THE SOMMERFELD EFFECT IN A SYSTEM WITH RANDOMLY VARYING NATURAL FREQUENCY

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Abstract

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M. F. DIMENTBERG, K. V. FROLOV

THE SOMMERFELD EFFECT IN A SYSTEM WITH RANDOMLY VARYING NATURAL FREQUENCY

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More than 60 years ago A. Sommerfeld ([1]) first described a phenomenon observed during the run-up of an unbalanced rotor of a motor mounted on elastic supports, and subsequently called the Sommerfeld effect. It was noted that, as the angular velocity of the rotor approaches the resonant state of the system, the amplitude of oscillations of the elastic support increases sharply, while the angular acceleration of the rotor simultaneously decreases; in order to pass through resonance, additional energy must be supplied to the motor, going toward increasing the speed; after passage through resonance, the angular velocity of the rotor rises sharply. Sommerfeld qualitatively explained this phenomenon by the fact that, near resonance, the elastic system begins to oscillate with large amplitudes, and the energy of the motor goes into "building up" these oscillations rather than into increasing the rotational speed of the rotor.

An analytical solution of the problem of the existence of stable and unstable regimes of oscillation was later given by I. I. Blekhman and G. Yu. Dzhanelidze ([2], [3]).

V. O. Kononenko ([4]) solved a class of problems on the interaction of various oscillatory systems (linear, nonlinear, parametric, self-oscillatory) with an energy source sustaining the oscillations of these systems.

The present article is devoted to the study of the resonant properties of a linear oscillatory system excited by the inertial forces of an unbalanced motor rotor, under random variations of the system' s natural frequency. The corresponding problem for the case of unlimited power of the excitation source was considered in ([5]), where sufficient conditions were found for reducing the amplitude of resonant oscillations by varying the natural frequency according to a random law. Here, as in ([4]), it is assumed that the power of the motor is limited, i.e., comparable with the power consumed by the oscillatory system.

The solution of the problem considered here shows that, if the parameters of the oscillatory system are varied according to a random law, then under certain conditions the Sommerfeld effect can be weakened; i.e., the rotor can pass through the critical resonant state without the supply of additional energy to the motor and with lower values of the amplitude of oscillations of the supports. The qualitative results of the theoretical investigations have been confirmed experimentally.

1. The equations of motion of a rotor mounted on an elastic support have the form ([4])

$$m_0\ddot{x} + 2\beta\dot{x} + cx = mr\dot{\varphi}^2\cos\varphi + mr\ddot{\varphi}\sin\varphi,$$
$$I\ddot{\varphi} + H(\dot{\varphi}) = L(\dot{\varphi}) + mr\ddot{x}\sin\varphi. \tag{1}$$

Here $x(t)$ is the displacement of the support; φ is the angular coordinate of the rotor; β is the coefficient of friction; mr is the unbalance moment; m_0 is the mass of the system; I –

moment of inertia of the rotor; $L(\dot{\varphi})$ and $H(\dot{\varphi})$ are, respectively, the motor torque and the torque of the resistance forces. Let the stiffness of the support c be equal to $c_0[1+\mu\xi(t)]$, where $\xi(t)$ is a stationary centered random function.

Following the ideas of paper ([4]), for the case of near-resonant oscillations under conditions of weak interaction of the rotor with the support, we introduce the notation:

$$\omega_1^2 = \frac{c_0}{m_0}, \qquad \frac{mr}{m_0} = \varepsilon q_2, \qquad \frac{mr}{I} = \varepsilon q_3, \qquad \frac{1}{I}\left[L(\dot{\varphi}) - H(\dot{\varphi})\right] = \varepsilon M_1(\dot{\varphi}), \qquad \frac{\beta}{m_0} = \varepsilon h,$$

where ε is a small parameter. We also set $\mu = \mu_1\sqrt{\varepsilon}$ and, by the change of variables

$$z = \left(x + \frac{\dot{x}}{i\omega_1}\right)e^{-i\varphi}, \qquad z_* = \left(x - \frac{\dot{x}}{i\omega_1}\right)e^{i\varphi}, \qquad \dot{\varphi} = \theta \tag{2}$$

we arrive at equations in standard form

$$\begin{aligned}
\dot{z} = & -\frac{\mu_1\sqrt{\varepsilon}\omega_1}{2i}\xi(t)(z + z_*e^{-2i\varphi}) + \varepsilon\left[-\frac{\Delta z}{i} + \frac{q_2\theta^2}{2i\omega_1}(1 + e^{-2i\varphi})\right.\\
& \left. -h(z - z_*e^{-2i\varphi}) + O(\varepsilon^2)\right] \qquad (\Delta = (\omega_1 - \theta)/\varepsilon),\\
\dot{z}_* = & \frac{\mu_1\sqrt{\varepsilon}\omega_1}{2i}\xi(t)(ze^{2i\varphi} + z_*) + \varepsilon\left[\frac{\Delta z_*}{i} - \frac{q_2\theta^2}{2i\omega_1}(1 + e^{2i\varphi})\right.\\
& \left. +h(ze^{2i\varphi} - z_*) + O(\varepsilon^2)\right],\\
\dot{\theta} = & \varepsilon\left[M_1(\theta) - \frac{q_3\omega_1^2}{4i}(ze^{i\varphi} + z_*e^{-i\varphi})(e^{i\varphi} - e^{-i\varphi})\right].
\end{aligned} \tag{3}$$

