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Abstract

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On the Damping of a Langmuir Wave Near the Threshold for the Emission of Ion Sound

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As is known, in an isotropic homogeneous plasma there exist Langmuir and ion-sound oscillations (l- and s-plasmons), for which the dependences of the frequencies Ω_k and ω_q on the wave vectors $\mathbf{k}$ and $\mathbf{q}$ are given, respectively, by the expressions (we consider long-wavelength l- and s-plasmons):

$$\Omega_k = \Omega_0(1 + {}^3\!/_2 k^2 R_D^2), \qquad \omega_q = c_s q, \tag{1}$$

where $\Omega_0 = \sqrt{4\pi e^2 n/m}$ is the Langmuir frequency; $R_D = \sqrt{T/4\pi e^2 n}$ is the Debye radius; c_s is the sound velocity. With the dispersion law expressed by formula (1), damping of an l-plasmon is possible, caused by the emission of an s-plasmon with transition to an l-plasmon of lower frequency. In this case the conservation laws for the frequency and wave vector must be satisfied,

$$\Omega_k = \Omega_{k-q} + \omega_q. \tag{2}$$

The indicated process is a threshold process, i.e., it is possible only for k greater than a certain threshold value k_p, since equation (2) has a solution with respect to q for $k > k_\text{p}$ and has no solution for $k < k_\text{p}$, where

$$k_\text{p} = \frac{1}{3R_D}\left(\frac{m}{M}\right)^{1/2}.$$

Above the threshold, $c_s < \partial\Omega_k/\partial k$, i.e., the condition of Cherenkov radiation of s-plasmons by l-plasmons is fulfilled. Here the situation is in many respects analogous to that which occurs for elementary excitations in the theory of condensed media ([1–5]).

We take as the basis of the treatment the complete Hamiltonian of plasma oscillations with allowance for the indicated three-plasmon process:

$$H = H_0 + \varepsilon H' = \sum_k \Omega_k a_k^+ a_k + \sum_q \omega_q c_q^+ c_q + \varepsilon \sum_{kq} \Phi_{q;k\,k-q} a_{k-q}^+ c_q^+ a_k + \text{h.c.} \tag{3}$$

Here a_k^+, a_k are, respectively, the creation and annihilation operators of an l-plasmon with energy Ω_k and momentum $\mathbf{k}$; c_q^+, c_q have the same meaning for s-plasmons; ε is a formally introduced expansion parameter.

$\Phi_{q;k\,k-q}$ is found by expanding the plasma Lagrangian in powers of the amplitudes of the collective plasma oscillations and has the form ([6,7]):

$$\Phi_{q;k\,k-q} = \left(\frac{\pi}{2}\right)^{1/2} \frac{ec_T c_s^{1/2}}{T} \frac{k^2 - \mathbf{kq}}{|\mathbf{k}|\,|\mathbf{k}-\mathbf{q}|}\, q^{1/2}. \tag{4}$$

Let us introduce the one-particle two-time retarded Green function ([8])

$$G_k(t,t') = -i\theta(t-t')\langle[a_k(t); a_k^+(t')]\rangle, \tag{5}$$

where

$$\langle\cdots\rangle = \mathrm{Sp}(e^{-H/T}\cdots)\{\mathrm{Sp}(e^{-H/T})\}^{-1}; \qquad \theta(t-t') = \begin{cases} 1, & t > t', \\ 0, & t < t', \end{cases}$$

$$[a_k(t); a_k^+(t')] = a_k(t)a_k^+(t') - a_k^+(t')a_k(t).$$

Let us write the equation of motion for the functions (5):

$$i\frac{dG_k(t,t')}{dt} = \delta(t-t') - i\theta(t-t')\left\langle\left[i\frac{da_k(t)}{dt};\ a_k^+(t')\right]\right\rangle.$$

Expanding this equation with the aid of the Hamiltonian (3), we obtain:

$$\begin{aligned} i\frac{dG_k(t,t')}{dt} &= \delta(t-t') + \Omega_{kG}k(t,t') + \varepsilon\sum_q \Phi_{q;k\,k-q}D_{k-q,q,k} \\ &\quad + \varepsilon\sum_q \Phi_{q;k\,k+q}D^{(1)}_{k+q,q,k}, \end{aligned} \tag{6}$$

where

$$\begin{aligned} D_{k-q,q,k} &= -i\theta(t-t')\left\langle\left[a_{k-q}c_q;\ a_k^+(t')\right]\right\rangle, \\ D^{(1)}_{k+q,q,k} &= -i\theta(t-t')\left\langle\left[a_{k+q}c_q^+;\ a_k^+(t')\right]\right\rangle. \end{aligned} \tag{7}$$

