THE INFLUENCE OF COULOMB COLLISIONS ON THE DRIFT INSTABILITY OF PLASMA

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Abstract

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THE INFLUENCE OF COULOMB COLLISIONS ON THE DRIFT INSTABILITY OF PLASMA

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The influence of collisions of charged particles on the drift instability of plasma was studied in ($^{1-8}$) with the aid of a model collision integral (^{9}). Unfortunately, such a collision integral does not make it possible to investigate effects connected with the nonuniformity of the plasma temperature and in a number of cases leads to an incorrect result ($^{11-14}$).

Below we use the Landau collision integral (^{10}). Restricting ourselves to the case of a low-pressure plasma, we shall consider only instability with respect to the excitation of potential oscillations. In addition, we assume the effective collision frequencies of the particles to be small in comparison with the Larmor frequencies. We carry out the analysis of the oscillation spectra in the geometrical-optics approximation (^{15}).

Potential oscillations are impossible under the conditions $|\omega + i\nu_e| \ll k_z v_{Te}$, $|\omega+i\nu_i| \ll k_z v_{Ti}$, where ω is the frequency of the oscillations; k_z is the projection of the wave vector $\mathbf{k}$ on the direction of the magnetic field; v_{Te} and v_{Ti} are the thermal velocities of electrons and ions; ν_e and ν_i are the collision frequencies. Therefore below we assume that $|\omega + i\nu_i| \gg k_z v_{Ti}$, and neglect the Larmor radius of the electrons.

Potential oscillations of an inhomogeneous plasma are described by the eikonal equation (^{15})

$$\varepsilon = 1 + \delta\varepsilon_e + \delta\varepsilon_i = 0, \tag{1}$$

where $\delta\varepsilon_i$ and $\delta\varepsilon_e$ are the contributions due to ions and electrons, respectively. In the frequency range $\omega \gg \nu_i$; $|\omega - n\Omega_i| \gg \nu_i,\ k_z v_{Ti}$

$$\delta\varepsilon_i = \frac{\omega_{Li}^2}{k^2 v_{Ti}^2}\left\{1 - \sum_n \frac{\omega}{\omega - n\Omega_i}\left(1 - \frac{k_y v_{Ti}^2}{\omega\Omega_i}\left[\frac{\partial \ln N}{\partial x} + \frac{\partial T_i}{\partial x}\frac{\partial}{\partial T_i}\right]\right)\right. \times$$

$$\times A_n\left(\frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2}\right)\left(1+\frac{k_z^2 v_{Ti}^2}{(\omega-n\Omega_i)^2}\right)\Bigg\} + \delta\varepsilon_{ii}. \tag{2}$$

Here ω_L is the Langmuir frequency, and Ω is the Larmor frequency of the particles; $k_\perp$ is the projection of the wave vector transverse to the magnetic field; $A_n(x) = e^{-x} I_n(x)$; $\delta\varepsilon_{ii}$ is the dissipative part associated with ion-ion collisions. In the limit $k_\perp v_{Ti} \ll \Omega_i$ we have

$$\delta\varepsilon_{ii} = i\cdot 0.1\,\frac{\nu_{ii}}{\omega}\frac{\omega_{Li}^2}{k^2 v_{Ti}^2}\left\{\left(16\frac{k_z^4}{\omega^4}+28\frac{k_z^2 u^2}{\omega^2\Omega_i^2}+7\frac{k_\perp^4}{\Omega_i^4}\right)\left(1-\frac{k_y v_{Ti}^2}{\omega\Omega_i}\frac{\partial \ln N}{\partial x}\right)-\right.$$

$$\left.-\frac{k_y v_{Ti}^2}{\omega\Omega_i}\frac{\partial \ln T_i}{\partial x}\left(24\frac{k_z^4}{\omega^4}+\frac{33}{2}\frac{k_z^2 k_\perp^2}{\omega^2\Omega_i^2}-\frac{3}{4}\frac{k_\perp^4}{\Omega_i^4}\right)\right\}, \tag{3}$$

where $\nu_{ii} = \frac{4}{3}\sqrt{\pi}\, M e^4 N L / T_i^{3/2}$; L is the Coulomb logarithm. In deriving formula (3) it was assumed that $\omega \ll \Omega_i$. In the limit when $k_\perp v_{Ti} \gg \Omega_i$ (but $\omega,\ \omega - n\Omega_i \gg \nu_i k_\perp^2 v_{Ti}^2/\Omega_i^2$), we have

