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Abstract

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A. F. KRAVCHENKO, Corresponding Member of the Academy of Sciences of the USSR A. V. RZHANOV,
V. S. SARDARYAN

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1. In semiconductors with a complex band structure, placed in a weak electric field **E** and in an intermediate magnetic field **H** perpendicular to it, not coinciding with an axis of symmetry of the cubic crystal, in addition to the usual Hall field there arises an electromotive force along the magnetic field. This e.m.f. can be represented as the sum of two terms, one of which is an even function of the magnetic field—the planar Hall effect—while the other is an odd function of H—the longitudinal Hall effect.

In work (^{1}) an estimate was made of the longitudinal Hall field for n-Ge under the assumption of a scalar relaxation time and one type of charge carrier. However, in many semiconductors ($^{2-4}$) the conductivity is due to two types of minima, and the relaxation time is anisotropic (^{5}); therefore, calculation of the longitudinal Hall effect for the most general case of two-zone conductivity and a tensor relaxation time is of undoubted practical and scientific interest.

2. The occurrence of the longitudinal Hall effect can be explained as follows. In an external electric field the equilibrium distribution function is displaced along the field; in other words, the surface of equal occupation probabilities becomes spherically asymmetric with respect to the former center; the population of isoenergetic surfaces for conduction electrons in the direction of the field becomes smaller than against the field, which is what gives rise to an electric current.

The action of a weak magnetic field is equivalent to rotation of the distribution function displaced in the electric field about an axis parallel to **H**. If the isoenergetic surfaces are spherical, then, obviously, the surfaces of equal occupation probabilities also have the form of a sphere. In this case rotation of the distribution function is equivalent to increasing the occupation probability in the direction of the positive angle of rotation ($+\varphi$) and decreasing it in the direction of the negative angle ($-\varphi$). Thus, in crossed electric and magnetic fields the isoenergetic surfaces are populated nonuniformly. This leads to the appearance of the ordinary Hall field, perpendicular to both **J** and **H**:

$$E_J \neq 0; \qquad E_{[\mathbf{H}\times\mathbf{J}]} \neq 0, \qquad E_H = 0.$$

In intermediate magnetic fields whose directions do not coincide with the symmetry axes of the cubic crystal, for semiconductors with a complex band structure the surfaces of equal occupation probabilities undergo simultaneous displacement along **E**, **H**, and $[\mathbf{E}\times\mathbf{H}]$, as can be verified by expanding the nonequilibrium part of the distribution function in $\mu H/c < 1$, retaining terms in H^3. Consequently, the Hall field that arises has components along **H** and $[\mathbf{H}\times\mathbf{J}]$:

$$E_H \neq 0, \qquad E_{[\mathbf{H}\times\mathbf{J}]} \neq 0.$$

In strong magnetic fields (classical region) $E_H \equiv 0$. It is obvious that for spherical isoenergetic surfaces in weak magnetic fields $E_H \equiv 0$.

3. Ohm' s law in weak electric and intermediate magnetic fields has the form

$$E_i = \rho_{ij}(H)J_j, \tag{1}$$

where J_j is the j-th component of the current density; ρ_{ij} is the generalized resistivity tensor satisfying the equality

$$\rho_{ij}(H) = \rho_{ji}(-H). \tag{2}$$

Represent $\rho_{ij}(H)$ in the form

$$\rho_{ij}(H) = \bar{\rho}_{ij}(H) + \tilde{\rho}_{ij}(H), \tag{3}$$

where $\bar{\rho}_{ij}(H)$ is a tensor function even in H and symmetric in i,j; $\tilde{\rho}_{ij}(H)$ is a tensor function odd in H and antisymmetric in i,j:

$$\bar{\rho}_{ij}(H) = \rho_{ij} + \rho_{ijkl}H_kH_l + \rho_{ijklmn}H_kH_lH_mH_n, \tag{4}$$

$$\tilde{\rho}_{ij}(H) = \rho_{ijk}H_k + \rho_{ijklm}H_kH_lH_m + \rho_{ijklmo}H_kH_lH_mH_nH_o. \tag{5}$$

Taking (3) into account, equation (1) will be

$$|\mathbf{E}|_i = |\bar{\mathbf{E}}|_i + |\tilde{\mathbf{E}}|_i = \bar{\rho}_{ij}(H)J_j + \tilde{\rho}_{ij}(H)J_j. \tag{6}$$

