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N. N. ROZANOV, A. V. TULUB

A CONTRIBUTION TO THE THEORY OF THE ZEEMAN EFFECT IN GAS LASERS

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1. A weak magnetic field, parallel to the z-axis of a neon–helium laser, splits the upper and lower working levels of the strong neon line $\lambda\ 1.15\,\mu$, respectively, into 3 and 5 sublevels; moreover, when observed along the z-axis, 3 transitions with left circular polarization ($\Delta m = +1$) and 3 with right circular polarization ($\Delta m = -1$) are found. In the most detailed work on the Zeeman effect in a laser ([1]), the beat frequency between oscillations with different polarizations was studied in the single-mode regime as a function of the magnetic-field intensity H. In this case, for certain parameters an anomalous region of H was found in which the beat frequency went to zero. At small H, the rotation of the plane of polarization was also studied.

2. To explain the anomalous region of coherence, it should be taken into account that the generation frequency, generally speaking, coincides neither with the frequency of the atomic transition nor with the "eigenfrequency" of the resonator. Therefore, although with increasing H the distance between the atomic Zeeman sublevels increases, this does not imply an increase in the difference of the generation frequencies as a consequence of the different role of the effects of frequency pulling and repulsion.

3. This assumption was confirmed by the calculation of nonlinear effects carried out in the present work. For this purpose Lamb' s method ([2]) was generalized to the case of two polarization states $\mu = 1, 1'$ of the electromagnetic field and three electronic levels a, b, and c, two of which $a(m = +1)$ and $b(m = -1)$ will belong to the upper level ($J = 1$), while $c(m = 0)$ belongs to the lower ($J = 0$). The electric vector of the radiation field $\mathbf{E}$ and the polarization of the medium $\mathbf{P}$, after passing to the scalar quantities $\sqrt{2}A^{(\pm)} = A_x \pm iA_y$, shall be written in the form

$$E^{(\pm)}(z,t) = \sum_{n\mu} E_{n\mu}(t)\sin(K_{nz})\exp\{\pm i(-1)^{\mu+1}(\nu_{n\mu}t + \varphi^0_{n\mu})\},$$

$$P^{(\pm)}(z,t) = \sum_{n\mu} \{C_{n\mu}(t) \pm i(-1)^{\mu} S_{n\mu}(t)\} \sin(K_{nz}) \times$$

$$\times \exp\{\pm i(-1)^{\mu+1}(\nu_{n\mu} t + \varphi^0_{n\mu})\},$$

where ([2]) $E_{n\mu}$, $C_{n\mu}$, and $S_{n\mu}$ are slowly varying functions of time, $\nu_{n\mu} = \Omega_n + \varphi_{n\mu}$, and $\varphi^0_{n\mu}$ are the initial phases. The transformed Maxwell equations give

$$(\nu_{n\mu} - \Omega_n) E_{n\mu} = -\tfrac{1}{2}(\nu/\varepsilon_0) C_{n\mu}; \qquad \dot{E}_{n\mu} + \tfrac{1}{2}(\nu/Q_n) E_{n\mu} = -\tfrac{1}{2}(\nu/\varepsilon_0) S_{n\mu}. \quad (1)$$

4. The polarization $\mathbf{P}$ is composed of the polarizations $\mathbf{P}_i = \langle i|e\mathbf{r}_i|i\rangle$ of the individual atoms (e is the electron charge). With the aid of the density matrix ([2]) ρ, we obtain $P^{(+)} = -d^* \rho_{ca} + d\rho_{bc}$ (the atomic index i has been omitted), where ([1,3]) $d = \langle j=1|e\mathbf{r}|j=0\rangle$ is the dipole moment. The equation of motion of the density matrix has the form

$$\dot{\rho} = (i\hbar)^{-1}[\hat{H}, \rho] + \tfrac{1}{2}(\Gamma\rho + \rho\Gamma) \quad (2)$$

