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Abstract

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CHANGE OF THE GENERATION CHANNEL IN A FOUR-LEVEL QUANTUM GENERATOR

In a number of works ([1,2]) it has been found that, when the operating conditions of a quantum generator are changed, generation passes from one channel to another. Simultaneous generation of several frequencies is also often observed, especially in gas lasers ([3]). In the present work, which is a continuation of ([4]), the conditions for transformation of the generation channel and for joint generation in several channels are considered. The analysis is carried out for the example of a four-level generator with unsplit levels. Similar arguments are also applicable to other cases.

To simplify the formulas, let us assume that the probabilities of nonoptical transitions $3 \to 4$, $2 \to 4$, $1 \to 4$, $2 \to 3$, and $1 \to 3$ are equal to zero. In addition, we shall assume that the accumulation of particles on the fourth level is very small, and one may put

$$n_1 + n_2 + n_3 = n. \tag{1}$$

As a rule, for large separations between the levels these conditions are satisfied.

Let us further assume that generation takes place in the channel $3 \to 2$. The populations of the levels may be found as the solution of a system of three equations: equation (1) and the equations

$$n_3 - \frac{g_3}{g_2} n_2 = n\delta', \tag{2}$$

$$\left[Bu_{\text{pump}} \frac{p_{42} + p_{43}}{p_{42} + p_{41} + p_{43}} + p_{12} \right] n_1 = n_2 p_{21} + n_3 p_{31}. \tag{3}$$

Here g_3 and g_2 are statistical weights,

$$\delta' = k_{23}^{\text{loss}} / \kappa_{32} \tag{4}$$

is the ratio of the loss coefficient in the channel $3 \to 2$ to the limiting gain coefficient realized when $n_3 = n$ (see ([4])); Bu_{pump} is the probability of transition in the pumping channel $1 \to 4$; u_{pump} is the radiation density of the pump; B

is the corresponding Einstein coefficient; p_{ij} are the probabilities of transitions $i \to j$. Equation (2) is the condition of stationary generation, i.e., equality of loss and gain. Equation (3) was obtained from the balance of particles on levels 1 and 4, which are not directly connected with the generation transition.

Solving equations (1)–(3), we obtain

$$n_1 = n\frac{(1-\delta) + \left(\frac{g_3}{g_2} + \delta\right)\frac{p_{31}}{p_{21}}}{1 + \left(1 + \frac{g_3}{g_2}\right)\frac{g_2}{g_1}e^{-h\nu_{21}/kT} + \frac{g_3}{g_2}\frac{p_{31}}{p_{32}} + \left(1 + \frac{g_3}{g_2}\right)\frac{\eta B u_{\text{pump}}}{p_{21}}\frac{p_{43}+p_{42}}{p_{43}}}, \tag{5}$$

$$n_2 = n\frac{(1-\delta)\frac{g_2}{g_1}e^{-h\nu_{21}/kT} - \delta\frac{p_{31}}{p_{32}} + (1-\delta)\frac{\eta B u_{\text{pump}}}{p_{21}}\frac{p_{42}+p_{43}}{p_{43}}}{1 + \left(1 + \frac{g_3}{g_2}\right)\frac{g_2}{g_1}e^{-h\nu_{21}/kT} + \frac{g_3}{g_2}\frac{p_{31}}{p_{32}} + \left(1 + \frac{g_3}{g_2}\right)\frac{\eta B u_{\text{pump}}}{p_{21}}\frac{p_{42}+p_{43}}{p_{43}}}. \tag{6}$$

$$n_3 = n\frac{\delta + \left(\frac{g_3}{g_2} + \delta\right)\frac{g_2}{g_1}e^{-h\nu_{21}/kT} + \left(\frac{g_3}{g_2} + \delta\right)\frac{\eta B u}{p_{21}}\frac{p_{42}+p_{43}}{p_{43}}}{1 + \left(1 + \frac{g_3}{g_2}\right)\frac{g_2}{g_1}e^{-h\nu_{21}/kT} + \frac{g_3}{g_2}\frac{p_{31}}{p_{32}} + \left(1 + \frac{g_3}{g_2}\right)\frac{\eta B u}{p_{21}}\frac{p_{42}+p_{43}}{p_{43}}}. \tag{7}$$

