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Abstract

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PHYSICS

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ON A REPRESENTATION FOR THE SCATTERING MATRIX IN QUANTUM FIELD THEORY

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Let $S = 1 + iR$ be the relativistic scattering matrix, specified in the interaction representation, and let $\mathcal{L}(p)$ be the Fourier transform of the interaction Lagrangian $\mathcal{L}(x)$ in the same representation:

$$\widetilde{\mathcal{L}}(p) = \int e^{-ipx} \mathcal{L}(x)\, dx. \tag{1}$$

According to (1),

$$R = R(\lambda\tau)\big|_{\tau=0}, \tag{2}$$

where τ is a one-dimensional invariant parameter, λ is a 4-vector satisfying the conditions

$$\lambda^2 = 1, \qquad \lambda_0 > 0,$$

and the matrix $R(\lambda\tau)$ is determined from the equation

$$R(\lambda\tau) = \widetilde{\mathcal{L}}(\lambda\tau) + \frac{1}{2\pi} \int_{-\infty}^{\infty} \widetilde{\mathcal{L}}(\lambda\tau - \lambda\tau')\, \frac{d\tau'}{\tau' - i\varepsilon}\, R(\lambda\tau'). \tag{3}$$

Instead of (3) one may consider the operator equation

$$\hat{R} = \hat{L} + \frac{1}{2\pi} \hat{L}\, \frac{1}{\tau - i\varepsilon}\, \hat{R}, \tag{4}$$

if, by definition, one takes

$$\tau' \delta(\tau' - \tau'') = \langle \tau' | \tau | \tau'' \rangle, \tag{5a}$$

$$\mathcal{L}(\lambda\tau' - \lambda\tau'') = \langle\tau'|\hat{L}|\tau''\rangle, \tag{5b}$$

$$R(\lambda\tau) = \langle\tau|\hat{R}|0\rangle. \tag{5c}$$

($|\tau\rangle$ is the state vector of certain "quasiparticles" possessing 4-momentum equal to $\lambda\tau$ (^{2}).) It is not difficult to verify that the solution of equation (4) has the form

$$\hat{R} = 2\pi \frac{1}{2\pi - \hat{L} - i\varepsilon} \hat{L}. \tag{6}$$

In order that formula (7) be effective, it is necessary to find a concrete realization of the state vectors $|\tau\rangle$ and of the operator $\hat{L}$. To this end we introduce into consideration the operator $a(\tau)$, assuming

$$\langle\tau'|a(\tau)|\tau''\rangle = \delta(\tau' + \tau - \tau''). \tag{7}$$

It is easy to establish that

$$a(\tau_1)a(\tau_2)\ldots a(\tau_n) = a(\tau_1 + \tau_2 + \ldots + \tau_n), \tag{8}$$

$$a^+(\tau) = a(-\tau), \qquad [a(\tau), a^+(\tau')] - = 0.$$

Moreover, with the aid of (5a) and (7), we find

$$[a(\tau'), \tau] = \tau' a(\tau'); \qquad [\tau, a^+(\tau')] = \tau' a^+(\tau'). \tag{9}$$

We shall now assume that

$$|\tau\rangle = a^+(\tau)|0\rangle. \tag{10}$$

Obviously, with such a definition,

$$\langle\tau'|\tau\rangle = \delta(\tau' - \tau), \qquad \tau|\tau'\rangle = \tau'|\tau'\rangle. \tag{11}$$

Indeed, according to (7), (8), and (9) we shall have

$$\langle\tau'|\tau\rangle = \langle 0|a(\tau')a(-\tau)|0\rangle = \langle 0|a(\tau' - \tau)|0\rangle = \delta(\tau' - \tau);$$

$$\tau|\tau'\rangle = \tau a^+(\tau')|0\rangle = a^+(\tau')\tau|0\rangle + \tau' a^+(\tau')|0\rangle = \tau'|\tau'\rangle,$$

if $\tau|0\rangle = 0$.

Taking into account definitions (5b) and (7), the operator L can be represented in the form

$$\hat{L} = \int_{-\infty}^{\infty} a(\tau)\,\widetilde{\mathcal{L}}(-\lambda\tau)\,d\tau. \tag{12}$$

Thus, the description of quasiparticles in terms of $a(\tau)$ and $a^+(\tau)$ resembles the formalism of second quantization used to describe physical particles, while the operator $\hat{L}$ plays the role of the Lagrangian of the interaction of physical particles with quasiparticles.

Substituting (12) into formula (6), we obtain the final expression for the operator $\hat{R}$:

$$\hat{R} = 2\pi\tau \frac{1}{2\pi\tau - \displaystyle\int_{-\infty}^{\infty} a(\tau)\widetilde{\mathcal{L}}(-\lambda\tau)\,d\tau - i\varepsilon} \int_{\infty-}^{\infty} a(\tau)\widetilde{\mathcal{L}}(-\lambda\tau)\,d\tau, \tag{13}$$

whence, on the basis of (2),

$$S = 1 + 2\pi i\langle 0|\tau \left[2\pi\tau - \int_{-\infty}^{\infty} a(\tau)\widetilde{\mathcal{L}}(-\lambda\tau)\,d\tau - i\varepsilon\right]^{-1} \int_{-\infty}^{\infty} a(\tau)\widetilde{\mathcal{L}}(-\lambda\tau)\,d\tau|0\rangle. \tag{14}$$

Let us emphasize that the Lagrangian $\widetilde{\mathcal{L}}(-\lambda\tau)$ in (13) and (14) is an operator only in the state space of the physical particles. For example, in the case

$$\mathcal{L}(x) = g : \varphi^3(x) :,$$

by virtue of (1) and under the condition that

$$\varphi(x) = \frac{1}{(2\pi)^{3/2}} \int \varphi(k)e^{ikx}\,dk,$$

we shall have

$$\widetilde{\mathcal{L}}(-\lambda\tau) = \frac{g}{\sqrt{2\pi}} \int \delta^{(4)}(\lambda\tau + k_1 + k_2 + k_3) : \varphi(k_1)\varphi(k_2)\varphi(k_3)\,dk_1 dk_2 dk_3. \tag{15}$$

Relations (13) and (14) may prove useful for obtaining chains of equations connecting the matrix elements S with one another, and also in studies using an expansion in inverse powers of the coupling constant g. In this case it is expedient to work precisely with the operator $\hat{R}$, and to carry out the averaging over the "quasiparticle vacuum" only at the end of all calculations.

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References

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2. V. G. Kadyshevskii, *ZhETF*, **46**, 872 (1964).

Note: Figure translations are in progress. See original paper for figures.