with Hamiltonian $\hat{H} = \hat{H}_0 + \hat{V}_0 + \hat{V}$, where $\hat{H}_0$ is the one-particle operator of the electron energy; $\hat{V}_0$ and $\hat{V}$ are, respectively, the operators of the energy of interaction with the given magnetic field and with the radiation field in the electric-dipole approximation; $\Gamma_{mn} = \gamma_m \delta_{mn}$; γ_m is the phenomenological decay constant of level m (a, b, or c); $\gamma_{mn} = (\gamma_m + \gamma_n)/2$. For the basic wave functions ψ_m we have

$$(\hat{H}_0 + \hat{V}_0)\psi_m(\mathbf{r}) = W_m \psi_m(\mathbf{r}), \qquad V_{ca} = -e\langle c|\mathbf{Er}|a\rangle = dE^{(+)},$$

$$V_{cb} = -dE^{(-)}, \qquad V_{ab} = 0.$$

5. We solve equation (2) by the method of successive approximations, taking the interaction of the atoms with the radiation field $\hat{V}$ to be small. We introduce $\hbar\omega_{mn} = W_m - W_n$, $x_{mc} = (\omega_{mc} - \Omega_n)/KU$, $y_{mc} = \gamma'_{mc}/KU$ (U is the rms velocity of the Maxwellian distribution). In the first approximation, analogously to (2), we obtain

$$C^{[1]}_{n\mathrm{I}}/E_{n\mathrm{I}} = -(\varepsilon_0/Q_n) u^{-1}(0, y_{ac}) \eta^{ac} v(x_{ac}, y_{ac}),$$

$$C^{[1]}_{n\mathrm{II}}/E_{n\mathrm{II}} = -(\varepsilon_0/Q_n) u^{-1}(0, y_{bc}) \eta^{bc} v(x_{bc}, y_{bc}),$$

$$S^{[1]}_{n\mathrm{I}}/E_{n\mathrm{I}} = -(\varepsilon_0/Q_n) u^{-1}(0, y_{ac}) \eta^{ac} u(x_{ac}, y_{ac}),$$

$$S^{[1]}_{n\mathrm{II}}/E_{n\mathrm{II}} = -(\varepsilon_0/Q_n)u^{-1}(0,y_{bc})\eta^{bc}u(x_{bc},y_{bc}). \tag{3}$$

Here η ([2,4]) is the relative (with respect to threshold) pumping, while $u(x,y)$ and $v(x,y)$ are the real and imaginary parts of the function ([5])

$$w(z) = \exp(-z^2)\left[1 + (2i/\sqrt{\pi})\int_0^z dt\,\exp(t^2)\right],$$

which is related to the "plasma dispersion function" ([6]) Z by the relation $Z(x+iy) = i\sqrt{\pi}w(x+iy)$. Note that $u(x,y)$ is an even function of the argument x, and $v(x,y)$ is odd.

6. In the third approximation the general formulas have a rather cumbersome form. We give only the final results for the case of generation in a single mode at exact tuning ($x_{ac} = -x_{bc} = x = g\mu_{BH}/KU$, μ_B is the Bohr magneton, the g-factor is equal to 1.3, with $\rho_{ab} = 0$). We also put $\eta^{ac} = \eta^{bc} = \eta$, $y_{ac} = y_{bc} = y$, and introduce the functions $M(\omega) = \omega L(\omega) = \omega/(\gamma^2 + \omega^2)$. Then

$$C^{[3]}_{n\mathrm{I}}/E_{n\mathrm{I}} = \frac{1}{2}u^{-1}(0,y)[\varepsilon_0|d|^2/\hbar^2\gamma_{cQ}n]\eta M(\omega_{ab}/2)\left[(\gamma/\gamma_a)E^2_{n\mathrm{I}} + \frac{1}{2}E^2_{n\mathrm{II}}\right],$$

$$C^{[3]}_{n\mathrm{II}}/E_{n\mathrm{II}} = -\frac{1}{2}u^{-1}(0,y)[\varepsilon_0|d|^2/\hbar^2\gamma_{cQ}n]\eta M(\omega_{ab}/2)\left[\frac{1}{2}E^2_{n\mathrm{I}} + (\gamma/\gamma_a)E^2_{n\mathrm{II}}\right], \tag{4}$$