Here it has been taken into account that

$$p_{12} = p_{21}\frac{g_2}{g_1}e^{-h\nu_{21}/kT}. \tag{8}$$

Let us first consider the influence of temperature. Initially, at low temperatures, $n_1 > n_3 > n_2$. Since $n_3 > n_2$, there is amplification in the channel $3 \to 2$, which also ensures generation. Subsequently n_1 falls, while n_2 and n_3 increase. At a certain value of the temperature the population of the third level begins to exceed the population of the first; amplification arises in the channel $3 \to 1$. At the same time, the difference $n_3 - n_1$ is always smaller than $n_3 - n_2$, except for the limiting value $T - \infty$, when they become equal. The larger δ' is, the lower the temperature at which inverted population of levels 3 and 1 is reached. For $\delta' > 1/2$ it is realized at all temperatures.

Table 1

Values of $\left(\frac{g_2}{g_1}e^{-h\nu_{21}/kT}\right)^*$

δ''	$\delta' = 0.2$	$\delta' = 0.3$	$\delta' = 0.4$	$\delta' = 0.5$	$\delta' = 0.6$
0	0.50	0.31	0.17	0	–*
0.2	0.70	0.45	0.25	0.07	–*
0.2	1	0.66	0.40	0.18	0
0.3		1	0.55	0.33	0.10

* Cases for which generation at frequency ν_{32} is impossible and, at any T, generation at frequency ν_{31} occurs.

The occurrence of inverted population at levels 3—1 does not yet mean the occurrence of generation at the frequency ν_{31}. For it to take place, it is necessary that the ratio of the gain coefficient to the loss coefficient at the frequency ν_{31} exceed the corresponding ratio at the frequency ν_{32} (i.e., unity). If $p_{31}+p_{32}$ is neglected in comparison with p_{21}, then this ratio is equal to

$$\frac{k_{31}}{k_{13}^{\text{loss}}} = \frac{n_3 - \frac{g_3}{g_1} n_1}{n\delta''} = \frac{\delta'\left(1+\frac{g_3}{g_1}\right) - \frac{g_3}{g_1} + \left(\frac{g_3}{g_2}+\delta'\right)\frac{g_2}{g_1} e^{-h\nu_{21}/kT}}{\left[1+\left(1+\frac{g_3}{g_2}\right)\frac{g_2}{g_1} e^{-h\nu_{21}/kT}\right]\delta''}, \quad (9)$$

where

$$\delta'' = k_{13}^{\text{loss}}/\varkappa_{31}. \quad (10)$$

Equating (9) to unity and solving with respect to the Boltzmann factor, we obtain

$$\left(\frac{g_2}{g_1} e^{-h\nu_{21}/kT}\right)^* = \frac{\delta'(1+g_3/g_1) - g_3/g_1 - \delta''}{(1+g_3/g_2)\,\delta'' - (g_3/g_2+\delta')}. \quad (11)$$

If the ratio of the gain coefficients to the loss coefficients for the channels $3\to 2$ and $3\to 1$ is equal to unity, then generation occurs in both channels simultaneously. This can be realized only at $T=T^*$, determined from (10). At this temperature the generation condition (2) is satisfied for the frequency ν_{32} and the corresponding condition

$$n_3 - \frac{g_3}{g_1} n_1 = n\delta'' \quad (12)$$

for the frequency ν_{31}. A change in temperature will lead to disruption of generation in one of the channels. For $T<T^*$ the frequency ν_{32} is generated; for $T>T^*$, the frequency ν_{31}.

Table 1 gives the results of calculating (11) for various δ' and δ''. For $\delta''>\delta'$, transformation of the generation channel is impossible.

