Astronomy

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1966

SovietRxiv

Abstract

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Astronomy

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ON THE MECHANISM OF FRAGMENTATION OF SMALL METEOR BODIES IN THE ATMOSPHERE

(Presented by Academician E. K. Fedorov, 13 V 1965)

The simplest physical theory of meteors (^{1}) gives the following distribution of the luminous intensity I along the meteor trail:

$$\frac{I}{I_m} = \frac{9}{4}\frac{\rho}{\rho_m}\left(1 - \frac{1}{3}\frac{\rho}{\rho_m}\right)^2, \tag{1}$$

where I_m is the maximum luminous intensity; ρ, ρ_m are the atmospheric densities at the given height and at the height of maximum luminous intensity. The light curve (1) agrees satisfactorily with observations of meteors brighter than approximately zero stellar magnitude.

Photographic observations with Super-Schmidt cameras (^{2},^{3}) reveal a number of features of faint photographic meteors of stellar magnitude 0 to +4: a) shortening of trails in comparison with (1); b) rapid increase of brightness near the point of appearance; c) anomalous increase of deceleration toward the end of the trail; d) blurring of interruptions in the second half of the trail on photographs obtained with an obturator. These features are usually (^{2},^{3}) explained by a very loose structure of meteor bodies, which leads to their fragmentation under the action of aerodynamic pressure. In the present work it is shown that these features can be explained without resorting to the hypothesis of an unusually loose structure of meteor bodies.

Let us consider the motion of a rotating spherical meteor body in the upper atmosphere. In this case the energy transferred to the meteor body by the air molecules colliding with it is distributed approximately uniformly over its surface. The heat-flux density through the surface of the body is

$$\varphi(t) = \frac{1}{8}\Lambda v^3 \rho = \frac{1}{8}\Lambda v^3 \rho_0 \exp\left(\frac{vt\cos z}{H}\right), \tag{2}$$

where t is time; Λ is the heat-transfer coefficient; v is the velocity of the body; ρ_0 is a constant; z is the zenith distance of the meteor radiant; H is the reduced height of a homogeneous atmosphere.

The heat-conduction equation in spherical coordinates has the form

$$\frac{\partial \tau}{\partial t} - b^2 \left(\frac{\partial^2 \tau}{\partial r^2} + \frac{2}{r} \frac{\partial \tau}{\partial r} \right) = 0, \qquad b^2 = \frac{\lambda}{c\delta}, \tag{3}$$

where $\tau(r, t)$ is the temperature measured from the initial temperature of the body T_0; λ is the coefficient of thermal conductivity; c is the heat capacity; δ is the density of the body. The boundary conditions are

$$\tau(r, -\infty) = 0, \qquad (\partial\tau/\partial r)_{r=0} = 0, \qquad \lambda(\partial\tau/\partial r)_{r=r_0} = \varphi(t), \tag{4}$$

(r_0 is the radius of the body).

Under these boundary conditions, if braking is not taken into account, the solution of equation (3) has the form (4)

$$\tau(r, \rho) = \frac{\Lambda r_0 v_0^3 \rho}{8\lambda \left[\frac{1}{x_0} \operatorname{ch}\left(\frac{r_0}{x_0}\right) - \frac{1}{r_0} \operatorname{sh}\left(\frac{r_0}{x_0}\right) \right]} \frac{1}{r} \operatorname{sh}\left(\frac{r}{x_0}\right); \qquad x_0 = b\sqrt{\frac{H}{v_0 \cos z}}. \tag{5}$$

The latent heat of melting of meteoric matter is approximately equal to the energy required to heat it from 0°K to $T_0 \approx 280$°K; therefore in (5), for $T > T_\mathrm{m}$ (T_m is the melting temperature), it is better to measure the temperature from 0°K. The temperature of the body surface is

$$\tau(r_0, \rho) = \frac{\Lambda r_0 v_0^3}{8\lambda \left[\frac{r_0}{x_0} \operatorname{cth}\left(\frac{r_0}{x_0}\right) - 1 \right]}. \tag{6}$$

The rate of evaporation of the substance in vacuum can be written as follows:

$$\lg(\Delta M) = -4.23 + C_1 + \frac{1}{2} \lg \mu - C_2/T - \frac{1}{2} \lg T. \tag{7}$$

Here ΔM is the mass of substance evaporating from a unit surface per unit time; C_1 and C_2 are constants; μ is the molecular weight.

