ON THE INFLUENCE OF A DOUBLE GRATING OF SPECIAL GEOMETRIC SHAPE ON THE RADIATION OF AN ACTIVE PLANE-PARALLEL LAYER

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Abstract

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ON THE INFLUENCE OF A DOUBLE GRATING OF SPECIAL GEOMETRIC SHAPE ON THE RADIATION OF AN ACTIVE PLANE-PARALLEL LAYER

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There are a number of works devoted to the study of the influence of metallic gratings on radiation by an active plane-parallel layer [1–3]. In the present paper, by the method developed in [4, 5], a rigorous solution is given of the electrodynamic problem on the influence of a grating of special geometric shape, applied symmetrically on both sides to a plane-parallel active layer, in the case when the vector **E** is parallel to the strips forming the grating. The paper derives the self-excitation condition, as well as the principal energy characteristics: the energy density and the output radiation power.

The analytical expressions obtained are convenient for numerical calculation and make it possible to conclude that the energy characteristics of the radiation can be controlled by means of gratings applied to the walls of the layer. Moreover, calculations performed for ordinary gratings allow one to conclude that this control can be quite effective.

1. FORMULATION OF THE PROBLEM. DETERMINATION OF THE FIELDS

Let gratings of special geometric shape be applied symmetrically on both sides to a plane-parallel active layer of thickness a with refractive index of the medium N (i.e., gratings in each period of which there are two perfectly conducting and infinitely thin strips of different widths) with alternating gaps d and d_1, of period l. Denote the space above the first grating ($z \gg 0$) by region I, the space between the gratings ($-a \leq z \leq 0$) by region II, and the space below the lower grating ($z \leq -a$) by region III. Without loss of generality we shall assume that the surrounding space (I, III) is vacuum.

The mathematical formulation of the problem consists in finding a unique solution of the Helmholtz equation for the n-th harmonic

$$\frac{\partial^2 E_{nx}^{(j)}}{\partial y^2} + \frac{\partial^2 E_{nx}^{(j)}}{\partial z^2} + k^2 \varepsilon_j E_{nx}^{(j)} = 0 \quad (j = 1, 2, 3), \tag{1}$$

in each of the regions (I-III) (see Fig.), satisfying the boundary conditions

$$
\begin{array}{lll}
E_{It} = E_{IIt} = 0 & & \text{(on the strip)} \\
E_{It} = E_{IIt}; \quad \left.\dfrac{1}{\varepsilon_1}\dfrac{\partial E_{It}}{\partial z} = \dfrac{1}{\varepsilon_2}\dfrac{\partial E_{IIt}}{\partial z}\right\}, & z = 0 & \text{(on the slit)}, \\
E_{IIt} = E_{IIIt} = 0 & & \text{(on the strip)} \\
E_{IIt} = E_{IIIt}; \quad \left.\dfrac{1}{\varepsilon_2}\dfrac{\partial E_{IIt}}{\partial z} = \dfrac{1}{\varepsilon_3}\dfrac{\partial E_{IIIt}}{\partial z}\right\}, & z = -a & \text{(on the slit)}.
\end{array}
\tag{2}
$$

The field components of regions (I-III) satisfy the radiation condition at infinity.

If the solution of equation (1) is sought in the form $e^{i\left(h_n^{(j)} z + \frac{2\pi n}{l} y\right)}$, then the propagation constant of the electromagnetic waves for each of the media will be equal to:

$$h_n = \sqrt{k^2 - \left(\frac{2\pi n}{l}\right)^2} \quad \text{for media (I, III)},$$

$$h_n = \sqrt{k^2 N^2 - \left(\frac{2\pi n}{l}\right)^2}$$

for medium (II).

