperature at the level of the surface $p = 200$ mb.

4) The boundary condition at the earth' s surface ($\eta = 1$) is the most difficult to formulate. A special study carried out by us showed that at present it is not possible to use the equation of the heat balance of the earth' s surface in the problem of forecasting cloudiness and fogs. This is explained by the fact that the parameters entering into it ···

(soil and air thermal-conductivity coefficients, the radiation balance, etc.) strongly affect the process of cloud formation, while in practice they are determined with an obviously insufficient degree of accuracy. In this connection, in developing the forecast scheme under discussion, dependences established empirically were used.

If the graphs of A. A. Bachurina [10] are used, then the temperature increment $(\Delta T)_{n+1} = T_{n+1} - T_n$ over the time interval $\Delta t_{n+1} = t_{n+1} - t_n$ for $\eta = 1$ can be represented in the form

$$(\Delta T)_{n+1} = (g_0/2\omega_z)\left[(T,\Phi)_n + (0.36 - 0.004\,c_g)_n(T,\Phi)_n\right]\Delta t_{n+1}, \tag{10}$$

where c_g is the modulus of the geostrophic wind at time t_n.

A statistical analysis of experimental materials led to the conclusion that, in processes of a nonperiodic (advective) type, there exists a linear dependence between changes in the dew point T_d and in the air temperature near the earth' s surface (the correlation coefficient between ΔT_d and ΔT is 0.80–0.86):

$$(\Delta T_d)_{n+1} = 1.26\,(\Delta T)_{n+1}, \qquad n = 0, 1, 2, ... \tag{11}$$

This relation was used as a boundary condition for humidity at $\eta = 1$. On the basis of (10) and (11) it is not difficult to write the conditions for the functions Π and s.

To solve equations (6)–(7) under conditions (8)–(11), a finite-difference method was employed. However, in its classical form the implementation of this method on an electronic computer is associated with high requirements on the size of the machine memory. This difficulty is overcome by the method for solving systems of equations which has become known as the method of splitting (factorization) of multidimensional operators into one-dimensional ones [11,12]. This method was also used by us in constructing the solution of equations (6)–(7).

In carrying out the computations, the horizontal grid step was taken equal to 300 km, the time step to 1 hour; initial values were taken at 18×14 grid points, and the computation of the required functions was performed for 14×10 points (3 rows of points in the west, 2 rows each in the south and north, and 1 row in the east were excluded). The objective analysis of the initial temperature and humidity fields was carried out according to the scheme of V. P. Meleshko [2]. From the forecast values of the functions Π and s, found from equations (6) and (7), the values of temperature and maximum specific humidity q_m at each grid point were determined with the aid of relations (4) and (5). The system (4)–(5) was solved by the method of successive approximations under the assumption that the air at the given point is saturated (there is cloudiness). By comparing q_m with the known moisture content s, first of all the fact of the presence or absence of cloudiness was determined and the temperature outside clouds was refined: for $s \geq fq_m$ the assumption of the presence of cloudiness is valid, and the temperature is determined correctly; for $s < fq_m$ there is no cloudiness at the point ($\delta = 0$); the actual specific humidity, according to (1), is equal to s, and the air temperature is found directly from formula (4). Here f is a certain coefficient which, in physical meaning, coincides with the relative humidity of cloudy air. It is equal to unity at positive temperatures ($T > 273$°K) and somewhat less than unity in supercooled clouds ($T < 273$°K), which is physically explained by the appearance of ice crystals in the cloud (q_m in (5) is always calculated with respect to water). Analysis of experimental data made it possible to establish the dependence of f on $T < 273$°K in the form $f = 0.008\,T - 1.184$.

Data on the skill of four 24-hour cloudiness forecasts for the 850, 700, and 500 mb levels are presented in Table 1, where u is the total forecast skill: $u = (N_{11} + N_{22})/N$; u_{11} is the skill of a "clear" forecast: $u_{11} = N_{11}/N_{01}$; u_{22} is the skill of a "cloudy" forecast: $u_{22} = N_{22}/N_{02}$; u_0 is the skill of random forecasts: $u_0 = (N_{01}N_{10} + N_{02}N_{20})/N^2$; ρ is the qualitative correlation coefficient:

Table 1

Forecast date	Cloudy: N_{02}	Cloudy: N_{22}	Cloudy: u_{22}	Clear: N_{01}	Clear: N_{11}	Clear: u_{11}	u	u_0	ρ	H_B	Q
14 I 1960	138	123	0,891	282	188	0,667	0,741	0,500	0,481	0,482	0,558
15 I 1960	186	153	0,823	234	178	0,761	0,788	0,500	0,576	0,572	0,584
10 I 1959	130	103	0,792	150	127	0,847	0,822	0,503	0,643	0,642	0,639
11 I 1959	165	151	0,916	255	189	0,742	0,810	0,495	0,620	0,623	0,656
Average			0,856			0,741	0,787	0,500	0,574	0,574	0,597

$$\rho = [(N_{11} + N_{22}) - (N_{12} + N_{21})]/N;$$

H_B is the reliability criterion of the method (according to N. A. Bagrov [13]):

$$H_{\text{B}} = (u - u_0)/(1 - u_0);$$

Q is the criterion of the skill of alternative forecasts (according to A. M. Obukhov [14]):

$$Q = 1 - (N_{12}/N_{02}) - (N_{21}/N_{01}).$$

The remaining designations given in Table 1 are summarized in Table 2.

Table 2

Forecast	Occurred cases: clear	Occurred cases: cloudy	Total cases
Clear	N_{11}	N_{12}	N_{10}
Cloudy	N_{21}	N_{22}	N_{20}
Total cases	N_{01}	N_{02}	N

The actual presence of cloudiness at grid points on the corresponding isobaric surfaces was determined from the dew-point deficit $\Delta = T - T_d$, taking into account data on cloudiness on surface weather maps. In doing so, the above-noted dependence of the deficit on air temperature for $T < 273$°K was taken into account. Forecasts for which $\Delta \leq 24{,}57 - 0{,}09\,T$ ("cloudy") or $\Delta > 24{,}57 - 0{,}09\,T$ ("clear") predicted "cloudy" or "clear," respectively. A total of 1540 points were analyzed. The overall skill of the method over all cases is, according to Table 1, about 79%. The sufficiently high value (on average greater than 0.57) of the reliability criterion H_{B} indicates the possibility of practical use of the proposed method (according to N. A. Bagrov [13], the method deserves attention if H_{B} is greater than 0.3).

Received
30 VI 1965

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Note: Figure translations are in progress. See original paper for figures.