To determine the slowly varying components of the mathematical expectations of the functions z, z_*, θ (denote them respectively by z_0, z_{0*}, Ω), we use the ideas of the Krylov-Bogoliubov method ([6]), applied to the investigation of statistical parametric problems in works ([7,8]). The functions z_0, z_{0*}, Ω are assumed to vary slowly in comparison with the correlation function $K(\tau)$ of the process $\xi(t)$. Applying the averaging method to the equations for the mathematical expectations of the functions z, z_*, θ, we obtain from (3)

$$\dot{z}_0 = \varepsilon\,[F(\Omega)z_0 + q_2\Omega^2/2i\omega_1], \qquad \dot{z}_{0*} = \varepsilon\,[F_*(\Omega)z_{0*} - q_2\Omega^2/2i\omega_1],$$

$$F(\Omega),\ F_*(\Omega) = -h - (\pi\mu_1^2\omega_1^2/4)[\Phi(0) - \Phi(2\Omega)]\pm$$

$$\pm i\,[\Delta_0 - (\pi\mu_1^2\omega_1^2/4)\Psi(2\Omega)], \tag{4}$$

$$\dot{\Omega} = \varepsilon\,[M_1(\Omega) - (z_{0*} - z_0)q_3\omega_1^2/4] \qquad (\Delta_0 = (\omega_1 - \Omega)/\varepsilon).$$

Here

$$\Phi(\omega) = \frac{1}{\pi}\int_0^\infty K(\tau)\cos\omega\tau\,d\tau, \qquad \Psi(\omega) = \frac{1}{\pi}\int_0^\infty K(\tau)\sin\omega\tau\,d\tau \tag{5}$$

($\Phi(\omega)$ is the spectral density of the process $\xi(t)$). On the basis of the first two equations (4), the mean amplitude of stationary ($\dot{z}_0 = \dot{z}_{0*} = 0$) oscillations is equal to

$$b = |z_0| = \sqrt{z_0 z_{0*}} = (mr\Omega^2/m_0)\left\{[2\omega_1\beta/m_0 + (\pi\mu^2\omega_1^3/2)(\Phi(0) - \Phi(2\Omega))]^2\right.$$

$$\left. + [2\omega_1(\omega_1 - \Omega) - (\pi\mu^2\omega_1^3/2)\Psi(2\Omega)]^2\right\}^{-1/2}. \tag{6}$$

As in the case of constant stiffness ($\mu = 0$) ([4]), the expression for the amplitude of the oscillations proves to be the same as for a fixed value of Ω.

The third equation (4) at $\dot{\Omega} = 0$ gives

$$L(\Omega) = H(\Omega) + \frac{b^2\omega_1^3}{\Omega}\left\{\beta + (\pi\mu^2 m_0\omega_1^2/4)[\Phi(0) - \Phi(2\Omega)]\right\}. \tag{7}$$

Relations (6), (7) indicate the possibility of reducing the maximum (in the near-resonance region) value of the power consumed by changing the natural frequency of the elastic support according to a random law. This is achieved

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

under the condition that $\Phi(0) > \Phi(2\omega)$ for all values of ω close to ω_1. As a result, a weakening of the Sommerfeld effect should occur.

Fig. 1

2. Figure 1 shows an experimentally obtained oscillogram illustrating the comparatively smooth passage of the system through resonance when the stiffness of the system is varied according to a random law. The experiment was carried out on a mechanical model consisting of an elastic cantilever with a low-power motor mounted at its free end. The oscillations of the elastic cantilever were excited by the inertia forces of the unbalanced rotor of the motor. In parallel with the cantilever beam, an additional stiffness was connected (the relative change in stiffness was about 15%). The connection was carried out according to the law of a repeating sequence of pulses with random duration and repetition frequency, by the method described in (9). As is known (10), such a process has a decreasing spectral density. Oscillogram 1 shows the moments of switching the additional stiffness on and off, as well as the rotational speed of the rotor. Oscillogram 2 illustrates the amplitude of the cantilever oscillations. In this case the power consumed by the motor in the resonant regime was $N_1 = 9.1$ W.

Fig. 2

Figure 2 presents an oscillogram for the case when resonant oscillations had become established in the system. At point a (see oscillogram 1)

a device for changing the stiffness according to a random law was switched on. As a result, there was a decrease in the amplitude of the oscillations, accompanied by an increase in the angular velocity of the motor and a decrease in the power consumed by the motor (a "breakdown" of the resonant oscillations). The motor power in the resonant regime was $N_2 = 12.9$ W, and after passing through resonance $N_3 = 9.6$ W. Consequently, when the stiffness of the system is changed according to a random law, passage of the system through the resonant state occurs without supplying additional energy to the motor, in contrast to a system with constant parameters. Restoration of the initial stiffness of the system does not lead to restoration of the resonant regime. The new steady-state regime (beyond resonance) is determined by the motor characteristic (4). Let us note that analogous phenomena were observed under random changes in the mass of the system (these changes were carried out by connecting and disconnecting an electromagnet).

Institute for Problems of Mechanics
Academy of Sciences of the USSR

State Scientific Research
Institute of Machine Science

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Note: Figure translations are in progress. See original paper for figures.