The equations of motion for the functions (7) will contain higher-order Green functions; for these Green functions, in turn, one can write equations of motion. Continuing this process, we obtain an infinite system of coupled equations. The decoupling of such a chain is, as a rule, justified each time by the physical aspect of the problem. Here we shall use the fact that the energy densities of the l- and s-plasmons greatly exceed the energy densities of their interaction; therefore, in the equations of motion for the functions (7), we single out and retain only those terms that contain the functions G_k, $D_{k-q,q,k}$, $D^{(1)}_{k+q,q,k}$. This is achieved by means of the following approximations:

$$\sum_{q_1} \Phi_{q_1;k-q_1,k-q+q_1} \left\{-i\theta(t-t') \left\langle \left[a_{k-q+q_1} c^+_{q_1} c_q;\; a^+_k(t')\right]\right\rangle\right\} \to \Phi_{q;k\,k-q} n_{qG} k,$$

$$\sum_{k_1} \Phi_{q;k_1\,k_1-q} \left\{-i\theta(t-t') \left\langle \left[a_{k-q} a^+_{k_1-q} a_{k_1};\; a^+_k(t')\right]\right\rangle\right\} \to \Phi_{q;k\,k-q}(1+N_{k-q})G_k,$$

$$\sum_{q_1} \Phi_{q_1;k+q_1\,k+q-q_1} \left\{-i\theta(t-t') \left\langle \left[c_{q_1} a_{k+q-q_1} c^+_q;\; a^+_k(t')\right]\right\rangle\right\} \to \Phi_{q;k\,k+q}(1+n_q)G_k,$$

$$\sum_{k_1} \Phi_{q;k_1\,k_1-q} \left\{-i\theta(t-t') \left\langle \left[a_{k+q} a^+_{k_1} a_{k_1-q};\; a^+_k(t')\right]\right\rangle\right\} \to \Phi_{q;k\,k+q}(1+N_{k+q})G_k,$$

$$N_k = \langle a^+_k a_k \rangle, \qquad n_q = \langle c^+_q c_q \rangle.$$

Thus we have:

$$i\frac{dD_{k-q,q,k}}{dt} = (\Omega_{k-q} + \omega_q) D_{k-q,q,k} + \varepsilon \Phi_{q;k\,k-q}(1 + n_q + N_{k-q}) G_k,$$

$$i\frac{dD^{(1)}_{k+q,q,k}}{dt} = (\Omega_{k+q} - \omega_q) D^{(1)}_{k+q,q,k} + \varepsilon \Phi_{q;k\,k+q}(n_q - N_{k+q}) G_k. \tag{8}$$

Passing to the Fourier representation $A = \int A(\omega) e^{-i\omega t}\, d\omega$, from expressions (6) and (8) we obtain

$$\left(\omega - \Omega_k - \varepsilon^2 \sum_q \Phi_{q;k\,k-q} \frac{1 + N_{k-q} + n_q}{\omega - \Omega_{k-q} - \omega_q} - \varepsilon^2 \sum_q \Phi_{q;k\,k+q} \frac{n_q - N_{k+q}}{\omega + \omega_q - \Omega_{k+q}}\right) G_k(\omega) = \frac{1}{2\pi}.$$

Let us continue this function into the upper half-plane $\omega \to \omega + i\alpha$ and use the symbolic identity:

$$\frac{1}{\omega-\omega_0 \pm i\alpha} = P\frac{1}{\omega-\omega_0} \mp i\pi\delta(\omega-\omega_0).$$

