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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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CYBERNETICS AND CONTROL THEORY

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ON THE SYNTHESIS OF NONLINEAR CONTROL LAWS

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The problem of applying nonlinear control laws in automation ([1]) has several aspects. One question in this problem is as follows: in an automatic system (usually in a servomechanism) there are certain nonlinearities, and in order to compensate for their harmful influence, as well as to improve the dynamic properties of the system, it is necessary to introduce special nonlinearities into the control law. Work ([2–5]) and others are devoted to solving this question for particular problems. In the present note, a method of solution based on exact analytical methods is considered as applied to an automatic system of the n-th order.

Fig. 1

Let it be required to synthesize a nonlinear control law for the original system (Fig. 1a) with equations

$$
\begin{gathered}
D(p)\eta = -b\xi, \\
p\xi = f(\sigma), \\
\sigma = \Delta[p]\eta - r\xi,
\end{gathered}
\tag{1}
$$

where

$$
\begin{gathered}
D(p) = p^{n-1} + a_1 p^{n-2} + \cdots \\
\cdots + a_{n-1}, \\
\Delta(p) = e_0 + e_1 p + \cdots \\
\cdots + e_{n-2} p^{n-2} \\
(p \equiv d/dt),
\end{gathered}
\tag{2}
$$

η, ξ, σ are dependent variables; a_i, e_i, b, r are real coefficients. Let us introduce correcting nonlinearities into the control law; for this purpose we include nonlinear elements in the scheme (Fig. 1b): the nonlinear element NE_1 in the local feedback loop, the element NE_2 in the main feedback loop, and the element NE_3 for transforming the control signal as a whole. Then the equations of the system take the form

$$D(p)\eta = -b\xi, \qquad p\xi = f(\sigma),$$

$$\sigma = f_3\{f_2[\sigma, \Delta(p)\eta] - f_1(\sigma, r\xi)\}, \tag{3}$$

where f_1, f_2, f_3 are as yet unknown nonlinear functions.

Let us use the canonical transformation

$$\eta = -b\left[\sum_{i=1}^{n-1} \frac{x_i}{\lambda'_{iD}(\lambda_i)} + \frac{x_n}{a_{n-1}}\right],$$

$$p^j\eta = -b\sum_{i=1}^{n-1} \frac{\lambda_i^{j-1}}{D'(\lambda_i)} x_i \quad (j = 1, \dots, n-2), \tag{4}$$

$$\xi = x_n,$$

where

$$D'(\lambda_i) = dD(p)/dp\big|_{p=\lambda_i}.$$

The transformation (4) will be nonsingular if $b \neq 0$ and the roots λ_i ($i = 1, \dots, n-1$) of the equation $D(p) = 0$ are simple and nonzero. This transformation brings system (1) to the form

$$px_k = \lambda_{kx}k + f(\sigma) \quad (k = 1, \dots, n-1);$$

$$px_n = f(\sigma), \qquad \sigma = \sum_{i=1}^{n} \gamma_i x_i, \tag{5}$$

where

$$\gamma_i = -b\frac{\Delta(\lambda_i)}{\lambda'_{iD}(\lambda_i)} \quad (i = 1, \dots, n-1); \qquad \gamma_n = -b\frac{e_0}{a_{n-1}} - r. \tag{6}$$

The transformation (4) brings system (3) to the form

$$px_k = \lambda_{kx} k + f(\sigma) \quad (k = 1, \dots, n-1); \qquad px_n = f(\sigma);$$

$$\sigma = f_3 \left[f_2 \left(\sigma, \sum_{i=1}^{n-1} \gamma_i x_i + \gamma_n' x_n \right) - f_1(\sigma, r x_n) \right], \tag{7}$$

where $\gamma_n' = -b e_0 / a_{n-1}$, and γ_i $(i = 1, \dots, n-1)$ are the same as in (5).

Consider, in the space C_n of the parameters e_i $(i = 0, \dots, n-2)$, r, the section determined by the equations:

$$e_0 + e_1 \lambda_i + e_2 \lambda_i^2 + \dots + e_{n-2} \lambda_i^{n-2} = \delta_{is} A_s \quad (i = 1, \dots, n-1), \tag{8}$$

where δ_{is} is the Kronecker symbol, A_s is an arbitrary real constant (λ_s, respectively). According to (6), we denote the section with equations (8) by $G_2^{(s,n)}$. By virtue of (8), this section in the space C_n is a plane of dimension 2 passing through the r-axis. If the equation $D(p) = 0$ has s real roots, then, by choosing different λ_s in (8), we construct in the space C_n s distinct sections $G_2^{(s,n)}$.

For system (1), under the conditions of the section $G_2^{(s,n)}$, the variable σ, according to (5), (6), (8), is determined from the system

$$px_s = \lambda_s x_s + f(\sigma); \qquad px_n = f(\sigma); \qquad \sigma = \gamma_{sx} s + \gamma_{nx} n, \tag{9}$$

and, after determining $\sigma(t)$, the remaining variables x_i are found, by virtue of (5), from linear nonhomogeneous equations of the first order. For system (3), under the conditions of the section $G_2^{(s,n)}$, by virtue of (6)–(8), the variable σ is determined from the system

$$px_s = \lambda_{sx} s + f(\sigma); \qquad px_n = f(\sigma);$$

$$\sigma = f_3 \left[f_2(\sigma, \gamma_{sx} s + \gamma_n' x_n) - f_1(\sigma, r x_n) \right], \tag{10}$$

and the variables x_i $(i \neq s)$, as before, from nonhomogeneous equations of the first-order

order. The dynamic behavior of system (3) under the conditions of the section $G_2^{(s,n)}$ is determined to a considerable extent by the behavior of system (10); therefore, the selection of the characteristics of the correcting nonlinearities will be carried out in the study of this latter system. In this way, the problem of synthesizing correcting nonlinearities for an n-th order system is reduced essentially to the analogous problem for a second-order system, which can be solved effectively with the aid of the phase plane. We emphasize that equations (8) determine the section $G_2^{(s,n)}$ both for system (1) and for system (3), for identical values of the coefficients of both systems. This circumstance makes it possible, by comparing the results of considering both systems, to observe directly the effect of introducing nonlinearities into the control law.

The method considered has practical application in certain problems of motion control, where high accuracy is required and well-developed control laws are used.

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CITED LITERATURE

[1] E. P. Popov, *Izv. AN SSSR, Energetika i avtomatika*, No. 5, 49 (1962). [2] G. S. Pospelov, in: *Collection. Certain Methods for Calculating Automatic-Control Systems and Their Elements*, 1959, p. 5. [3] K. N. Shen, *Proc. 1st International Congress of the International Federation of Automatic Control*, 1, Publishing House of the USSR Academy of Sciences, 1960, p. 413. [4] M. I. Rabinovich, *Izv. vyssh. uchebn. zav., Radiofizika*, 5, No. 5, 998 (1962). [5] V. A. Zemskov, B. M. Makar' ev, *Izv. AN SSSR, Tekhnicheskaya kibernetika*, No. 6, 60 (1963). [6] R. A. Nelepin, *DAN*, 161, No. 4, 771 (1965).

Note: Figure translations are in progress. See original paper for figures.