Reports of the Academy of Sciences of the USSR

1965

SovietRxiv

Abstract

Full Text

Reports of the Academy of Sciences of the USSR

1965. Volume 163, No. 2

Mechanics

Ya. N. Roitenberg

A Corrected Gyrocompass

(Presented by Academician A. Yu. Ishlinskii, 29 XII 1964)

A gyroscopic compass whose sensitive element is mounted on a platform stabilized in the horizontal plane and is brought into the meridian by means of correcting torques applied to the gyroscope ([1]) may be called a corrected gyrocompass.

Let us consider a single-rotor corrected gyrocompass, consisting of an astatized gyroscope whose outer gimbal-ring axis is mounted on a platform stabilized in the horizontal plane and, consequently, coincides with the axis ξ directed along the radius of the terrestrial sphere (Fig. 1). The angles α and β determine the position of the gyroscope rotor axis z relative to the reference system $\xi\eta\zeta$. We shall denote the angular velocity of the gyroscope' s proper rotation by Φ'.

The projections of the instantaneous angular velocity of the reference system $\xi\eta\zeta$ onto its axes ξ, η, ζ will be

$$u_1 = -\frac{v_N}{R}, \qquad u_2 = U\cos\varphi + \frac{v_E}{R}, \qquad u_3 = U\sin\varphi + \frac{v_E}{R}\operatorname{tg}\varphi, \tag{1}$$

where v_E and v_N are the eastward and northward components of the ship' s velocity relative to the terrestrial sphere, R is the radius of the terrestrial sphere, U is the angular velocity of the daily rotation of the terrestrial sphere, and φ is the latitude of the ship' s location.

Restricting ourselves to the study of the precessional motion of the gyrocompass, one may take for its kinetic energy the approximate expression

$$T \approx {}^1\!/_2 C r^2, \tag{2}$$

where C is the moment of inertia of the gyroscope rotor relative to its axis z, and r is the projection of the absolute angular velocity of the gyroscope onto the axis z. It is not hard to see that

Fig. 1

Figure 1: Fig. 1

$$r = \alpha' \sin\beta + \Phi' - u_1 \sin\alpha \cos\beta + +u_2 \cos\alpha \cos\beta + u_3 \sin\beta. \quad (3)$$

Fig. 1

Assuming that the moment of the resistance forces about the gyroscope rotor axis z is balanced by the active driving torque, we take $Q_\Phi \equiv 0$. Since $\partial T/\partial\Phi' = Cr,\ \partial T/\partial\Phi = 0$, we shall have

$$Cr = H = \text{const.} \quad (4)$$

Taking into account that

$$\partial T/\partial\alpha' = H \sin\beta, \qquad \partial T/\partial\alpha = -H\left(u_1 \cos\alpha \cos\beta + u_2 \sin\alpha \cos\beta\right),$$

$$\partial T/\partial\beta' = 0, \quad (5)$$

$$\partial T/\partial\beta = H\left(\alpha' \cos\beta + u_1 \sin\alpha \sin\beta - u_2 \cos\alpha \sin\beta + u_3 \cos\beta\right),$$

we find the following equations of motion of the gyroscopic compass:

$$\begin{aligned} H\beta' \cos\beta + H(u_1 \cos\alpha \cos\beta + u_2 \sin\alpha \cos\beta) &= M_\xi, \\ H(\alpha' \cos\beta + u_1 \sin\alpha \sin\beta - u_2 \cos\alpha \sin\beta + u_3 \cos\beta) &= M_{x^*}. \end{aligned} \quad (6)$$

Here M_ξ and M_{x^*} are the correcting torques applied to the gyrocompass about the axes ξ and x^*, respectively.

Since the instrument is mounted on a platform stabilized in the horizon, the elevation angle β of the z-axis of the gyroscope rotor above the plane of the horizon can be measured.

