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Abstract

Full Text

THEORY OF ELASTICITY

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FLUTTER OF A THREE-LAYER CIRCULAR CONICAL SHELL

The stability is considered of an elastic thin three-layer straight circular conical shell, supported at its ends, with a rigid filler resistant to transverse shear, and subjected on the outside to a potential supersonic flow. The aerodynamic pressure of the flow is taken into account according to linear piston theory. The determination of critical pressures and flow velocities is reduced to the investigation of the eigenvalues of the matrix of a system of homogeneous equations obtained by applying the Bubnov method to the linear dynamic equations of the theory of shallow shells (^{1}).

1. Formulation of the problem. Small transverse vibrations of a conical circular three-layer shell of a structure asymmetric in thickness are described, according to (^{1}), by the system

$$L_1 = D\nu\nabla^2\nabla^2 w + \frac{\operatorname{ctg}\alpha}{r}N_2 + \left[N_1^0\frac{\partial^2}{\partial r^2} + N_2^0\left(\frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2\sin^2\alpha}\frac{\partial^2}{\partial\varphi^2}\right)\right]w +$$

$$+\Omega\frac{\partial^2 w}{\partial t^2} + q_1\frac{\partial w}{\partial t} + q_2\frac{\partial w}{\partial x} + D(1-\nu)\nabla^2\nabla^2\chi = 0, \qquad (1,1)$$

$$L_2 = \left(1 - \frac{h_2}{\beta}\nabla^2\right)\chi - w = 0.$$

Here

$$\nabla^2 = \frac{\partial^2(\ )}{\partial r^2} + \frac{\partial(\ )}{r\,\partial r} + \frac{\partial^2(\ )}{r^2\sin^2\alpha\,\partial\varphi^2},$$

$N_1^0 = pr\ \operatorname{tg}\alpha/2$, $N_2^0 = pr\ \operatorname{tg}\alpha$ are the forces of the momentless state caused by the static pressure p (the difference between the pressures outside and inside the shell; for $p > 0$, compression);

$$N_2 = \frac{Eh}{1-\nu^2}\left[\frac{u}{r} + \frac{\operatorname{ctg}\alpha}{r}w + \frac{1}{r\sin\alpha}\frac{\partial v}{\partial\varphi} + \nu\frac{\partial u}{\partial r}\right]$$

is the membrane force in the direction tangent to the lines $r = \text{const}$; r, φ, α are spherical coordinates (α is the semi-vertex angle of the cone, r is the distance from the vertex to the current point of the middle surface); u, v, w are, respectively, the displacements from the middle surface along the generator, in the circumferential direction, and along the normal to the middle surface; D is the cylindrical stiffness of the package relative to the middle surface; E is the reduced Young' s modulus, ν the reduced Poisson coefficient (^{1}); $q_1 = \varkappa p_\infty / c$; $q_2 = \varkappa p_\infty M$; $\varkappa$ is the adiabatic exponent; p_∞, M, c are the pressure, Mach number, and speed of sound of the undisturbed flow, whose density is equal to ρ;

$$\Omega = \sum_{i=1}^{3} \rho_i h_i = h\bar{\rho};$$

ρ_i, h_i are the specific density and thickness of the i-th layer; $h = h_1 + h_2 + h_3$; $\bar{\rho}$ is the averaged density; θ, ν, β are taken from (^{1}).

At the ends of the cone $r = r_0,\ r = r_1$, the boundary conditions must be satisfied

$$\chi = w = \frac{\partial^2 w}{\partial r^2} + \nu \frac{1}{r} \frac{\partial w}{\partial r} + \nu \frac{1}{r^2 \sin^2 \alpha} \frac{\partial^2 w}{\partial \varphi^2} = 0; \qquad N_1 = v = 0, \tag{1,2}$$

where

$$N_1 = \frac{Eh}{1 - \nu^2} \left[\frac{\partial u}{\partial r} + \frac{\nu}{r \sin \alpha} \frac{\partial v}{\partial \varphi} + \nu \frac{u}{r} + \frac{\nu \operatorname{ctg} \alpha}{r} w \right]. \tag{1,3}$$

The displacements u, v are related to the deflection w by the system of differential equations

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{1}{r^2} u + \frac{1 - \nu}{2} \frac{1}{r^2 \sin^2 \alpha} \frac{\partial^2 u}{\partial \varphi^2} + \frac{1 + \nu}{2} \frac{1}{r \sin \alpha} \frac{\partial^2 v}{\partial r \, \partial \varphi} -$$

$$- \frac{3 - \nu}{2} \frac{1}{r^2 \sin^2 \alpha} \frac{\partial v}{\partial \varphi} + \frac{\nu \operatorname{ctg} \alpha}{r} \frac{\partial w}{\partial r} - \frac{\operatorname{ctg} \alpha}{r^2} w = 0;$$

$$\frac{1 + \nu}{2} \frac{1}{r \sin \alpha} \frac{\partial^2 u}{\partial r \, \partial \varphi} + \frac{3 - \nu}{2} \frac{1}{r^2 \sin \alpha} \frac{\partial u}{\partial \varphi} + \frac{1 - \nu}{2} \frac{\partial^2 v}{\partial r^2} + \frac{1 - \nu}{2} \frac{1}{r} \frac{\partial v}{\partial r} +$$

$$+ \frac{1}{r^2 \sin^2 \alpha} \frac{\partial^2 v}{\partial \varphi^2} - \frac{1 - \nu}{2} \frac{v}{r^2} + \frac{\operatorname{ctg} \alpha}{r^2 \sin \alpha} \frac{\partial w}{\partial \varphi} = 0,$$

which are solved subject to conditions $(1,3)$. Then, determining u, v from $(1,4)$ in terms of w, we find an expression for N_2, which we substitute into the first equation $(1,1)$.

