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Abstract

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A. P. VASIL' EV, V. I. KOGAN

ON THE THEORY OF RADIATION TRANSFER IN PLASMA

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1. In an attempt to extend the theory of transfer of line (resonance) radiation ([1−3]) to the case of transfer of recombination radiation, undertaken by the authors, it became clear that, along with the natural features of similarity, there is also an essential difference between the two phenomena: the transfer of recombination radiation is qualitatively similar to ordinary diffusion (see the remark in ([4])). In ([1−3]) it was shown that, under complete redistribution over frequency in the act of re-emission (and with identical emission and absorption profiles), the transfer of resonance radiation has, in contrast to ordinary diffusion, an essentially nonlocal character. The reason for the nondiffusive character is the slow decrease of the kernel of the integral equation of resonance-radiation transfer with distance, and the consequent nonreducibility of this equation to a differential one. Physically this means that quanta arriving from remote parts of the system make a substantial contribution to the local population of the excited level.

2. We shall show that complete redistribution over frequency in the act of re-emission by itself, without an additional restriction on the form of the relation between the emission and absorption profiles, does not necessarily lead to the nondiffusive character of the radiation-transfer process noted in item 1. Without yet specifying whether we are dealing with a spectral line or a continuum, and using the function

$$T(\rho) = \int_0^\infty P(\omega) e^{-\chi(\omega)\rho} \, d\omega \tag{1}$$

—the weighted probability that a quantum traverses a path ρ without absorption ($P(\omega)$ is the emission profile, normalized to unity; $\chi(\omega)$ is the absorption coefficient), we have for the mean free path of a quantum

$$\bar{\lambda} = \int_0^\infty \rho \left(-\frac{dT}{d\rho}\right) d\rho = \int_0^\infty \frac{P(\omega)}{\chi(\omega)} \, d\omega = \int_0^\infty \lambda(\omega) P(\omega) \, d\omega. \tag{2}$$

By virtue of (2), for the finiteness of $\bar{\lambda}$ (i.e., also for the diffusiveness of radiation transfer) it is necessary and sufficient that $T(\rho)$ as $\rho \to \infty$ decrease no more slowly than $\rho^{-1-\nu}$ ($\nu > 0$), or, equivalently, that $P(\omega)/\chi(\omega)$ as $\omega \to \infty$ decrease no more slowly than $\omega^{-1-\nu}$. In the usually considered case $P(\omega)/\chi(\omega) = \text{const}$, characteristic of the transfer of line radiation, it follows from (2) that $\bar{\lambda} = \infty$ ([2]). However, other cases are also possible. Thus, in the transfer of recombination (or bremsstrahlung) radiation, $P(\omega)/\chi(\omega)$ decreases sufficiently rapidly (item 3), and $\bar{\lambda}$ turns out to be finite.

3. Let us consider the ionization equilibrium of an impurity in a plasma of finite dimensions, starting from the simplest model of a hydrogen-like atom (or ion), whose electron may be either on a single discrete level or in the continuous spectrum. We shall assume spatial homogeneity of the temperature T, the electron density n_e, and the "unio-

of ionized atoms" n_0, and consequently also $\varkappa(\omega)$. The assumption of uniformity of n_e is acceptable, for example, in cases of small admixture or a high degree of ionization of the "intrinsic" gas. Uniformity of n_0 is ensured when $kT \ll \chi$ (χ is the ionization energy). Under the same condition induced emission may be neglected.

The transfer of recombination radiation, with still greater justification than that of line radiation, may be regarded as occurring with complete redistribution over frequency in the act of "re-emission." Indeed, owing to the high probability of electron–electron collisions, the electron released as a result of a photoionization act, long before the photorecombination that is to occur, has time to become "incorporated" into the energy distribution function existing in the plasma (assumed below to be Maxwellian), which also determines the form of $P(\omega)$.

In view of what has been said, it is not difficult to see that the equation of ionization equilibrium is reduced to the same form as the balance equation for the number of excited atoms in the two-level model ([1,5]):

$$(1+\beta)y(\mathbf{r}) = \int_V G(\mathbf{r},\mathbf{r}')y(\mathbf{r}')\,d\mathbf{r}' + \beta, \tag{3}$$

where now $y(\mathbf{r})$ is the density of "ionized atoms" $n_1(\mathbf{r})$ in units of their thermally equilibrium density n_1^C (determined by Saha' s formula), $\beta \equiv n_e w\ /\langle v\sigma_\phi\rangle$ is the ratio of the probabilities of three-body and photorecombination, and the kernel G, as usual, is the probability of absorption of a quantum emitted at one of the points $\mathbf{r}, \mathbf{r}'$, per unit volume near the other:

$$G(\mathbf{r},\mathbf{r}') = -\frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|^2}\frac{dT(|\mathbf{r}-\mathbf{r}'|)}{d|\mathbf{r}-\mathbf{r}'|}. \tag{4}$$