In order to express the coefficients of the generalized resistivity tensor through experimentally measured quantities, let us write the antisymmetric tensor $\tilde{\rho}_{ij}(H)$ in the form

$$\tilde{\rho}_{ij}(H) = e_{ijk} R_k(H), \tag{7}$$

where $R_k(H)$ is the k-th component of the generalized Hall vector,

$$R_k(H) = \frac{1}{2} e_{ijk} \rho_{ij}(H). \tag{8}$$

Comparing (6) and (7), we have

$$\tilde{\mathbf{E}} = [\mathbf{R}(H) \times \mathbf{J}]. \tag{9}$$

To determine $\tilde{\mathbf{E}}$, it is necessary to calculate the coefficients of the antisymmetric resistivity tensor $\tilde{\rho}_{ij}(H)$. The method for calculating them reduces to the following:

a) taking into account the symmetry of the cubic crystal and Neumann' s principle, we select the coefficients of the antisymmetric resistivity tensor that differ from one another and are nonzero;

b) assuming a definite position of the energy minima, we determine the transformation matrices of the tensor $\tilde{\rho}_{ij}(H)$ from the principal axes of the ellipsoids to the crystallographic axes associated with the edges of the cube;

c) specifying a definite dispersion law, distribution function, scattering mechanism, and density of states, we calculate the nonzero coefficients, and then perform their summation for tensors of the same rank.

Having calculated by the indicated method the coefficients of the antisymmetric tensor and substituting them into (8) and (9), we obtain

$$\tilde{\mathbf{E}} = -R_0^{(1,2)}[\mathbf{J} \times \mathbf{H}] + SH^2[\mathbf{J} \times \mathbf{H}] + r[\mathbf{J} \times \mathbf{M}], \tag{10}$$

where $\mathbf{M}$ is a conditional vector with projections on the selected crystallographic axes

$$M_i = H_i^3,$$

$$\mathbf{R}(H) = -R_0^{(1,2)}\mathbf{H} + SH^2\mathbf{H} + r\mathbf{M}, \tag{11}$$

Fig. 1. Angular dependence of the longitudinal Hall field.

Figure 1: Fig. 1. Angular dependence of the longitudinal Hall field.

$$r = \begin{pmatrix} -1/3 \\ 2/9 \\ 1/12 \end{pmatrix} \rho \frac{n^{(2)} e_0^4}{c_0^2} \left[\left(\left\langle\left\langle \frac{\tau_\perp^3}{m_\perp^3} \right\rangle\right\rangle - 2 \left\langle\left\langle \frac{\tau_\parallel \tau_\perp^2}{m_\parallel m_\perp^2} \right\rangle\right\rangle + \left\langle\left\langle \frac{\tau_\parallel^2 \tau_\perp}{m_\parallel^2 m_\perp} \right\rangle\right\rangle \right) R_0^{(1,2)} \right.$$
$$\left. + \frac{e_0}{c_0} \rho \left(\left\langle\left\langle \frac{\tau^4}{m_\perp^4} \right\rangle\right\rangle - 2 \left\langle\left\langle \frac{\tau_\parallel \tau_\perp^3}{m_\parallel m_\perp^3} \right\rangle\right\rangle + \left\langle\left\langle \frac{\tau_\parallel^2 \tau_\perp^2}{m_\parallel^2 m_\perp^2} \right\rangle\right\rangle \right) \right], \qquad (12)$$

$$S = (\sigma^{(1,2)})^2 (R_0^{(1,2)})^3 - \rho R_0^{(1,2)} \frac{e_0^4}{c_0^2} \left\{ \frac{2n^{(1)} \langle \tau^3 \rangle}{m^{*3}} + \right.$$