2. Solution of the original system. Approximating w and χ by series (w is the frequency of oscillations of the shell; t is time; n is the number of circumferential waves)

$$w = e^{\omega t} \cos n\varphi \sum_{l=1}^{N} A_l w_l(r), \qquad \chi = e^{\omega t} \cos n\varphi \sum_{l=1}^{N} B_l \chi_l(r), \tag{2,1}$$

and applying Bubnov' s method $(^2)$, we obtain a system of linear algebraic equations for determining the constants A_l and B_l

$$\int_{r_0}^{r_1} \int_0^{2\pi} L_1(A_l, B_l)\, \delta w_j r\, dr\, d\varphi = 0, \qquad \int_{r_0}^{r_1} \int_0^{2\pi} L_2(A_l, B_l)\, \delta \chi_j r\, dr\, d\varphi = 0. \tag{2,2}$$

Introduce the substitution

$$r = r_1 e^{\lambda x}, \qquad \lambda = \ln r_0 - \ln r_1$$

and take w_l and χ_l in the form

$$w_l = e^{\mu_1 \lambda x} \sin l\pi x; \qquad \chi_l = e^{\mu_1 \lambda x} \sin l\pi x.$$

The conditions $(1,2)$ are satisfied for $\mu_1 = \frac{1}{2}(1-\nu)$. Eliminating A_l, B_l from $(2,2)$ leads to a system whose matrix must have zero determinant. Here A, B are square matrices of order N, whose elements depend on the dimensionless parameters of the problem

$$\mathrm{M},\ k,\ \nu,\ \mu^2,\ \delta,\ \sin\alpha,\ f,\ f_\infty,\ \Gamma,\ n; \tag{2,3}$$

$$\Lambda = -(\overline{w} + a)^2 + a^2; \tag{2,4}$$

$$k = H^2\pi^2/\beta, \quad \mu^2 = 1/z^2\pi^4 H^2, \quad \delta = \lambda\pi/\sin\alpha, \quad m = n\delta/\pi, \quad f = \chi\pi^4 p_\infty z/E, \tag{2,5}$$

$$f_\infty = \chi\pi^4 p z/E, \quad \Gamma = \frac{\rho}{\rho} z, \quad z = \sqrt{\theta/12(1-\nu^2)}, \quad H = n/R_1, \quad R_1 = r_1 \operatorname{tg}\alpha,$$

Fig. 3

Figure 1: Fig. 3

where $\overline{w}$ is proportional to the frequency w and has the same sign as it.

The dependence of the elements of the matrices A and B on the parameters $(2,3)$ is readily established when carrying out the integration in the system $(2,2)$; in this case the number of circumferential waves n proves to be arbitrary.

Fig. 1

Labels in the figure: "Tension" ; "Compression" ; $f \cdot 10^2$; f_∞; M^*; $n = 2$; $n = 6$.

Curve labels: 3, 4, 5, 6, 7.

Fig. 2

Labels in the figure: M^*; κ; $n = 8$; $n = 9$; $\nu = 0$; $\nu = 0.1$; 0.05; $\sin\alpha = 0.45$; $\sin\alpha = 0.25$.

Curve labels and axis markings shown: $4, 5, 6, 7, 8, 9, 10, 11, 12$; $\kappa = 0.2, 0.4, 0.6, 0.8, 1.0$.

3. Numerical investigations of critical states.

To the imaginary axis in the complex plane $\overline{w}$ there corresponds the so-called "stability parabola" in the plane $\Lambda = p + iq$, $p = -q^2/4a^2$.

Calculations show that the eigenvalues Λ_s are located inside the parabola for $f = 0$ and $M = 0$ and intersect it for certain values of f or M, depending on n; the least was taken as the critical value f^* or M^*.

If M^* at $N = 10$ is conditionally taken as the true value, then, as calculations for various parameters show, the error of the "second approximation" $(N = 2)$ is 50-100%, of the fourth 8-11%, and of the sixth and eighth 3-6%. (For f^* the convergence is considerably better.) Convergence is also improved when the conicity is decreased, when the shear parameter of the filler k is increased, and also in the neighborhood of the minimum M^*. In what follows, numerical results obtained for $N = 6$ by means of Danilevsky' s and Routh' s methods are used for the most part.

Fig. 3

The curve of the dependence $f(M^*)$ consists, for each n, of a divergent part, elongated along the M^* axis, and a flutter part, crossing the M^* axis (Fig. 1; $k = 0$, $\delta = -9.6$, $\mu^2 = 1000$, $\sin\alpha = 0.45$). As is seen from this figure, the greater the internal pressure in the shell $(f < 0)$, the fewer circumferential waves the buckling mode has.

Fig. 4

Fig. 4

Figure 2: Fig. 4

Figure 2 gives the dependence $\min M^*(k, f_\infty(k))$ for various ν for $f_\infty = 5f^*(k)$, $\sin\alpha = 0.25$ and 0.45, $\mu^2 = 1000$, $\delta = -9.6$, $\Gamma = 10^{-3}$. With further increase of k the curves reach horizontal asymptotes. The increase of M^* with increasing k is explained by the sharp decrease of $f^*(k)$, and consequently of f_∞. Figure 3 shows the dependence $M^*(\mu^2)$ for $\sin\alpha = 0.45$, $\nu = 0.1$, $\delta = -96$ at $f_\infty = 5f^*$ (it should also be taken into account that $f^*(\mu^2)$ decreases sharply). The dependence $M^*(\sin\alpha)$ for various k; $\nu = 0.1$; $\delta = -9.6$, $\mu^2 = 1000$, $\Gamma = 10^{-3}$ is given in Fig. 4.

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Note: Figure translations are in progress. See original paper for figures.