ON THE INTEGRATION OF THE EQUATIONS OF PLANE FLOW OF IDEALLY PLASTIC BODIES

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Abstract

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B. A. DRUYANOV

ON THE INTEGRATION OF THE EQUATIONS OF PLANE FLOW OF IDEALLY PLASTIC BODIES

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In plastic regions, where both families of characteristics are curvilinear, the following functions of the curvilinear coordinates α, β are to be determined: u, v—the projections of the velocity on the directions of the characteristics; R, S —the radii of curvature of the characteristics; the quantities $\bar{x}, \bar{y}$, related to the Cartesian coordinates x, y by the formulas $\bar{x} = x\cos\varphi + y\sin\varphi$, $\bar{y} = -x\sin\varphi + y\cos\varphi$ (φ is the angle of inclination of the characteristics of the family α to the axis of abscissas, $\varphi = \alpha + \beta$). The functions $u, v, R, S, \bar{x}, \bar{y}$ satisfy the equation $\partial^2 f/\partial\alpha\,\partial\beta + f = 0$, for which the Riemann function is the Bessel function of the first kind of zero order

$$J_0\left[2\sqrt{(\alpha-a)(\beta-b)}\right] \tag{1}$$

(a, b are parameters).

The application of analytical methods, however, is complicated by the necessity of solving a chain of boundary-value problems, since usually the plastic region has a "patchwork" form. A successive solution of boundary-value problems leads to lengthy calculations. The method presented below by way of an example considerably simplifies the matter.

Consider the problem of the initial flow of a strip under indentation by a convex smooth punch. Let the punch move translationally. The field of characteristics, generalizing the known fields ([2]), is shown in Fig. 1. In the region $A_{34}A_{04}A_{15}A_{25}$ the characteristics of the family β are rectilinear; in the region $A_{43}A_{40}A_{51}A_{52}$ the characteristics α are rectilinear. In the remaining plastic regions the characteristics are curvilinear. The velocity distribution can be found after the actual construction of the net of characteristics in the physical plane ([2]). The impermeability condition makes it possible to determine the distributed pressures along the contact arc $A_{52}A_{25}$ and the forming straight characteristics $A_{04}A_{15}$ and $A_{40}A_{51}$. The opening angle of the sector $A_{24}A_{25}A_{15}A_{14}$, as well as of the

Fig. 1

Figure 1: Fig. 1

sector $A_{42}A_{52}A_{51}A_{41}$, is determined only roughly. Therefore one may assume that along the lines $A_{40}A_{43}$, $A_{43}A_{34}$, $A_{34}A_{04}$ the quantities $\bar{x}, \bar{y}$ and their derivatives are known as functions of the coordinates α, β. This makes it possible to determine $\bar{x}, \bar{y}$, and consequently the net of characteristics in the whole plastic region.

Fig. 1

Let us determine $\bar{x}, \bar{y}$ at the point M with coordinates $\alpha = a$, $\beta = b$. For this it is necessary successively to determine $\bar{x}, \bar{y}$ in the regions $A_{43}A_{33}A_{34}$, $A_{33}A_{34}A_{04}A_{03}$, $A_{33}A_{43}A_{40}A_{30}$, $A_{33}A_{03}A_{00}A_{30}$. Another possibility consists in the following. Let

$$dU_x = \left(G\,\frac{\partial \bar{x}}{\partial \alpha} - \bar{x}\,\frac{\partial G}{\partial \alpha}\right) d\alpha + \left(\bar{x}\,\frac{\partial G}{\partial \beta} - G\,\frac{\partial \bar{x}}{\partial \beta}\right) d\beta. \tag{1}$$

Integrating (1) along $A_{33}A_{34}A_{43}A_{33}$, $A_{33}A_{43}ABA_{33}$, $A_{33}CDA_{34}A_{33}$, $A_{33}BMCA_{33}$, we shall have

$$\int_{A_{43}A_{33}} dU_{\bar{x}} + \int_{A_{33}A_{34}} dU_{\bar{x}} + \int_{A_{34}A_{43}} dU_{\bar{x}} = 0, \tag{2}$$

$$\int_{A_{33}A_{43}} dU_{\bar{x}} + \int_{A_{43}N} dU_{\bar{x}} + \int_{NK} dU_{\bar{x}} + \int_{KA_{33}} dU_{\bar{x}} = 0$$

and so on.

Adding these equalities, we obtain an equality containing the integral of $dU_{\bar{x}}$ along $MDA_{34}A_{43}AM$ and integrals along the internal boundaries of these plastic regions, containing jumps of the functions $\bar{x}$, $\partial\bar{x}/\partial\alpha$, $\partial\bar{x}/\partial\beta$.

Since x, y are continuous, $\bar{x}, \bar{y}$ are also continuous. In passing across the boundary of two regions, only the following quantities can undergo a discontinuity: $\partial\bar{x}/\partial\alpha$ when crossing β, and $\partial\bar{y}/\partial\beta$ when crossing α. However, from (1) it is seen that $dU_{\bar{x}}$ does not contain $\partial\bar{x}/\partial\alpha$ on the lines β. Similarly, $dU_{\bar{y}}$ does not contain $\partial\bar{y}/\partial\beta$ on the lines α. Consequently, after adding the equalities (2) we obtain

$$\int_{MD} dU_{\bar{x}} + \int_{DA_{34}} dU_{\bar{x}} + \int_{A_{34}A_{43}} dU_{\bar{x}} + \int_{A_{43}A} dU_{\bar{x}} + \int_{AM} dU_{\bar{x}} = 0. \tag{3}$$

Hence

$$\bar{x}_M = \frac{1}{2}\left(\bar{x}_A + \bar{x}_D\right) + \frac{1}{2}\int_{DA_{34}A_{43}A} dU_{\bar{x}}. \quad (4)$$

An analogous formula is obtained for $\bar{y}_M$.

For $a = b = 0$, at the point A_{00}, $\bar{x} = x, \bar{y} = y$. Then

$$\eta_{00} = \frac{1}{\sqrt{2}}(y_{00} - x_{00}) = \frac{1}{2\sqrt{2}}\left(\bar{y}_{40} - \bar{x}_{40} + \bar{y}_{04} - \bar{x}_{04}\right) + \frac{1}{2\sqrt{2}}\int_{A_{04}A_{34}A_{43}A_{40}} dU_{(\bar{y}-\bar{x})/\sqrt{2}}.$$

Since it must be that $\eta_{00} = 0$, we obtain an equation imposing a condition on the parameters of the problem, for example, on the position of the points A_{43}, A_{34}.

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[1] R. Hill, *Mathematical Theory of Plasticity*, 1956. [2] B. Druyanov, *Journal of Applied Mechanics and Technical Physics*, No. 6 (1961).

Note: Figure translations are in progress. See original paper for figures.