In partially overlapping regions: A) $\beta \gg 1$ and B) $T(a) \approx 1$ (a is the characteristic size of the system), the integral term in (3) is relatively small and, consequently, the solution is spatially uniform: $y \approx \beta/(\beta+1)$. Case A) corresponds to thermally equilibrium ionization: $n_1 \approx n_1^C$; case B), under the additional condition $\beta \ll 1$, to the Elwert-Shklovskii ionization equilibrium: $n_1/n_0 \approx \langle v\sigma_i\rangle/\langle v\sigma_\phi\rangle$ (the equality $y \approx \beta$, transformed with the aid of the principle of detailed balance; σ_i, σ_ϕ are the cross sections for ionization by electron impact and for photorecombination).

In the general case, especially in the region $\beta \ll 1$, $T(a) \ll 1$, the solution $y(\mathbf{r})$ is substantially nonuniform and depends on the specific form of the kernel G, determined by formulas (4) and (1). For recombination radiation:

$$P(\omega) = \text{const}\,[f\quad(\varepsilon)\sqrt{\varepsilon}\,\sigma_\phi(\varepsilon)]_{\varepsilon=\hbar\omega-\chi} = \frac{1}{\text{Ei}(\chi/kT)}\frac{e^{-\hbar\omega/kT}}{\omega} \tag{5}$$

for $\omega \geq \omega_0 \equiv \chi/\hbar$; $P(\omega) \equiv 0$ for $\omega < \omega_0$. Here Ei is the exponential integral, and σ_ϕ has been taken in the Kramers approximation. Accordingly $\varkappa(\omega) \approx \varkappa_0(\omega_0/\omega)^3$, and from (2), taking into account $\chi \gg kT$, we obtain $\lambda \approx 1/\varkappa_0$. Thus the spectral mean free path of a quantum of recombination radiation, in sharp contrast to the case of line radiation, is close to the mean free path of a quantum of the most probable frequency. Substitution into (1) gives

$$T(\rho) = \frac{1}{\text{Ei}(s)}\int_1^\infty \exp\left(-st - \frac{\varkappa_0\rho}{t^3}\right)\frac{dt}{t}, \tag{6}$$

where $s \equiv \chi/kT$. The asymptotic form of $T(\rho)$ (obtained by the saddle-point method) is

$$T(\rho) \simeq \sqrt{\frac{\pi}{2}}\,\frac{1}{\text{Ei}(s)}\,\frac{1}{(3\varkappa_0\rho s^3)^{1/8}}\exp\left[-\frac{4s}{3}\left(\frac{3\varkappa_0\rho}{s}\right)^{1/4}\right], \qquad \varkappa_0\rho > \frac{s}{3} \gg 1. \tag{7}$$

The same law of decrease of $T(\rho)$ also holds for bremsstrahlung (cf. (6)), since in this case $P(\omega)$ and $\chi(\omega)$ have (in the Kramers approximation) the same frequency dependence.

The question discussed in Sec. 3 is considered from another point of view in (7).

4. Consideration of the question of the escape of recombination radiation from a finite volume of nonequilibrium plasma is analogous to that carried out in (5) for line radiation (integral relations, the transition from volume radiation to surface radiation, and the role of quenching—the criterion $\beta \sim T(a)$, the diagram of the regions of β and $\varkappa_0 a$, etc.). The function $y(\mathbf{r})$—a measure of nonequilibrium—makes it possible formally to extend the use of Kirchhoff' s law to a nonequilibrium region:

$$\eta(\omega,\mathbf{r})/\chi(\omega) = y(\mathbf{r})B(\omega)$$

(η is the emissivity, B is the Planck function).

5. The qualitative difference between the steep law of decrease (7) and the corresponding kernel (4) and the slowly decreasing T and G for line radiation leads to the question of the influence of the rate of decrease of the kernel G on the character of the solutions of equation (3). In this connection we shall analyze the approximate method proposed in (8, 9) and connected (at least in its idea) precisely with the use of a sufficiently steep falloff of the kernel G. We present the corresponding approximate solution in a form convenient for us:

$$y(\mathbf{r}) \approx \beta/(\beta + \bar{T}(\mathbf{r})) \equiv \tilde{y}(\mathbf{r}), \tag{8}$$

where

$$\bar{T}(\mathbf{r}) = 1 - \int_V G(\mathbf{r}, \mathbf{r}')\, d\mathbf{r}'$$

is the probability that a quantum emitted from the point $\mathbf{r}$ will escape beyond the limits of the system without absorption; for the relation of $\bar{T}(\mathbf{r})$ to $T(\rho)$, see (5).