$$S^{[3]}_{n\mathrm{I}}/E_{n\mathrm{I}} = \frac{1}{2}u^{-1}(0,y)[\varepsilon_0|d|^2/\hbar^2\gamma\gamma_{cQ}n]\eta\left[1 + \gamma^2 L(\omega_{ab}/2)\right]\left[(\gamma/\gamma_a)E^2_{n\mathrm{I}} + \frac{1}{2}E^2_{n\mathrm{II}}\right],$$

$$S^{[3]}_{n\mathrm{II}}/E_{n\mathrm{II}} = \frac{1}{2}u^{-1}(0,y)[\varepsilon_0|d|^2/\hbar^2\gamma\gamma_{cQ}n]\eta\left[1 + \gamma^2 L(\omega_{ab}/2)\right]\left[\frac{1}{2}E^2_{n\mathrm{I}} + (\gamma/\gamma_a)E^2_{n\mathrm{II}}\right].$$

Let us determine the intensities of the oscillations in the stationary regime, putting in (1) $\dot{E}_{n\mu} = 0$. From (3) and (4) we obtain $E^2_{n\mathrm{I}} = E^2_{n\mathrm{II}} = E^2_n$,

$$\frac{1}{2}\left[|d|^2/\hbar^2\gamma_a\gamma_c\right]\left[1 + \frac{1}{2}(\gamma_a/\gamma)\right]E^2_n = \left\{\left[u(x,y)/u(0,y)\right] - \eta^{-1}\right\}/\left[1 + \gamma^2 L(\omega_{ab}/2)\right]. \tag{5}$$

The equality of the intensities is natural by virtue of symmetry considerations. Using (5), we calculate the generation frequencies

Fig. 1

Figure 1: Fig. 1

$$\nu_{n\mu} - \Omega_n = (-1)^{\mu+1} A\eta v(x,y) + (-1)^{\mu}\frac{1}{4}A\left[xy/(x^2+2y^2)\right]\times$$

$$\times\left\{\eta[u(x,y)/u(0,y)] - 1\right\}, \tag{6}$$

where $A = (\nu/Q_n)u^{-1}(0,y)$. The first term on the right-hand side of (6) corresponds to the contribution of the first approximation (pulling of oscillations), and the second to the contribution of the third (repulsion).

7. In Ref. ([4]), for the quantity $y = \gamma/KU$ the values 0.04 and 0.08 are given. For the calculations we take $y = 0.06$. The graphs $\Delta\nu(x) = \nu_{n\mathrm{I}} - \nu_{n\mathrm{II}}$ are shown in Fig. 1 for various parameters η; the beat frequency is equal to the modulus of this function. One should single out the critical value of the relative excitation

$$\eta_k \cong [1 - 4\gamma/KU]^{-1} = 1.32$$

(the approximate formulas here and below are obtained from (6) as a result of the substitution, permissible in the region under consideration, $v(x) \to x$, $[u(x,y)/u(0,y)] \to 1$). For $\eta < \eta_k$, for all x the first order of perturbation theory is greater than the third and the function $\Delta\nu(x)$ increases monotonically. For $\eta > \eta_k$, as is seen from Fig. 1, there appears a region $x = 0 \div x_0$ where the ratio of the orders of perturbation theory is the reverse. At $x = x_0$, determined by the formula

Fig. 1

$$x_0^2 \cong 0.5y(1-\eta^{-1}) - 2y^2 = 3\cdot 10^{-2}(1-\eta^{-1}) - 7.2\cdot 10^{-3}, \tag{7}$$

the beat frequency vanishes. (7) also gives the upper boundary $x_0 : x_1 \cong \sqrt{0.5y - 2y^2} \cong 0.15$, corresponding to $\eta^{-1} \to 0$. This same point characterizes the magnetic-field strength at which the beat frequency does not depend on η. For $x > x_1$, an increase in η leads to an increase of $\Delta\nu(x)$; for x somewhat smaller than x_1, the dependence is the opposite. For appreciable x, the beat frequency is completely determined by the first order of perturbation theory ($\Delta\nu \sim \eta v(x,y)$). Let us note that application of formula (6) for excessively large x may prove formal, since with strong separation of the levels generation on several modes will become possible.

