REFLECTION OF FAST IONS FROM A DENSE MEDIUM AT GLANCING ANGLES

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O. B. Firsov

REFLECTION OF FAST IONS FROM A DENSE MEDIUM AT GLANCING ANGLES

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If ions with an energy far exceeding the binding energy of the atoms of a medium fall on the surface of a dense medium, then some fraction of the ions, after undergoing a number of scatterings inside the medium, always emerges again from the medium through the same surface. In some cases the coefficient of such reflection may be close to unity (a small ratio of the ion mass to the mass of the atoms of the medium; angles of incidence close to glancing).

To determine the law of reflection of fast ions from the surface of a medium it is necessary to solve the Boltzmann kinetic equation with boundary conditions according to which, at the surface, there are no particles moving into the medium except for particles of the beam entering at a specified angle with a specified velocity.

In the case of ion energies substantially exceeding the binding energy of the particles of the medium, collisions may be regarded as binary. If, moreover, the ion energy is nevertheless not very large, the scattering in each collision differs little from spherically symmetric scattering. In this case the angular distribution of the reflected particles does not depend on the angle of incidence and obeys the usual cosine law. The only quantity of interest is the velocity distribution of the reflected particles. A similar problem has been solved in neutron physics. Here the case of extremely anisotropic scattering will be considered, which is approximately realized at sufficiently high ion energies, i.e., when the main role in the scattering is played by an interaction law close to the Coulomb law. Scatterings through very small angles then play the principal role. In this case one may approximately replace the collision integral by an angular Laplace operator describing diffusion in the directions of the particle-velocity vector, and by a term describing uniform slowing down of the particles. Thus the Boltzmann equation takes the form

$$\frac{\partial f}{\partial t} + \mathbf{v}\nabla f + \dot{v}\frac{\partial f}{\partial v} - g\nabla_{\varphi}^2 f = 0, \tag{1}$$

where f is the distribution function of the particles, $\mathbf{v}$ is the velocity, $\dot{v}$ is the slowing down, and g is the mean square scattering angle per unit time.

For elastic scattering the relation

$$\dot{v} = -\frac{1}{2}\frac{m_1}{m_2}vg, \tag{2}$$

holds, where m_1 is the mass of the scattered particles and m_2 is the mass of the atoms of the medium.

The specific features of the scattering under consideration should be expressed mainly at angles of incidence and reflection close to glancing. In this case $\nabla^2_\varphi \approx \partial^2/\partial\vartheta_1^2 + \partial^2/\partial\vartheta_2^2$, where ϑ_1 and ϑ_2 are deviations in two perpendicular directions. Let $\vartheta_1 = \vartheta \ll 1$ be the glancing angle in the plane of incidence. Equation (1) can be integrated with respect to ϑ_2, and the integral of f again denoted by f. Then, assuming that

$f = f(z, v, \vartheta)$, equation (1) can be written in the form

$$\vartheta\frac{\partial fv}{\partial v} - \frac{\partial \dot{f}v}{\partial v} - \frac{g}{v}\frac{\partial^2 fv}{\partial\vartheta^2} = 0. \tag{3}$$

The number of particles leaving the surface is determined by the formula

$$I(\vartheta) = \vartheta\int_{v_0}^{0} f(0, v, \vartheta)v\,dv = -\vartheta\psi(0, \vartheta). \tag{4}$$

For $\psi(z, \vartheta) = \int_0^{v_0} f(z, v, \vartheta)v\,dv$, from (3) one obtains the equation

$$\vartheta\,\partial\psi/\partial z - \bar{\mu}\,\partial^2\psi/\partial\vartheta^2 = 0, \tag{5}$$

where it is assumed that $\dot{v}f(z, v, \vartheta) = 0$ at $v = v_0$, since the particles are slowing down, and at $z \neq 0$ or $\vartheta \neq \vartheta_0$ (the angle at which the particles enter the surface) there are no particles with velocity v_0. In addition, it is put that $\dot{v}f(z, 0, \vartheta) = 0$ at $v = 0$. If $\dot{v}|_{v=0} \neq 0$, this assumes that all particles leave the medium before they have time to slow down to zero velocity. Further,

$$\bar{\mu} = \int_0^{v_0}\frac{g}{v}\frac{\partial^2 f\cdot v}{\partial\vartheta^2}\,dv \Big/ \int_0^{v_0}\frac{\partial^2 f\cdot v}{\partial\vartheta^2}\,dv = \bar{\mu}(z, \vartheta).$$