Equating the energy expended on evaporation at the temperature given by formula (6), $\varphi(t)$, from (2), (6), and (7) we find the atmospheric density at the height h_n of the onset of intense evaporation of the meteoric body

$$\rho_\mathrm{n} = \frac{8\lambda C_2 \left[\frac{r_0}{x_0} \operatorname{cth}\left(\frac{r_0}{x_0}\right) - 1 \right]}{\Lambda r_0 v_0^3 \left\{ -4.23 + C_1 + \lg Q + \frac{1}{2} \lg \mu + \lg \frac{r_0}{\lambda} + \lg \left[\frac{r_0}{x_0} \operatorname{cth} \frac{r_0}{x_0} - 1 - \frac{3}{2} \lg T_\mathrm{n} \right] \right\}}, \tag{8}$$

where Q is the energy of evaporation of 1 g of meteoric matter; T_n is the temperature at the onset of intense evaporation (T_n is considerably higher than T_m).

For $r_0 \leqslant 2x_0$, according to (5), the body is heated practically throughout, and at the height h_n it has already completely melted. Under the action of aerodynamic pressure and surface-tension forces, the molten body acquires a flattened shape. Let us estimate the amount of deformation, assuming that the drop has the form of an ellipsoid of revolution with the minor axis in the direction of motion.

The pressure difference produced by the surface-tension forces in the planes of the longitudinal and transverse sections of the drop must be equal to the aerodynamic pressure. With the aid of Laplace' s formula we obtain

$$\sigma\left(\frac{a^2}{b^2}+\frac{1}{a}\right)-\frac{2\sigma}{a}=\Gamma\rho v^2, \tag{9}$$

where σ is the coefficient of surface tension; Γ is the drag coefficient; a, b are the major and minor semiaxes of the ellipsoid; $a^2 b = r^3$ (r is the radius of the drop). Then

$$\left(\frac{a}{r}\right)^5-\frac{r}{a}=\frac{\Gamma r v^2\rho}{\sigma}. \tag{10}$$

Neglecting, in the left-hand side of equation (10), the term r/a (which leads to a certain underestimation of the deformation), we obtain

$$a=(\Gamma v^2\rho/\sigma)^{1/5} r^{6/5}. \tag{11}$$

Equation of evaporation of a meteoric body

$$\frac{dM}{dt}=-\frac{\Lambda}{2Q}Sv^3\rho=-\frac{\pi\Lambda}{2(Q-Q_\mathrm{H})}v^{11/5}\left(\frac{\Gamma}{\sigma}\right)^{2/5} r^{12/5}\rho^{7/5}, \tag{12}$$

where M is the mass of the body. For an isothermal atmosphere, (12) can be rewritten in the form

$$\frac{dr}{r^{2/5}}=-\frac{\Lambda H v^{14/5}}{8(Q-Q_\mathrm{H})\delta\cos z}\left(\frac{\Gamma}{\sigma}\right)^{2/5}\rho^{2/5}d\rho. \tag{13}$$

Neglecting the deceleration of the meteoric body during the evaporation process, we obtain

$$r^{3/5}=-\frac{3\Lambda H v_0^{14/5}}{56(Q-Q_\mathrm{H})\delta\cos z}\left(\frac{\Gamma}{\sigma}\right)^{2/5}(\rho^{7/5}-\rho_\mathrm{H}^{7/5})+r_0^{3/5}. \tag{14}$$

Fig. 1

Figure 1: Fig. 1

From (12) and (14) we find the meteor light-intensity curve

$$I = -\frac{kv_0^2}{16\pi}\frac{dM}{dt} =$$

$$= \frac{k\Lambda}{16(Q - Q_{\mathrm{H}})}\left(\frac{\Gamma}{\sigma}\right)^{2/5} v_0^{22/5}\rho^{7/5}\left[r_0^{3/5} - \frac{3\Lambda H v_0^{14/5}}{56(Q - Q_{\mathrm{H}})\delta\cos z}\left(\frac{\Gamma}{\sigma}\right)^{2/5}(\rho^{7/5} - \rho_{\mathrm{H}}^{7/5})\right]^4, \tag{15}$$

where k is the luminosity coefficient.