A change of the generation channel can also occur at constant temperature, but by changing the loss coefficients. For this it is necessary to decrease δ'' or increase δ'. The limiting values of δ'' and δ' are determined from (9):

$$(\delta'')^* = \frac{\delta'\left(1+\frac{g_3}{g_1}\right) - \frac{g_3}{g_1} + \left(\frac{g_3}{g_2}+\delta'\right)\frac{g_2}{g_1} e^{-h\nu_{21}/kT}}{1+\left(1+\frac{g_3}{g_2}\right)\frac{g_2}{g_1} e^{-h\nu_{21}/kT}}, \quad (13)$$

$$(\delta')^* = \frac{\left[1 + \left(1 + \frac{g_3}{g_2}\right)\frac{g_2}{g_1} e^{-h\nu_{21}/kT}\right]\delta'' - \frac{g_3}{g_1}\left(1 - e^{-h\nu_{21}/kT}\right)}{1 + \left(1 + \frac{g_3}{g_2}\right)\frac{g_2}{g_1} e^{-h\nu_{21}/kT}}. \tag{14}$$

The onset of generation at the frequency ν_{31} can sometimes also occur with an increase in the pumping intensity. This is possible, however, only in the case when the probability p_{21} is relatively small and comparable both with the probabilities p_{31} and p_{32}, and with the probability $\eta B u_{\text{nak}}$, i.e., when particles are delayed at level 2. The probability p_{21} is usually unknown and is hardly always large. In view of this, the given case may occur in practice.

In the case under consideration,

$$\frac{k_{31}}{k_{31}^{\text{pot}}} = \frac{\delta' + \frac{\eta B u_{\text{nak}}}{p_{21}}\left(\frac{g_3}{g_2} + \delta'\right)\frac{p_{42}+p_{43}}{p_{43}} - \frac{g_3}{g_1}\left[(1-\delta') + \left(\frac{g_3}{g_2} + \delta'\right)\frac{p_{31}}{p_{21}}\right]}{\left[1 + \frac{g_3}{g_2}\frac{p_{31}}{p_{32}} + \left(1 + \frac{g_3}{g_2}\right)\frac{\eta B u_{\text{nak}}}{p_{21}}\frac{p_{42}+p_{43}}{p_{42}}\right]\delta''}. \tag{15}$$

For simplicity it has been assumed that the Boltzmann factor is equal to zero. Equating (15) to unity, it is not difficult to determine $\eta B u^*_{\text{nak}}$, at which generation arises in the $3 \to 1$ channel:

$$\eta B u^*_{\text{nak}} = p_{21}\frac{\frac{g_3}{g_1} + \delta'' - \delta'\left(1 + \frac{g_3}{g_2}\right) + p_{31}\left(\frac{g_3^2}{g_1 g_2} + \frac{g_3}{g_1}\delta' + \frac{g_3}{g_2}\delta''\right)}{\left[\frac{g_3}{g_2} + \delta' - \left(1 + \frac{g_3}{g_2}\right)\delta''\right]\frac{p_{42}+p_{43}}{p_{43}}}. \tag{16}$$

Table 2 gives the ratios u^*_{nak} to the threshold value $u^{\text{por}}_{\text{nak}}$ at the frequency ν_{32}, calculated by the usual formulas (4) for different combinations of δ', δ'', p_{31}, p_{32}, p_{21}, and $g_1 = g_2 = g_3$.

Table 2

Conditions	δ'	$\delta'' = 0$	$\delta'' = 0.1$	$\delta'' = 0.2$	$\delta'' = 0.4$	$\delta'' = 0.5$
$p_{32} = p_{31}$; $p_{21} = 1.5p_{31}$	0.01	120	48	193	540	∞
$p_{32} = p_{31}$; $p_{21} = 1.5p_{31}$	0.1	95	11	14	3.5	∞
$p_{32} = p_{31}$; $p_{21} = 1.5p_{31}$	0.6	0.27	0.30	0.33	0.47	∞

Conditions	δ'	$\delta''=0$	$\delta''=0.1$	$\delta''=0.2$	$\delta''=0.4$	$\delta''=0.5$
$p_{32} = 6p_{31}$; $p_{21} = 1.5p_{31}$	0.01	35	42	55	154	∞
$p_{32} = 6p_{31}$; $p_{21} = 1.5p_{31}$	0.1	2.7	3.2	4	9	∞
$p_{32} = 6p_{31}$; $p_{21} = 1.5p_{31}$	0.6	0.76	0.86	0.95	1.3	∞

It is clear from the table that, for small δ', the value of u^{*}_{nak} is many times greater than $u^{\text{por}}_{\text{nak}}$. However, if δ' is large and δ'' is small, then generation at the frequency ν_{31} can arise with only a small increase in the pumping intensity.