$$\delta\varepsilon_{ii} = i\frac{\omega_{Li}^2}{k^2 v_{Ti}^2}\frac{k_\perp v_{Ti}}{\Omega_i}\frac{3(\pi+1)}{8\sqrt{\pi}}\sum_n \frac{\nu_{ii}\omega}{(\omega-n\Omega_i)^2}\left[1-\frac{k_y v_{Ti}^2}{\omega\Omega_i}\left(\frac{\partial \ln N}{\partial x}-\frac{3\pi+2}{4\pi+4}\frac{\partial \ln T_i}{\partial x}\right)\right]. \tag{4}$$

If, however, $\omega,\ \omega - n\Omega_i \ll \nu_{ii} k_\perp^2 v_{Ti}^2/\Omega_i^2$, then ($C_0 \approx 0.914,\ C_1 \approx 0.225$),

$$\delta\varepsilon_i = \frac{\omega_{Li}^2}{k^2 v_{Ti}^2}\left\{1 + i\frac{\omega\Omega_i^3}{\nu_{ii} k_\perp^3 v_{Ti}^3}\frac{2}{3\pi}C_0\left[1-\frac{k_y v_{Ti}^2}{\omega\Omega_i}\left(\frac{\partial \ln N}{\partial x}-C_1\frac{\partial \ln T_i}{\partial x}\right)\right]\right\}. \tag{5}$$

Below we shall give solutions of the eikonal equation in the form of local oscillation spectra (^{17}).

In the frequency region $|\omega + i\nu_e| \ll k_z v_{Te} \ll \Omega_e$, $\delta\varepsilon_e$ may be written in the form

$$\delta\varepsilon_e = \frac{\omega_{Le}^2}{k^2 v_{Te}^2}\left\{1 + i\sqrt{\frac{\pi}{2}}\frac{\omega}{|k_z| v_{Te}}\left(1-\frac{k_y v_{Te}^2}{\omega\Omega_e}\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_e}}\right)\right\}. \tag{6}$$

Let $\omega \ll \Omega_i$. In the long-wavelength limit, when $k_\perp v_{Ti} \ll \Omega_i$, such oscillations can be unstable only under the condition $\omega_{\text{dr}} \sim (k_y v_T^2/\Omega)(1/L_0) \gg k_z v_s$, where $v_s = \sqrt{T_e/M}$ is the sound velocity. Then ($\omega \to \omega + i\gamma,\ \gamma \ll \omega$)

$$\omega_1 = -\frac{1}{1+k^2 r_{De}^2}\,\frac{k_y v_s^2}{\Omega_i}\,\frac{\partial \ln N}{\partial x},$$

$$\gamma_1 = \sqrt{\frac{\pi}{2}}\,\frac{\omega_1^2}{|k_z| v_{Te}}\left(\frac{k^2 r_{De}^2 + k_\perp^2 v_s^2/\Omega_i^2\,(1 + T_i\,\partial \ln N T_i / T_e\,\partial \ln N)}{1+k^2 r_{De}^2} - \frac{1}{2}\frac{\partial \ln T_e}{\partial \ln N}\right) -$$

$$-\frac{7}{10}\nu_{ii}\,\frac{k_\perp^4 v_{Ti}^4}{\Omega_i^4}\left\{\frac{T_e}{T_i} + (1+k^2 r_{De}^2)\left(1 - \frac{3}{28}\frac{\partial \ln T_i}{\partial \ln N}\right)\right\}; \tag{7}$$

$$\omega_2 = k_z^2 \Omega_i / k_y\,\frac{\partial \ln N}{\partial x},$$

$$\gamma_2 = -\sqrt{\frac{\pi}{2}}\,\frac{\omega_2^2}{|k_z| v_{Te}}\left(1 - \frac{1}{2}\frac{\partial \ln T_e}{\partial \ln N}\right) - \frac{8}{5}\nu_{ii}\,\frac{k_z^2 v_{Ti}^2}{\omega_2^2}. \tag{8}$$

Here r_D is the Debye radius of the particles. The second branch of oscillations is possible only in a plasma in which $T_e \gg T_i$. It is seen that ion collisions exert a stabilizing influence on this branch of oscillations, whereas the Cerenkov effect on the electrons can lead to instability, provided that $\partial \ln T_e / \partial \ln N > 2$ (see ([15])). For formula (7), electron dissipation at $\partial \ln T_e / \partial \ln N > 2(k^2 r_{De}^2 + k_\perp^2 v_s^2/\Omega_i^2)$ stabilizes the oscillations, while ion-ion collisions, on the contrary, can lead to excitation under the condition $\partial \ln T_i / \partial \ln N > 28/3$.