The dependence of $\bar{\mu}$ on z is immaterial, since below a new variable will be introduced,

$$z_1 = \int_0^z \bar{\mu}\, dz.$$

However, below we also neglect the dependence of $\bar{\mu}$ on ϑ. In essence this means that the particles leave the medium mainly before they have slowed down appreciably. Since the restriction of smallness of the angles ϑ, ϑ_0 has been imposed, a certain justification for this assumption is provided by the inequality

$$\frac{v - v_0}{v_0} < \frac{m_1}{2m_2}\,\vartheta^2,$$

valid for elastic collisions. Thus, the equation to be solved is

$$\vartheta\, \partial\psi/\partial z = \partial^2\psi/\partial\vartheta^2; \tag{6}$$

the boundary conditions are: at $z = 0$, $\psi_{\vartheta>0} = \frac{1}{v}\delta(\vartheta - \vartheta_0)$, while at $z = 0$, $\psi_{\vartheta<0}$ is to be determined. There is no unlimited increase of ψ as z increases.

The solution of equation (6) is sought in the form

$$\psi = \int_{-\infty}^{\infty} C(\lambda) e^{-\lambda^3 z}\, \mathcal{E}(\lambda\vartheta)\lambda\, d\lambda, \tag{7}$$

where

$$\partial^2\mathcal{E}(x)/\partial x^2 + x\mathcal{E}(x) = 0, \qquad \int_{-\infty}^{\infty} \mathcal{E}(ax)\mathcal{E}(bx)x\, dx = \frac{1}{a}\delta(a - b).$$

Obviously, one must have $C(\lambda) = 0$ for $\lambda \leq 0$. This gives an integral equation for determining $\psi(0, \vartheta)$ at $\vartheta < 0$. Multiplication of (7) by $\mathcal{E}(\lambda_1\vartheta)\vartheta\, d\vartheta$ and integration over the limits $\pm\infty$ gives

$$C(\lambda_1) = \int_{-\infty}^{0} \psi(0, \vartheta)\mathcal{E}(\lambda_1\vartheta)\vartheta\, d\vartheta + \mathcal{E}(\lambda_1\vartheta_0)$$

or, if $\vartheta = -\theta$, $\lambda_1 = -\nu$, where $\theta > 0$ and $\nu > 0$, $\Phi(\theta) = \psi(0, -\theta)$,

$$\int_0^{\infty} \Phi(\theta)\mathcal{E}(\nu\theta)\theta\, d\theta = \mathcal{E}(-\nu\vartheta_0). \tag{8}$$

The functions $\mathcal{E}(x)$ are expressed in terms of Bessel functions

$$\mathcal{E}(x) \sim J_{-1/3}\left({}^2\!/_3 x^{3/2}\right) + J_{1/3}\left({}^2\!/_3 x^{3/2}\right), \qquad x > 0,$$
$$\mathcal{E}(x) \sim \frac{1}{\pi} K_{1/3}\left({}^2\!/_3 x^{3/2}\right), \qquad x < 0, \tag{9}$$

with one and the same proportionality coefficient.

The transformation of (8) to the corresponding variables and Bessel functions, by virtue of relation (1),

$$\frac{\sin \frac{\pi}{2}\mu}{\sin \pi\mu} \int_0^\infty \left(J_{-\mu}(ax) + J_\mu(ax)\right) \frac{x\,dx}{1+x^2} = K_\mu(a) \tag{10}$$

solves equation (8) and gives, for $\Phi(\theta) = \psi(0, -\theta)$,

$$\Phi(\theta) = \frac{3}{2\pi} \frac{\theta^{1/2} \vartheta_0^{1/2}}{\theta^3 + \vartheta_0^3}. \tag{11}$$

Accordingly, for the flux density of the reflected particles, by definition,

$$I(\theta) = \theta\psi(0, -\theta) = \theta\Phi(\theta) = \frac{3}{2\pi} \frac{\theta^{3/2} \vartheta_0^{1/2}}{\theta^3 + \vartheta_0^3}. \tag{12}$$

The maximum of $I(\theta)$ corresponds to the angle of specular reflection, and the form of $I(\theta)$ does not depend on the particular scattering law, provided scattering through small angles predominates.

The fact that in (12) an albedo equal to unity is obtained is connected with the condition $\vartheta f|_{v=0} = 0$. The latter is all the more admissible the smaller the angle ϑ_0 and the ratio m_1/m_2 (formula (2)), when the particles leave the medium before they have time to slow down. Let us recall that the function f was integrated over the angle ϑ_2. It should be expected that the initial $f(z, v, \vartheta_1 \vartheta_2)$ had an effective width in ϑ_2 also of order ϑ_0. Therefore comparison of (12) with experiment should be made only in the case when the angular width of the slit for the angle ϑ_2 was greater than ϑ_0.

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