In region I the electromagnetic field is outgoing in the direction of the axis $z > 0$. In the second region the field is a superposition of plane electromagnetic waves that propagate or decay in the direction from one grating of special geometric form to the other; in the third region there is a field outgoing in the direction opposite to the axis $z < -a$. The electromagnetic field arising due to the induced radiation of the substance is periodic along the axis OY; therefore the field **E** for each region may be written as a Fourier series in the same functions $e^{i\frac{2\pi n}{l}y}$. Then the field **E** for the corresponding regions takes the following form:

$$
\begin{aligned}
\mathbf{E}_I &= \mathbf{i} \sum_{n=-\infty}^{\infty} A_n e^{ih'_n z} e^{i\frac{2\pi n}{l}y} \qquad (z > 0), \\
\mathbf{E}_{II} &= \mathbf{i} \sum_{n=-\infty}^{\infty} \left\{ B_n e^{-ih_n z} + C_n e^{ih_n (z+a)} \right\} e^{i\frac{2\pi n}{l}y} \qquad (0 > z > -a), \\
\mathbf{E}_{III} &= \mathbf{i} \sum_{n=-\infty}^{\infty} D_n e^{-ih_n (z+a)} e^{i\frac{2\pi n}{l}y} \qquad (z < -a).
\end{aligned}
\tag{3}
$$

From Maxwell' s equations we find the components **H** for each of the regions, respectively (we omit the time factor $e^{-i\omega t}$):

$$\mathbf{H}_I = \mathbf{j} \sum_{n=-\infty}^{\infty} A_n \frac{h'_n}{k} e^{ih'_n z} e^{i\frac{2\pi n}{l} y} - \mathbf{k} \sum_{n=-\infty}^{\infty} A_n \frac{n}{\chi} e^{ih'_n z} e^{i\frac{2\pi n}{l} y} \qquad (z > 0),$$

$$\mathbf{H}_{\mathrm{II}} = \mathbf{j} \sum_{n=-\infty}^{\infty} \left\{-B_n e^{ih_n z} + C_n e^{ih_n(z+a)}\right\} \frac{h_n}{k} e^{i\frac{2\pi n}{l} y} - \tag{4}$$

$$- \mathbf{k} \sum_{n=-\infty}^{\infty} \left\{B_n e^{-ih_n z} + C_n e^{ih_n(z+a)}\right\} \frac{n}{\varkappa} e^{i\frac{2\pi n}{l} y} \qquad (0 > z > -a),$$

$$\mathbf{H}_{\mathrm{III}} = \mathbf{j} \sum_{n=-\infty}^{\infty} (-D_n) \frac{h'_n}{k} e^{-ih'_n(z+a)} e^{i\frac{2\pi n}{l} y} -$$

$$- \mathbf{k} \sum_{n=-\infty}^{\infty} D_n \frac{n}{\varkappa} e^{ih_n(z+a)} e^{i\frac{2\pi n}{l} y} \qquad (z < -a),$$

where A_n, B_n, C_n, D_n are unknown Fourier coefficients in the field expressions (3), (4).

Owing to the periodicity of our structure, in order to determine the field in the whole space it is sufficient to find it in one of the periods. The unknown coefficients of the expansions (3), (4) are found from the boundary conditions (2).

Substituting the field expressions (3), (4) into the boundary conditions (2), we obtain a number of relations between the diffraction harmonics, which are satisfied over the entire period and over its parts—the slit and the metal:

$$A_n = B_n + C_n e^{ih'_n a}, \qquad D_n = B_n e^{ih_n a} + C_n;$$

$$\sum_{n=-\infty}^{\infty} \left\{B_n + C_n e^{ih'_n a}\right\} e^{i\frac{2\pi n}{l} y} = 0 \quad \text{for } z = 0 \text{ (on the strip)}, \tag{4a}$$

$$\sum_{n=-\infty}^{\infty} \left\{B_n e^{ih'_n a} + C_n\right\} e^{i\frac{2\pi n}{l} y} = 0 \quad \text{for } z = -a \text{ (on the strip)},$$

$$\sum_{n=-\infty}^{\infty} \left\{B_n (h'_n + h_n) - C_n (h'_n - h_n) e^{ih'_n a}\right\} e^{i\frac{2\pi n}{l} y} = 0 \quad \text{for } z = 0 \text{ (on the slit)},$$

(4)

$$\sum_{n=-\infty}^{\infty} \left\{ B_n (h'_n - h_n) e^{ih'_n a} + C_n (h'_n + h_n) \right\} e^{i\frac{2\pi n}{l} y} = 0 \quad \text{for } z = -a \text{ (on the slit).}$$