If $k_\perp v_{Ti} \gg \Omega_i$, then in the frequency region under consideration we find

$$\omega = -\frac{T_e}{T_e + T_i(1+k^2 r_{De}^2)}\,\frac{k_y v_{Ti}}{\sqrt{2\pi}\,k_\perp}\,\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_i}},$$

$$\gamma = \frac{T_e T_i}{[T_e + T_i(1+k^2 r_{De}^2)]^2}\,\frac{k_y^2 v_s^2 v_{Ti}}{2|k_z| k_\perp v_{Te} \Omega_i}\left(\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_e}}\right)\left(\frac{\partial}{\partial x}\ln\frac{N}{\sqrt{T_i}}\right) -$$

$$-\sqrt{2\pi}\,\nu_{ii}\,\frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2}\,\frac{3(\pi+1)}{8\sqrt{\pi}}\,\frac{\partial \ln N T_i^{-0.69}}{\partial \ln N T_i^{-0.5}}. \tag{9}$$

Here the Cerenkov effect leads to excitation of the oscillations, while ion-ion collisions can stabilize them, except for the region $2 > \partial \ln T_i / \partial \ln N > 1.45$, in which they enhance the instability.

Under the condition $|\omega + i\nu_e| \ll k_z v_{Te}$, short-wavelength drift-cyclotron oscillations are also possible (cf. ([16])) $\omega \approx n\Omega_i$, provided that $\Delta = \omega - n\Omega_i \gg \nu_{ii} k_\perp^2 v_{Ti}^2 / \Omega_i^2$. For $\omega \ll \omega_{\mathrm{dr}}$ we have

$$\operatorname{Re}\Delta = -\frac{T_e}{T_i + T_e(1 + k^2 r_{De}^2)} \frac{k_y v_{Ti}}{\sqrt{2\pi}\, k_\perp} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_i}},$$

$$\gamma = \operatorname{Im}\Delta = \sqrt{\frac{\pi}{2}} \frac{\operatorname{Re}\Delta^2}{|k_z| v_{Te}} \frac{\partial \ln N/\sqrt{T_e}}{\partial \ln N/\sqrt{T_i}} - \nu_{ii} \frac{3\sqrt{2}(\pi+1)}{8} \frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2} \frac{\partial \ln T_i^{-0.69}}{\partial \ln T_i^{-0.5}}.$$

Here too the Cherenkov effect on electrons always leads to the growth of oscillations, whereas ion collisions play a destabilizing role only under conditions when $2 > \partial \ln T_i / \partial \ln N > 1.45$. Let us now consider

Fig. 1
Fig. 1

Fig. 2
Fig. 2

oscillations in the frequency range $|\omega + i\nu_e| \gg k_z v_{Te}$. If, in addition, $\omega \gg \nu_e$, then

$$\delta\varepsilon_e = \frac{\omega_{Le}^2}{k^2 v_{Te}^2} \left\{ \frac{k^2_{yv_{Te}}}{\omega\Omega_e} \frac{\partial \ln N}{\partial x} + i \frac{k_z^2 v_{Te}^2}{\omega^2} \frac{\nu_{\text{eff}}}{\omega} \left(1 - \frac{k^2_{yv_{Te}}}{\omega\Omega_e} \frac{\partial}{\partial x} \ln \frac{N}{\sqrt{T_e}} \right) \right\}. \tag{11}$$

If, however, $\omega \ll \nu_e$, then

$$\delta\varepsilon_e = \frac{\omega_{Le}^2}{k^2 v_{Te}^2} \left\{ \frac{k^2_{yv_{Te}}}{\omega\Omega_e} \frac{\partial \ln N}{\partial x} + i1.96 \frac{k_z^2 v_{Te}^2}{\omega\nu_{\text{eff}}} \left(1 - \frac{k^2_{yv_{Te}}}{\omega\Omega_e} \frac{\partial}{\partial x} \ln N T_e^{1.71} \right) \right\}. \tag{12}$$

Here $\nu_{\text{eff}} = {}^4/_3 \sqrt{2\pi}/m e^4 N L / T_e^{3/2}$.