$$+ n^{(2)} \left\langle\left\langle \frac{\tau_\parallel^2 \tau_\perp}{m_\parallel^2 m_\perp} \right\rangle\right\rangle \left[\begin{pmatrix} 1/3 \\ 2/3 \\ 7/12 \end{pmatrix} \left(\left\langle\left\langle \frac{m_\parallel \tau_\perp}{m_\perp \tau_\parallel} \right\rangle\right\rangle \right)^2 + \begin{pmatrix} 1/3 \\ 2/3 \\ 5/6 \end{pmatrix} \left\langle\left\langle \frac{m_\parallel \tau_\perp}{m_\perp \tau_\parallel} \right\rangle\right\rangle + \begin{pmatrix} 1/3 \\ 2/3 \\ 7/12 \end{pmatrix} \right] \left. \right\}$$
$$- \rho^2 \frac{e_0^5}{c_0^3} \left\{ \frac{n^{(1)}}{m^{*4}} \langle \tau^4 \rangle + n^{(2)} \left\langle\left\langle \frac{\tau_\parallel^4}{m_\parallel^4} \right\rangle\right\rangle \left[\begin{pmatrix} 0 \\ 1/3 \\ 1/4 \end{pmatrix} \left\langle\left\langle \frac{m_\parallel^2}{m_\perp^2} \frac{\tau_\perp^2}{\tau_\parallel^2} \right\rangle\right\rangle + \begin{pmatrix} 2/3 \\ 0 \\ 1/6 \end{pmatrix} \left\langle\left\langle \frac{m_\parallel}{m_\perp} \frac{\tau_\perp}{\tau_\parallel} \right\rangle\right\rangle + \begin{pmatrix} 1/3 \\ 2/3 \\ 7/12 \end{pmatrix} \right] \right\}, \qquad (13)$$

$$\langle g(\varepsilon) \rangle = \frac{(2m^*)^{3/2}}{3\pi^2 \hbar^3 n^{(1)}} \int_0^\infty g(\varepsilon) \left(-\frac{\partial f_0}{\partial \varepsilon} \right) \varepsilon^{3/2} \, d\varepsilon,$$

$$\langle\langle g(\varepsilon) \rangle\rangle = \frac{2\sqrt{2}\,(m_\parallel^2 m_\perp)^{1/2}}{3\pi^2 \hbar^3 n^{(2)}} N \int_0^\infty g(\varepsilon) \left(-\frac{\partial f_0}{\partial \varepsilon} \right) \varepsilon^{3/2} \, d\varepsilon,$$

$$R_0^{(1,2)} = \left(\frac{\sigma^{(1)}}{\sigma^{(1,2)}} \right)^2 R_0^{(1)} + \left(\frac{\sigma^{(2)}}{\sigma^{(1,2)}} \right)^2 R_0^{(2)};$$

ρ is the specific resistivity of the semiconductor; $n^{(1)}, n^{(2)}$ are the concentrations of charge carriers in the corresponding bands; $R_0^{(12)}$ are the Hall coefficients in a weak magnetic field in the cases of two conduction bands.

One-dimensional matrices mean that the first element refers to the model in which the spherical minimum is at $K = 0$, and the anisotropic minima lie along ⟨100⟩; the second matrix element refers to the case in which the anisotropic minima are located along ⟨111⟩, and the third along ⟨110⟩.

Fig. 1. Angular dependence of the longitudinal Hall field. a—**J** along $\langle 110\rangle$, **H** in the (110) plane, the angle φ is measured from the $\langle 001\rangle$ direction; b—**J** along $\langle 100\rangle$, **H** in the (100) plane, the angle φ is measured from the $\langle 001\rangle$ direction

The projection $\widetilde{\mathbf{E}}$ onto the direction of the magnetic field gives us the longitudinal Hall effect

$$\widetilde{E}_H = rJH^3\{\eta_1\eta_3\xi_2(\eta_3^2-\eta_1^2)+\eta_1\eta_2\xi_3(\eta_1^2-\eta_2^2)+\eta_2\eta_3\xi_2(\eta_2^2-\eta_3^2)\}, \qquad (14)$$

where η_i, ξ_i are the direction cosines, respectively, of the magnetic field and of the current density relative to the cube edges. The longitudinal Hall field for different orientations of the magnetic field is shown in Fig. 1.

The study of the longitudinal Hall effect can provide very valuable information about the anisotropic energy minima of charge carriers. The particular advantages of this method for studying band structure appear when two types of minima participate in the conductivity and the ordinary Hall effect or magnetoresistance does not make it possible to determine the role of each type of minimum in the conductivity.

Institute of Semiconductor Physics,
Siberian Branch of the Academy of Sciences of the USSR

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Note: Figure translations are in progress. See original paper for figures.