Here P denotes an integral in the sense of the principal value, and $(\omega-\omega_0)$ is regarded as a real quantity. We then obtain

$$[\omega-\Omega_k-\varepsilon^2(Q_k^{(1)}(\omega)-iQ_k^{(2)}(\omega))]G_k(\omega)=\frac{1}{2\pi}, \tag{9}$$

where

$$Q_k^{(1)}(\Omega_k)=P\sum_q\left(\Phi^2_{q;k\,k-q}\frac{1+n_q+N_{k-q}}{\Omega_k-\Omega_{k-q}-\omega_q}+\Phi^2_{q;k\,k+q}\frac{n_q-N_{k+q}}{\Omega_k+\omega_q-\Omega_{k+q}}\right);$$

$$Q_k^{(2)}(\Omega_k)=\gamma_k^{(1)}(\Omega_k)+\gamma_k^{(2)}(\Omega_k)=\pi\sum_q\Phi^2_{q;k\,k-q}(1+n_q+N_{k-q})\times \tag{10}$$

$$\times\delta(\Omega_k-\Omega_{k-q}-\omega_q)+\pi\sum_q\Phi^2_{q;k\,k+q}(n_q-N_{k+q})\delta(\Omega_k+\omega_q-\Omega_{k-q}).$$

The frequency shift and damping of the l-plasmon caused by interaction with the s-plasmon are expressed, respectively, through $Q_k^{(1)}(\Omega_k)$ and $Q_k^{(2)}(\Omega_k)$; here $\gamma_k^{(1)}(\Omega_k)$ describes the damping due to decay, while $\gamma_k^{(2)}(\Omega_k)$ describes the damping due to scattering accompanied by absorption of an s-plasmon. The latter is not a threshold effect, and therefore is of no interest to us.

Let us proceed to the concrete calculation of the damping quantity

$$\gamma_k^{(1)}(\Omega_k)=\pi\sum_q\Phi^2_{q;k\,k-q}(1+n_q+N_{k-q})\delta(\Omega_k-\Omega_{k-q}-\omega_q).$$

The argument of the δ-function appearing in this expression has the roots:

$$q_1=0, \qquad q_2=2k(t-z),$$

where

$$t=\frac{(\mathbf{kq})}{kq}, \qquad z=\frac{k_\Pi}{k}.$$

Replacing n_q and N_{k-q} by Rayleigh–Jeans distributions and retaining the leading terms, we obtain

$$\gamma_k^{(1)}(\Omega_k) = \frac{1}{8\pi^2}\frac{T}{c_s}\int \frac{\Phi^2_{q;k\,k-q}}{q}\sum_i \frac{\delta(q-q_i)}{\left|\frac{\partial}{\partial q}(\Omega_k - \Omega_{k-q} - \omega)\right|_{q=q_i}}\,dq.$$

Using (4) and integrating the last expression over q with the aid of the δ-function, we shall have:

$$\gamma_k^{(1)}(\Omega_k) = \frac{1}{24\pi}\frac{\Omega_0}{N_D}(kR_D)\int_z^1 \frac{t^4 - 2t^3 z - t^2(1-z^2) + zt + \frac{1}{4}}{zt - z^2 - \frac{1}{4}}(z-1)\,dt, \quad (11)$$

where $N_D = nR_D^3$. From formula (11) it follows that near the threshold $z \simeq 1$, l-plasmons damp due to emission of s-plasmons with decrement:

$$\gamma_k^{(1)}(\Omega_k) \simeq 10^{-2}\frac{\Omega_0}{N_D}(kR_D)(1-z^2). \quad (12)$$

Thus, for $z \ll 1$,

$$\gamma_k^{(1)} \simeq 10^{-2}\frac{\Omega_0}{N_D}(kR_D).$$

Note that $\gamma_k^{(1)} \ll \gamma_{\text{st}}$, where $\gamma_{\text{st}} \sim \dfrac{\Omega_0}{N_D}\ln N_D$ is the damping decrement of l-plasmons caused by collisions of electrons with ions.

$(\gamma_k^{(1)}/\gamma_{\text{st}} = 10^{-2}(kR_D)/\ln N_D \ll 1)$. For the Landau damping decrement we have the expression: $\gamma_L \sim \dfrac{\Omega_0}{(kR_D)^3}e^{-1/(kR_D)^2}$; therefore

$$\frac{\gamma_k^{(1)}}{\gamma_L} = \frac{(kR_D)^4}{10^2 N_D}e^{1/(kR_D)^2}.$$

It follows from this that $\gamma_k^{(1)} \gtrsim \gamma_L$ for waves whose wave vectors satisfy the condition $k \lesssim k_0$, where k_0 is found from the equation

$$e^{1/(k_0R_D)^2} = 10^2 N_D\frac{1}{(k_0R_D)^4}.$$

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Note: Figure translations are in progress. See original paper for figures.