With sufficiently strong deformation the drop becomes unstable. Questions of the stability of liquid drops in a gas flow have been considered in a number of experimental and theoretical works ($^{5-9}$). The stability of a drop is characterized by the Weber number

$$\mathrm{We} = \Gamma\rho v^2 r/\sigma.$$

For a viscous liquid, the critical value of We, according to (^{8,9}), is $\mathrm{We}_0 \approx 6.5$. When $\mathrm{We} > \mathrm{We}_0$, the drop is fragmented.

From (8) and (14) we find the limiting value of the initial radius of a drop, r_0', that preserves stability during evaporation:

$$r_0' = {}^3\!/_4\,\sigma \mathrm{We}_0/\Gamma\rho_{\mathrm{H}} v_0^2.$$

When $r_0 < r_0'$, the drop is not fragmented. In this case some shortening of the meteor trail occurs as a result of an increase in the midsection area due to deformation of the drop.

All meteoric bodies with initial radii $r_0 \geq r_0'$ are fragmented, moreover: 1) if ${}^4\!/_3 r_0' \leq r_0 \leq 2x_0$, fragmentation occurs already at the height at which intensive evaporation begins; 2) if $r_0' \leq r_0 < {}^4\!/_3 r_0'$ and $r_0 \leq 2x_0$, fragmentation occurs when the radius of the drop reaches the critical value corresponding to the condition $\mathrm{We} = \mathrm{We}_0$; 3) if $r_0 \geq {}^4\!/_3 r_0'$ and $r_0 > 2x_0$, fragmentation occurs after the body has melted through; 4) if $r_0' \leq r < {}^4\!/_3 r_0'$ and $r_0 > 2x_0$, fragmentation occurs after the body has melted through and the radius of the drop reaches the critical value corresponding to the condition $\mathrm{We} = \mathrm{We}_0$.

Fig. 1

As an example, let us consider the evaporation of a dense stony meteoric body with $r_0 = 2x_0$ and $v_0 = 15$ km/sec. According to ([10]), for stony meteor-

bodies, $\lambda = 2 \cdot 10^5$ erg/cm • sec • deg, $b = 0.08$ cm/sec$^{1/2}$, $\delta = 3.4$ g/cm^3, $Q = 6 \cdot 10^{10}$ erg/g, $\sigma = 360$ dyn/cm, $k = 1.1 \cdot 10^{-3}$. Taking $\Lambda = \Gamma = 1$ and the mean value $\cos z = 2/3$, we find $r_0 = 4/3 r_0'$, i.e., the body fragments already at the altitude at which intense evaporation begins. The calculations carried out show that the form of the light-intensity curve depends comparatively weakly on the assumed sizes of the droplets into which the body fragments. If the droplet sizes are sufficiently large, the droplets soon become unstable and in turn fragment into parts.

In Fig. 1, light-intensity curve I was computed under the assumption that the drop fragments into the minimum number of droplets, each of which remains stable during evaporation. For comparison, the following are given: II—the light-intensity curve computed from (15) without taking fragmentation into account; III—the light-intensity curve (1) given by the simplest physical theory of meteors.

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Received
10 V 1965

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[1] N. Herlofson, Rep. Progr. Phys., **11**, 444 (1948).
[2] G. S. Hawkins, R. B. Southworth, Smithsonian Contrib. Astrophys., **2**, 349 (1958).
[3] Z. G. Jaccia, Astrophys. J., **121**, 521 (1955).
[4] Z. Ceplecha, V. Padevět, Bull. Astr. Inst. Czechoslovakia, **12**, No. 5, 491 (1961).
[5] M. S. Volynskii, DAN, **62**, No. 3, 301 (1948).
[6] M. S. Volynskii, DAN, **68**, No. 2, 237 (1949).
[7] V. A. Borodin, Yu. F. Dityakin, V. I. Yagodkin, Zhurn. Prikl. Mekh. i Tekhn. Fiz., No. 1, 85 (1962).
[8] J. O. Hinze, Am. Inst. Chem. Eng. J., **1**, 200 (1955).
[9] N. Isshiki, Rep. Trans. Techn. Res. Inst., No. 35 (1959).
[10] E. J. Öpik, *Physics of Meteor Flight in the Atmosphere*, N. Y., 1958.

Note: Figure translations are in progress. See original paper for figures.

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