The grating of special geometric form considered by us is symmetric with respect to the plane $y = 0$; therefore the range of variation of y for the strips is one and the same in the planes $z = 0$ and also $z = -a$. Similar reasoning applies to the slit region as well. Consequently, if one adds and subtracts the relations that are satisfied on the strip in (4a) and (4), then the new relations can be written with a single summation index and the common factor exp $\left\{i\frac{2\pi n}{l}y\right\}$ can be taken out. Carrying out the analogous operation with the relations on the slits, we obtain

$$\sum_{n=-\infty}^{\infty} (B_n + C_n)(1 + e^{ih'_n a}) e^{i\frac{2\pi n}{l} y} = 0$$

$$\sum_{n=-\infty}^{\infty} (B_n - C_n)(1 - e^{ih'_n a}) e^{i\frac{2\pi n}{l} y} = 0. \tag{5}$$

$$\sum_{n=-\infty}^{\infty} (B_n + C_n)(h'_n + h_n) \left(1 - \frac{h'_n - h_n}{h'_n + h_n} e^{ih'_n a}\right) e^{i\frac{2\pi n}{l} y} = 0$$

$$\sum_{n=-\infty}^{\infty} (B_n - C_n)(h'_n - h_n) \left(1 + \frac{h'_n - h_n}{h'_n + h_n} e^{ih'_n a}\right) e^{i\frac{2\pi n}{l} y} = 0. \tag{6}$$

It should be noted that the physical meaning is possessed by that value of h_n and h'_n for which $\operatorname{Im} h_n > 0$, $\operatorname{Im} h'_n > 0$, while for $\operatorname{Im} h_n = 0$, $\operatorname{Im} h'_n = 0$, $\operatorname{Re} h_n > 0$, $\operatorname{Re} h'_n > 0$ (in our case the radiation conditions are equivalent to the absorption principle).

II. SOLUTION OF THE BOUNDARY-VALUE PROBLEM

Make the substitution:

$$a_n = (B_n + C_n)(1 + e^{ih'_n a}); \qquad \beta_n = (B_n - C_n)(1 - e^{ih'_n a});$$

$$\varphi = \frac{2\pi y}{l}; \qquad \theta = \frac{\pi(l-d)}{l}, \qquad \theta_1 = \frac{\pi d_1}{l}.$$

Then

$$\sum_{n \neq 0} X_n e^{in\varphi} = 0, \qquad |\varphi| < \theta, \quad \theta_1 < |\varphi| < \pi, \tag{7}$$

$$\sum_{n\neq 0} X_n \frac{|n|}{n} e^{in\varphi} = i\chi \widehat{G}_0^x a_0 + \sum_{n\neq 0} X_n \frac{|n|}{n} \psi'_n e^{in\varphi}, \qquad \theta_1 < |\varphi| < \theta, \tag{8}$$

$$\sum_{n\neq 0} Y_n e^{in\varphi} = 0, \qquad |\varphi| < \theta, \quad \theta_1 < |\varphi| < \pi, \tag{9}$$

$$\sum_{n\neq 0} Y_n \frac{|n|}{n} e^{in\varphi} = i\chi \widehat{G}_0^y \beta_0 + \sum_{n\neq 0} Y_n \frac{|n|}{n} \xi'_n e^{in\varphi}, \qquad \theta_1 < |\varphi| < \theta, \tag{10}$$

where

$$\psi'_n = 1 + \frac{1}{2}\frac{\chi}{|n|}\frac{\gamma_n + \gamma'_n - (\gamma_n - \gamma'_n)e^{ih_n a}}{1 + e^{ih_n a}}; \qquad \gamma_n = \sqrt{\frac{\chi^2}{n^2}N^2 - 1};$$

$$\xi'_n = 1 + \frac{1}{2}\frac{\chi}{|n|}\frac{\gamma_n + \gamma'_n - (\gamma_n - \gamma'_n)e^{ih_n a}}{1 - e^{ih_n a}}; \qquad \gamma'_n = \sqrt{\frac{\chi^2}{n^2} - 1}.$$

The systems of equations (7)–(10) can be solved by the method proposed in [4, 5].