Then, in the region of low frequencies ($\omega \ll \Omega_i$) and long waves ($k_\perp v_{Ti} \ll \Omega_i$),

$$\omega_1 = \frac{1}{1 + v_A^2/c^2} \frac{k^2_{yv_{Ti}}}{\Omega_i} \frac{\partial \ln N T_i}{\partial x}, \qquad \text{for } \omega \sim \omega_{\text{dr}} \gg \omega_s$$

$$\gamma_1 = \frac{1.96\omega_s}{1 + v_A^2/c^2} \left\{ 1 + \frac{T_e}{T_i} \left(1 + \frac{v_A^2}{c^2} \right) \frac{\partial \ln N T_e^{1.71}}{\partial \ln N T_i} \right\} -$$

$$-\frac{7}{10} \nu_{ii} \frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2} \left\{ \frac{1}{1 + v_A^2/c^2} - \frac{\partial \ln N}{\partial \ln N T_i} + \frac{3}{28} \frac{\partial \ln T_i}{\partial \ln N T_i} \right\}, \tag{13}$$

$$\omega_2 = -\frac{k_{yv} s^2}{\Omega_i} \frac{\partial}{\partial x} \ln N T_e^{1.71}, \qquad \text{for } \omega \sim \omega_{\mathrm{dr}} \ll \omega_s \tag{14}$$

$$\gamma_2 = \frac{\omega_2^2}{1.96 \omega_s} \left(1 + \frac{v_A^2}{c^2} + \frac{T_i}{T_e} \frac{\partial \ln N T_i}{\partial \ln N T_e^{1.71}} \right)$$

$$\omega_3 = i 1.96\, \omega_s \frac{T_e}{T_i} \frac{\partial \ln N T_e^{1.71}}{\partial \ln N T_i}, \qquad \text{for } \omega \sim \omega_s \ll \omega_{\mathrm{dr}}. \tag{15}$$

Here v_A is the Alfvén velocity,

$$\omega_s = \frac{k_z^2}{k_\perp^2} \frac{M}{m} \frac{\Omega_i^2}{\nu_{\mathrm{eff}}}.$$

It is seen that oscillations in the frequency range $\omega \lesssim \omega_s$ are always unstable because of electron collisions. In the frequency range $\omega \gg \omega_s$, electron collisions stabilize the instability; ion collisions, for $k_z \approx 0$, can lead to growth when $(31 + 3 v_A^2/c^2) \partial \ln T_i / \partial \ln N < 28 v_A^2/c^2$.

In the short-wavelength limit $(\omega \ll \Omega_i, \nu_e)$ we have

$$\omega = \frac{1}{1 + k^2 r_{Di}^2} \frac{k_{yv_{Ti}}^2}{\Omega_i} \frac{\partial \ln N}{\partial x}, \tag{16}$$

$$\gamma = -1.96 \frac{T_i}{T_e} \frac{k_z^2 v_{Te}^2}{\nu_{\mathrm{eff}}} \left\{ \frac{1}{1 + k^2 r_{Di}^2} + \frac{T_e}{T_i} \left(1 + 1.71 \frac{\partial \ln T_e}{\partial \ln N} \right) \right\} + \gamma_i,$$

$$\gamma_i = \begin{cases} -\nu_{ii} \dfrac{k_\perp v_{Ti}}{\Omega_i} \dfrac{3(3\pi + 2)}{32\sqrt{\pi}} \left(\dfrac{\partial \ln T_i}{\partial \ln N} - \dfrac{1.45 k^2 r_{Di}^2}{1 + k^2 r_{Di}^2} \right), & \text{for } \omega \gg \nu_{ii} \frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2}; \quad (17) \\ -\dfrac{\omega^2 \Omega_i^3}{\nu_{ii} k_\perp^3 v_{Ti}^3} \dfrac{2}{3\pi} C_0 C_1 \left(\dfrac{\partial \ln T_i}{\partial \ln N} - \dfrac{4.45 k^2 r_{Di}^2}{1 + k^2 r_{Di}^2} \right), & \text{for } \omega \ll \nu_{ii} \frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2}. \quad (18) \end{cases}$$

Here electron collisions stabilize, while ion collisions lead to the growth of oscillations, provided only that
$\partial \ln T_i / \partial \ln N < 4.45 k^2 r_{Di}^2$.