Solution (8) is obtained by taking $y(\mathbf{r}')$ out from under the integral (3) at the point $\mathbf{r}' = \mathbf{r}$, and consequently is valid only when y varies sufficiently slowly in comparison with G. As is not hard to see from equation (3), rewritten in the form

$$y(\mathbf{r}) = \frac{\beta}{\beta + \bar{T}(\mathbf{r}) + \int_V [1 - y(\mathbf{r}')/y(\mathbf{r})] G(\mathbf{r}, \mathbf{r}')\, d\mathbf{r}'}, \tag{9}$$

$y < \tilde{y}$ at the center and $y > \tilde{y}$ at the edge of the system,* i.e., the true solution $y(\mathbf{r})$ varies more slowly than $\tilde{y}(\mathbf{r})$, and, consequently, wherever $\tilde{y}$ already varies slowly, the regularity of approximation (8) is ensured. This applies above all to regions A) and B) (Sec. 3), in which solution (8) refines the spatially homogeneous solution.

In the region $\beta \ll 1$, $T(a) \ll 1$ the situation is more complicated. Let us fix $\beta \ll 1$ and vary $T(a)$, changing the form of the function $T(\rho)$ at fixed $\varkappa_0 a$, and conversely. Then for $T(a) \ll \beta$ approximation (8) is valid inside the system, where $\bar{T} \ll \beta$, but certainly not in the outer (indeed, increasingly narrowing as $T(a)$ decreases) layer $\beta \lesssim \bar{T} \leq 1/2$, where the rates of variation of $\tilde{y} \approx \beta/\bar{T}$ and G are comparable. For $\beta \lesssim T(a) \ll 1$ approximation (8) breaks down throughout the entire volume.

For a slowly decreasing power-law falloff of $T(\rho)$, the indicated violation of approximation (8) is numerically not very large, since in this case the kernel G varies, by virtue of (4), still noticeably more strongly than $\tilde{y} \approx \beta/\bar{T}$; this also explains the success of approximation (8) in the analysis of radiation transfer in lines, summarized in (10). On the contrary, for steeply decreasing $T(\rho)$, for example of the form (7), approximation (8) may be violated very strongly. Therefore the use of this approximation in the analysis of recombination-radiation transfer (7) appears unjustified in a number of cases.

6. Let us illustrate what has been said by a model example of a one-dimensional kernel

$$G(x,x') = \frac{1}{2}\varkappa_0 e^{-\varkappa_0|x-x'|},$$

which permits an exact solution of equation (3) by reducing it to a second-order differential equation—

* We note the equality of the volume integrals of y and $\tilde{y}$ with weight $(\beta+\bar{T})$ (5).

(3). This exact solution has the form

$$y(x) = 1-(1-\gamma^2)\,\frac{e^{-\chi_0\gamma x}+e^{-\chi_0\gamma(l-x)}}{1+\gamma+(1-\gamma)e^{-\chi_0\gamma l}}, \tag{10}$$

where $\gamma=\sqrt{\beta/(\beta+1)}$, and l is the thickness of the "layer." Approximation (8), however, gives

$$\tilde{y}(x) = \frac{\beta}{\beta+\frac{1}{2}\left[e^{-\chi_0 x}+e^{-\chi_0(l-x)}\right]}. \tag{11}$$

Comparison of (10) with (11) shows that in unfavorable cases the approximation $\tilde{y}$ is incorrect even as to order of magnitude. Thus, at the edge of the "layer" we have, for $\chi_0 l \gg 1/\sqrt{\beta} \gg 1$: $\tilde{y}(0)/y(0) \approx 2\sqrt{\beta} \ll 1$, while at its center, for $\chi_0 l \approx 2\ln(1/\beta) \gg 1$: $\tilde{y}(l/2)/y(l/2) \approx 2/\beta\ln^2(1/\beta) \gg 1$.

Thus, contrary to initial expectation, precisely for sharply decreasing kernels G the approximation $\tilde{y}$ is unsatisfactory in those regions where it depends sufficiently strongly on the coordinates. On the other hand, precisely for such kernels $\lambda \neq \infty$ (items 2, 3), which opens up the possibility of solving equation (3) in the diffusion approximation.

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Note: Figure translations are in progress. See original paper for figures.