Comparison of our formulas with the experimental data is difficult, since in ([1]) the laser parameters and the experimental conditions are not given with

Fig. 2

Figure 2: Fig. 2

sufficient accuracy. Therefore one has to speak mainly about the shape of the curves and the orders of magnitude. In Fig. 2 the solid line represents the function $|\Delta\nu/2A|$ for $\eta = 4$; the dashed line gives $\Delta\nu/2A$, the points are taken from the experiment ([1]). The agreement should be regarded as good, which indicates the smallness of the higher orders of perturbation theory. A small systematic deviation at large x (magnetic fields) may be caused by a change in the parameter KU owing to heating of the laser by the solenoid current.

8. It is convenient to consider the state of polarization of light by means of a simple vector diagram ([7]). The superposition of oscillations with frequencies $\nu_{n\mathrm{I}}$ and $\nu_{n\mathrm{II}}$,

polarized respectively left- and right-circularly, can be described as a "plane-polarized wave"with frequency $(\nu_{n\mathrm{I}}+\nu_{n\mathrm{II}})/2 = \Omega_n$, whose "plane of polarization" rotates with frequency $(\nu_{n\mathrm{I}} - \nu_{n\mathrm{II}})/2 = \Delta\nu/2$. For $\nu_{n\mathrm{I}} = \nu_{n\mathrm{II}}$, which holds for $H = 0$ and at the point x_0, the resulting oscillation is ordinarily plane-polarized. For sawtooth-like [(1)] variations of H, the rotation of the "plane of polarization" will be a quadratic function of time or of H (the slope is connected with the pump magnitude—the parameter η, formula (6) and Fig. 1). The difference between the point x_0 and $x = 0$ consists in the change of the direction of rotation of the "plane of polarization" as x passes through x_0. An order-of-magnitude estimate shows that such a mechanism can make the main contribution to the effect [(1)] of "rotation of the plane of polarization" in a magnetic field. Confirmation of the treatment is the admissibility, in contrast to the classical Hanle effect, of an unlimited rotation, which was also observed in [(1)].

Fig. 2

9. The treatment carried out of the Zeeman effect clearly shows the important role of nonlinear phenomena and the existence of a region where the third approximation exceeds the first. Agreement with experiment makes it possible to regard Lamb' s method as applicable also in this region. The results obtained could serve for determining, from the shape of the corresponding curves, such parameters of gas lasers as γ and KU (see also [(4)]). It would be of interest to test experimentally the existence of the intersection point of the curves x_1, and also the proposed mechanism of temporal rotation of the "plane of polarization."

It should also be noted that it is necessary to obtain additional information on the influence of pressure on the beat frequency $\Delta\nu(H)$. Thus, the graphs in Figs. 1 and 2 were constructed under the assumption $\rho_{ab} = 0$, i.e., it was assumed that collision effects destroy coherence sufficiently rapidly. In the opposite limiting case of complete neglect of collisions, the matrix elements ρ_{ab} calculated from equation (2) give a contribution to the third approximation of perturbation theory (we do not give the corresponding formulas here). Although qualitatively the curves $\Delta\nu(H)$ have the same form, some parameters change substantially (thus, one obtains a clearly underestimated critical pump $\eta_{\mathrm{k}} \cong 1.03$). In Fig. 1 the curve $\Delta\nu(H)$ calculated with allowance for ρ_{ab} is shown; it possesses the same asymmetry as the experimental one; it is seen that in this case the values of x do not agree with the field strengths H used in the experiment [(1)]. A final conclusion on the influence of collisions on the shape of the curves, however, requires further investigations.

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Note: Figure translations are in progress. See original paper for figures.