The condition that the system of linear homogeneous algebraic equations obtained from (7), (8), as well as from (9)–(10), be equal to zero gives the self-excitation conditions of the system and takes the following form:

$$\hat{G}_0^x = i\,\frac{D_{0\psi'}}{\chi\Delta_{0\psi'}}, \tag{11}$$

$$\hat{G}_0^y = i\,\frac{D_{0\xi'}}{\chi\Delta_{0\xi'}}, \tag{12}$$

where $D_{0\psi'}, \Delta_{0\psi'}$ and $D_{0\xi'}, \Delta_{0\xi'}$ are the determinants of the system, which were calculated in [5] and have the following form:

$$\Delta = \Delta_0, \qquad \text{and} \qquad D = D_0$$

(for $n = 0$, $\xi'_n = \psi'_n = 0$ the long-wavelength approximation is considered, when $\chi = \dfrac{kl}{2\pi} \ll 1$). Then

$$\Delta_0 = -4\left(R_{[\sigma]} \cdot \tilde{R}^1_{[\sigma]} - R_{[\sigma]} \cdot R^1_{[\sigma]}\right),$$

$$D_{\eta 0} = -2\left(R^1_{[\sigma]} - \tilde{R}^1_{[\sigma]}\right), \tag{13}$$

where

$$\tilde{R}^1_{[\sigma]} = \frac{1}{2\sqrt{2(1-u)}} \ln \frac{1-v}{32} \frac{\left(1-u+\sqrt{2(1-u)}\right)^2}{(1-u)^2},$$

$$R^1_{[\sigma]} = \frac{1}{\sqrt{2(1-u)}} \ln \frac{\sqrt{1-u^2}}{1-u+\sqrt{2(1-u)}},$$

$$\tilde{R}_{[\sigma]} = \tilde{R}^1_{[\sigma]} - \ln\left(1+\sqrt{\frac{1-u}{2}}\right); \qquad R_{[\sigma]} = R^1_{[\sigma]} - \frac{1}{2}\ln\frac{1+u}{2},$$

(here the grating parameters are expressed through $u = \cos\theta_1$ and $v = \cos\theta$).

Thus, it is clear that the simultaneous solution of the dispersion equations (11)–(12) gives the self-excitation condition of the layer with the adjoining gratings.

In [6] it was shown that in a plane-parallel layer the flux of emitted radiation is directed from the middle of the layer toward the edges; therefore the amplitudes of the diffraction harmonics B_n and C_n will be equal in modulus and related by the relation $B_n = (-1)^s C_n$ $(s = 1, 2, ...)$.

In the case when s is even, only expression (11) is used, while when s is odd, equation (12) plays the role of the self-excitation condition. It should be noted that, in general form, equations (11)–(12) are cumbersome transcendental equations requiring solution on a computer.

Let us examine the case when s is even.

III. RADIATION CONDITION. ENERGY CHARACTERISTICS

Transform equation (11) to the form

$$\frac{N+1}{2}\,\frac{1-\sqrt{r}\,e^{i(kNa-\delta)}}{1+e^{ikNa}} = \frac{i}{\chi}\,\frac{Q_0}{\ln\frac{1+u}{2}}, \tag{14}$$

where

$$Q_0 = \frac{1-L}{\left\{-L + \dfrac{2\ln\left(1+\sqrt{\dfrac{1-u}{2}}\right)}{\ln\dfrac{1+u}{2}}\right\}}; \qquad L = \frac{\widetilde{R}_{[\sigma]}}{R_{[\sigma]}};$$

$$\frac{N-N_1}{N+N_1}=\sqrt{r}\,e^{-i\delta}.$$

After a number of calculations, (14) is somewhat simplified:

$$\frac{1-\sqrt{r}\,e^{i(kNa-\delta)}}{1+e^{ikNa}}=\frac{i}{\chi}\,\frac{2Q_0}{(N+1)\ln\dfrac{1+u}{2}}=i\widetilde{Q}_0te^{-i\delta_1}. \tag{15}$$

Let us introduce the notation:

$$te^{-i\delta_1}=\frac{2}{\chi(N+1)\ln\dfrac{1+u}{2}};\qquad \operatorname{tg}\delta=\frac{-2N_0\chi}{N_0^2-1+\chi^2};\qquad \operatorname{tg}\delta_1=\frac{\chi}{N_0+1}.$$

Taking into account that δ and δ_1 are small, we represent (15) in the following form:

$$\sqrt{R}\,e^{\,i(2\pi N\frac{a}{\lambda}+\hat{\gamma})}=1, \tag{16}$$

where

$$R=\frac{r+\widetilde{Q}_0^2t^2}{1+\widetilde{Q}_0^2t^2},$$

$$\hat{\gamma}=\operatorname{arctg}\frac{\widetilde{Q}_0t}{\sqrt{r}}+\operatorname{arctg}\widetilde{Q}_0t+\tilde{\gamma}',$$