The angle between the direction of the horizontal projection of the z-axis of the gyroscope and the vector $\mathbf{v}$ of the ship' s velocity relative to the terrestrial sphere (Fig. 1) is equal to $\psi + \alpha$, where ψ is the ship' s course. Since the angle $\psi + \alpha$ can be measured, then, if an instrument is available which determines the ship' s velocity relative to the terrestrial sphere, the components of the ship' s velocity $v \cos(\psi + \alpha)$ and $v \sin(\psi + \alpha)$ can be determined.

Also assuming the latitude φ of the ship' s position to be known, one can apply to the gyroscope correcting torques formed according to the law

$$M_\xi = -H\frac{v}{R}\cos(\psi+\alpha)\cos\beta - \mu K\sin\beta,$$
$$M_{x^*} = -H\frac{v}{R}\sin(\psi+\alpha)\sin\beta +$$
$$+H\left[U\sin\varphi + \frac{v}{R}\sin(\psi+\alpha)\operatorname{tg}\varphi\right]\cos\beta + K\sin\beta, \tag{7}$$

where μ and K are certain constant coefficients. Here it is naturally assumed that the latitude of the ship' s position $\varphi < 90^\circ$.

Taking into account that

$$v\cos(\psi+\alpha) = v_N\cos\alpha - v_E\sin\alpha,$$
$$v\sin(\psi+\alpha) = v_E\cos\alpha + v_N\sin\alpha, \tag{8}$$

where

$$v_N = v\cos\psi, \qquad v_E = v\sin\psi, \tag{9}$$

and replacing M_ξ and M_{x^*} by their expressions (7), the equations of motion of the gyroscopic compass (6) can be reduced to the form

$$H\beta'\cos\beta + HU\cos\varphi\sin\alpha\cos\beta + \mu K\sin\beta = 0,$$
$$H\alpha'\cos\beta - HU\cos\varphi\cos\alpha\sin\beta +$$
$$+H\frac{v_E}{R}\operatorname{tg}\varphi\cos\beta(1-\cos\alpha) - H\frac{v_N}{R}\operatorname{tg}\varphi\sin\alpha\cos\beta - K\sin\beta = 0. \tag{10}$$

It is not difficult to see that the system of differential equations (10) has the particular solution

$$\alpha = 0, \qquad \beta = 0. \tag{11}$$

Thus, for any law of motion of the ship $\mathbf{v} = \mathbf{v}(t)$, the direction toward the north is an equilibrium position of the z-axis of the gyrocompass, i.e. the corrected gyrocompass has no speed deviation.

For satisfactory operation of the gyrocompass, its oscillations relative to the equilibrium position $\alpha = \beta = 0$ must be damped.

The variational equations, which can be obtained from (10) by taking α and β to be small angles, can be brought to the form

$$\alpha' - \frac{v_N}{R}\operatorname{tg}\varphi\cdot\alpha - \left(\frac{K}{H} + U\cos\varphi\right)\beta = 0, \qquad \beta' + \frac{\mu K}{H}\beta + U\cos\varphi\cdot\alpha = 0. \tag{12}$$

Since $v_N = v_N(t)$, $\varphi = \varphi(t)$, equations (12) constitute a system of linear differential equations with variable coefficients.

Sufficient conditions for the asymptotic stability of the particular solution (11), which determines the equilibrium position of the z-axis of the gyroscopic compass, will be found with the aid of a Lyapunov function, which can be constructed by the method proposed in (2).

Denoting

$$f_1(t) = \frac{v_N}{R} \operatorname{tg} \varphi, \qquad f_2(t) = U \cos \varphi, \tag{13}$$

we reduce equations (12) to the form

$$\alpha' - \frac{K}{H}\beta = f_1(t)\alpha + f_2(t)\beta, \qquad \beta' + \frac{\mu K}{H}\beta + s\alpha = [s - f_2(t)]\alpha. \tag{14}$$

Here the coefficient s in the second equation (14) is chosen so that, for the system of equations

$$\alpha' - \frac{K}{H}\beta = 0, \qquad \beta' + \frac{\mu K}{H}\beta + s\alpha = 0 \tag{15}$$

the characteristic equation

$$\gamma^2 + \frac{\mu K}{H}\gamma + \frac{sK}{H} = 0 \tag{16}$$

has a pair of complex roots

$$\gamma_1, \gamma_2 = \varepsilon \pm \omega i, \tag{17}$$

where

$$\varepsilon = -\frac{\mu K}{2H}, \qquad \omega = \left(\frac{sK}{H} - \varepsilon^2\right)^{1/2}. \tag{18}$$