The spectrum of oscillations (16) is preserved for $\Omega_i \gg \omega \gg \nu_e$. In this case

$$\gamma = -\frac{T_e}{T_i} \frac{k_z^2 v_{Te}^2 \nu_{\mathrm{eff}}}{\omega^2} \left[\frac{1}{1 + k^2 r_{Di}^2} + \frac{T_e}{T_i} \frac{\partial \ln N / \sqrt{T_e}}{\partial \ln N} \right] + \gamma_i. \tag{19}$$

Here electron collisions can already become the cause of instability, provided only that
$\partial \ln T_e / \partial \ln N > 2$.

Let now $|\omega + i\nu_e| \gg k_z v_{Te}$. Then, for
$\Delta = \omega - n\Omega_i \ll \nu_{ii} k_\perp^2 v_{Ti}^2 / \Omega_i^2$, the spectrum of drift-cyclotron oscillations is determined by expressions (16) and (19) for $n\Omega_i \ll \nu_e$ and $n\Omega_i \gg \nu_e$, respectively; in both cases γ_i is to be understood as expression (18). If, however,
$\Delta \gg \nu_{ii} k_\perp^2 v_{Ti}^2 / \Omega_i^2$, then at the intersection of the drift and cyclotron branches of the oscillations hydrodynamic growth of the oscillations is possible both for $\omega \gg \nu_e$ and for $\omega \ll \nu_e$. This growth is of collisionless nature [16], with growth increment
$\gamma \sim \sqrt[4]{m/M}$. When collisions are taken into account, a new instability appears with spectrum ($\omega \ll \omega_{\text{dr}}$)

$$\operatorname{Re} \Delta = \frac{n\Omega_i^2}{\sqrt{2\pi}\, k_\perp v_{Ti}},$$

$$\gamma = \operatorname{Im} \Delta = -\nu_{ii} \frac{k_\perp^2 v_{Ti}^2}{\Omega_i^2} \frac{3(\pi+1)}{4\sqrt{2}} \frac{\partial \ln N T_i^{-0.69}}{\partial \ln N T_i^{-0.5}} + \gamma_e, \tag{20}$$

$$\gamma_e = \begin{cases} \dfrac{\nu_{\text{eff}} \operatorname{Re} \Delta^2}{\omega^4} \sqrt{2\pi}\, k_z^2 v_{Te}^2 \dfrac{k_\perp v_{Ti}}{\Omega_i} \dfrac{\partial \ln N/\sqrt{T_e}}{\partial \ln N/\sqrt{T_i}}, & \text{for } \omega \gg \nu_{\text{eff}}, \\ 1.96 \dfrac{\operatorname{Re} \Delta^2}{\nu_{\text{eff}}} \dfrac{k_z^2 v_{Te}^2}{\omega^2} \sqrt{2\pi} \dfrac{k_\perp v_{Ti}^2}{\Omega_i} \dfrac{\partial \ln N T_e^{1.71}}{\partial \ln N \sqrt{T_i}}, & \text{for } \omega \ll \nu_{\text{eff}}. \end{cases} \tag{21}$$

It follows from this that for $\omega \gg \nu_{\text{eff}}$ electron collisions lead to the growth of oscillations practically always, while for $\omega \ll \nu_{\text{eff}}$ only when
$\partial \ln T_i / \partial \ln N < 2$. Ion collisions, however, stabilize the instability, except for the region where
$2 > \partial \ln T_i / \partial \ln N > 1.45$.

In Figs. 1 and 2 the boundaries of the plasma instability region are given, respectively, for
$\partial \ln T / \partial \ln N = 0$ and $\partial \ln T / \partial \ln N = 1.6$,
$\sigma = k_z L_0$,
$z = k_\perp^2 v_{Ti}^2 / \Omega_i^2$
$(L_0^{-1} = \partial \ln N / \partial x,\ T_e = T_i,\ v_A = 0.1c,\ \alpha = \nu_{ii} / k_z v_{Ti})$.

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Note: Figure translations are in progress. See original paper for figures.

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