$\tilde{\gamma}'$ is a small correction. Then the self-excitation condition (16) has the form

$$Re^{-\kappa a}=1,\qquad kN_0a+\hat{\gamma}=2\pi s\quad(s=1,2,...). \tag{17}$$

Knowing the self-excitation condition for a plane-parallel layer, one can determine the energy characteristics for the electric and magnetic fields, as well as the output radiation power. For a layer with gratings of special geometric shape the energy characteristics are as follows:

$$U_{\mathrm{e}}=\frac{1}{2}U_0\left[1-\left(\frac{\lambda\ln R}{4\pi N_0a}\right)^2\right]\left\{\chi\left(\frac{\xi'}{a}\ln R\right)+\cos\left[(2\pi s-\hat{\gamma})\frac{\xi'}{a}\right]\right\};$$

$$U_{\mathrm{m}} = \frac{1}{2}U_0 \left[1 + \left(\frac{\lambda \ln R}{4\pi N_0 a}\right)^2\right] \left\{\chi\left(\frac{\xi'}{a}\ln R\right) - \cos\left[(2\pi s - \hat{\gamma})\frac{\xi'}{a}\right]\right\}; \tag{18}$$

$$U_{\mathrm{tot}} = U_{\mathrm{e}} + U_{\mathrm{m}} = U_0 \left\{\chi\left(\frac{\xi'}{a}\ln R\right) - \left(\frac{\lambda \ln R}{4\pi N_0 a}\right)^2 \cos\left[(2\pi s - \hat{\gamma})\frac{\xi'}{a}\right]\right\} \tag{19}$$

$$W = -kU_0 \frac{c}{N_0} \left\{\operatorname{sh}\left(\frac{\xi'}{a}\ln R\right) + \frac{\lambda \ln R}{4\pi N_0 a} \sin\left[(2\pi s - \hat{\gamma})\frac{\xi'}{a}\right]\right\}, \tag{20}$$

where

$$\xi' = z + \frac{a}{2}; \qquad U_0 = |B_0'|^2 \frac{N_0^2}{4\pi}.$$

The quantity U_0 remains undetermined; it can be found by invoking the theory of nonlinear optics [6]. After this, neglecting the oscillatory terms, (19), (20) will respectively take the form

$$W = -\frac{c}{N_0 a} \frac{K_0 a - \ln R}{R - 1} \sqrt{R} \operatorname{sh}\left[\left(\frac{z}{a} + \frac{1}{2}\right)\ln R\right], \tag{21}$$

$$U_{\mathrm{tot}} = \frac{1}{a} \frac{K_0 a - \ln R}{R - 1} \sqrt{R} \operatorname{ch}\left[\left(\frac{z}{a} + \frac{1}{2}\right)\ln R\right]. \tag{22}$$

IV. ANALYSIS OF THE EXPRESSIONS OBTAINED

From relation (17) it is not difficult to show that it passes into expression (1) of [6] if one assumes that the gratings are absent ($-u = v = 1$), since in this case $R = r$, $\tilde{\gamma} = -\delta$. One may also verify that the internal-radiation density and the output radiation power for an active isotropic layer with adjoining gratings of special geometric shape, obtained in the present work, correspond to their values calculated in [6]. Therefore the considerations set forth in [6] with respect to r apply fully also to R. Since R (see (16)) contains the geometric dimensions of the grating of special form: l, d, d_1, and $x = \frac{l}{\lambda}$, while, in turn, the energy reflection coefficient R is contained in the analytic expressions (21), (22), which are convenient for calculations, then, by specifying the parameters of the double grating of special geometric form, one can, by varying the quantities r, $p = \frac{x}{2}(N_0 + 1)$, $u = \cos\frac{\pi d}{l}$, $v = \cos\frac{\pi d_1}{l}$, control the radiation of a plane-parallel layer for the corresponding polarization.

Numerical calculations carried out in [2] show that the presence of adjoining gratings increases the energy reflection coefficient (and, consequently, increases the output radiation power, especially for small r) and thus confirms the idea of the practical possibility of effective control of the radiation of the layer.

These considerations also extend fully to those results that can be obtained from (21), (22).

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