We now pass to new variables x_1 and x_2, which we introduce by means of the relations

$$\alpha = x_1, \qquad \beta = \frac{\varepsilon H}{K}x_1 + \frac{\omega H}{K}x_2. \tag{19}$$

In accordance with (14), the functions x_1 and x_2 will satisfy the differential equations

$$x_1' = \left[\varepsilon + f_1(t) + \frac{\varepsilon H}{K} f_2(t)\right] x_1 + \omega \left[1 + \frac{H}{K} f_2(t)\right] x_2,$$

$$x_2' = \varepsilon \left[1 - \frac{H}{K} f_2(t)\right] x_2 - \left\{\omega - \frac{K}{\omega H}[s - f_2(t)] + \frac{\varepsilon}{\omega}\left[f_1(t) + \frac{\varepsilon H}{K} f_2(t)\right]\right\} x_1. \tag{20}$$

As a Lyapunov function we take the negative definite function

$$V = -\frac{1}{2}(x_1^2 + x_2^2). \tag{21}$$

Its derivative with respect to time, by virtue of the differential equations (20), will have the form

$$V' = a_{11} x_1^2 + 2a_{12} x_1 x_2 + a_{22} x_2^2, \tag{22}$$

where

$$a_{11} = -\left[\varepsilon + f_1(t) + \frac{\varepsilon H}{K} f_2(t)\right],$$

$$a_{12} = \frac{1}{2}\left\{\frac{\varepsilon}{\omega} f_1(t) - \frac{H}{\omega K}(\omega^2 - \varepsilon^2) f_2(t) - \frac{K}{\omega H}[s - f_2(t)]\right\}. \tag{23}$$

$$a_{22} = -\varepsilon\left[1 - \frac{H}{K} f_2(t)\right].$$

According to Sylvester' s theorem, the quadratic form (22) will be positive definite if, for any instant of time t, the conditions

$$a_{11}(t) > 0, \qquad a_{11}(t) a_{22}(t) - [a_{12}(t)]^2 > 0 \tag{24}$$

are satisfied.

Conditions (24) are sufficient conditions for the asymptotic stability of the equilibrium position of the gyroscopic compass.

As an example, consider a wide-range compass whose parameters have the values: $K/H = 3.6$ sec^{-1}, $\mu = 0.005$. The coefficient s, by which the transformation (14) is determined, and accordingly the parameters of the Lyapunov function, will be taken as $s = 4 \cdot 10^{-5}$ sec^{-1}.

For the indicated parameter values, the sufficient stability conditions (24) are satisfied at any point of the rectangle

$$-\varphi_m \leqslant \varphi \leqslant \varphi_m, \qquad -v_m \leqslant v_N \leqslant v_m, \tag{25}$$

where $\varphi_m = 85^\circ$, $v_m = 600$ m $\cdot$ sec^{-1}, so that the corrected gyroscopic compass can be used in aviation, as was also noted in work (1).

Let us note that, since $\varphi' = v_N/R$, the latitude of the vessel' s location is determined by the expression

$$\varphi(t) = \varphi(0) + \int_0^t \frac{v_N(\tau)}{R}\, d\tau. \tag{26}$$

It follows from what has been set forth that the equilibrium position of the gyroscopic compass preserves stability under any law of variation of the northern component of the vessel' s velocity $v_N = v_N(t)$, for which $v_N(t)$ and $\varphi(t)$ do not leave the region (25).

Moscow State University
named after M. V. Lomonosov

Received
16 XII 1964

CITED LITERATURE

[1] P. H. Savet (Ed.), *Gyroscopes: Theory and Design*, N. Y., 1961, p. 80. [2] Ya. N. Roitenberg, *Prikl. matem. i mekh.*, **22**, 2 (1958).

Note: Figure translations are in progress. See original paper for figures.