If $\eta B u_{\text{nak}} > \eta B u^{*}_{\text{nak}}$, then generation in the channels $3 \to 2$ and $3 \to 1$ will occur simultaneously. In this case it is possible to satisfy at once both condition (2) and condition (12). Solving jointly the system of equations (2),

From (9) and (12), we obtain

$$n_1 = n\frac{g_1}{g_1+g_2+g_3}\left[1-\left(1+\frac{g_2}{g_3}\right)\delta''+\frac{g_2}{g_3}\delta'\right], \tag{17}$$

$$n_2 = n\frac{g_2}{g_1+g_2+g_3}\left[1-\left(1+\frac{g_1}{g_3}\right)\delta'+\frac{g_1}{g_3}\delta''\right], \tag{18}$$

$$n_3 = n\frac{g_3}{g_1+g_2+g_3}\left[1+\frac{g_1}{g_3}\delta''+\frac{g_2}{g_3}\delta'\right]. \tag{19}$$

With simultaneous generation at two frequencies, the level populations cease to depend on the pump intensity.

The densities of the radiation generated in the channels $3 \to 2$ and $3 \to 1$ are readily found from the particle-balance equations at level 2 and, respectively, level 1. The calculation gives

$$B_{32}u^{\text{gen}}_{32} = \frac{p_{21}n_2 - p_{32}n_3 - p_{42}n_4}{n\delta'}, \tag{20}$$

$$B_{31}u^{\text{gen}}_{31} = \frac{\eta B u_{\text{pump}} n_1 - p_{31}n_3 - p_{21}n_2}{n\delta''}, \tag{21}$$

where n_1, n_2, and n_3 are determined by formulas (17)–(19), while n_4 is found from the balance equation for the fourth level,

$$n_4 = n_1 \frac{Bu_{\text{pump}}}{p_{41} + p_{42} + p_{43}}. \tag{22}$$

If p_{42} is small, then, in accordance with (20), the density (and power) of generation in the channel $3 \to 2$ does not depend on the pump intensity. At the same time, the density of the generated radiation in the channel $3 \to 1$ increases linearly and, at sufficiently high pumps, may exceed the generation density in the channel $3 \to 2$. The value (20) for $p_{42} = 0$ determines the limit u_{32}^{gen} that can be reached by increasing the pump. If, however, $p_{42} \neq 0$, then increasing the pump above u^*_{pump} leads to a decrease of u_{32}^{gen}. The ratio of the generation powers u_{31}^{gen} and u_{32}^{gen} depends substantially on δ' and δ''. The larger δ'', the smaller u_{31}^{gen}, but the larger u_{32}^{gen}. An increase in δ' is accompanied by a simultaneous decrease in u_{32}^{gen} and u_{31}^{gen}.

In some cases (for large δ' and small $\delta''' = k_{21}^{\text{loss}}/\varkappa_{21}$), increasing the pump can lead to the appearance of a second generation channel in the transition $2 \to 1$, rather than $3 \to 1$. This occurs if the value

$$\eta Bu^{**}_{\text{pump}} = p_{21} \frac{\left(1 + \frac{g_3}{g_2}\frac{p_{31}}{p_{32}}\right)\delta'' + \delta\,\frac{p_{31}}{p_{32}} + \frac{g_2}{g_1}(1-\delta') + \frac{g_2}{g_1}\left(\frac{g_3}{g_2} + \delta'\right)\frac{p_{31}}{p_{21}}}{\left[-\left(1 + \frac{g_3}{g_2}\right)\delta''' + (1-\delta')\right]\frac{p_{42}+p_{43}}{p_{43}}} \tag{23}$$

is less than (16).

Simultaneous generation in three channels: $3 \to 2$, $3 \to 1$, and $2 \to 1$ is practically impossible and arises only in exceptional cases, for accidental combinations of the loss coefficients. Simultaneous generation in three channels at once arises only in a system of four levels for which $n_1 + n_2 + n_3 + n_4 = n$.

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Note: Figure translations are in progress. See original paper for figures.