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Abstract

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ON THE LOWER STABILITY OF A TWO-AXIS GYROSCOPIC STABILIZER IN COMPARISON WITH A SINGLE-AXIS ONE

In an experimental study of gyroscopic stabilizers it was observed that two-axis and three-axis stabilizers are less stable (as a result of which they are more prone to self-oscillations) than single-axis ones with the same design parameters. Using a characteristic example of a two-axis gyroscopic stabilizer, this property of stabilizers is investigated theoretically.

1. In what follows, in order not to complicate the exposition, we shall neglect the elastic compliance of the elements of the stabilizer suspension, the time constant of the stabilizing amplifier, the self-inductance of the output circuit of the amplifier, and shall assume that the stabilizing motor is a direct-current motor with independent excitation. We shall also assume that the gyroscopic stabilizer is located on a fixed base and that there is no friction in the axes of its suspension.

The stability condition of a two-axis gyroscopic stabilizer, as will be shown below, has in this case the form of a simple algebraic inequality similar to the known stability condition of the corresponding single-axis stabilizer.

2. The system of differential equations describing the small motions of a single-axis gyroscopic stabilizer (its scheme is shown in Fig. 1), under the assumptions enumerated above, is as follows:

$$\Theta\frac{d^2\theta}{dt^2}+H\frac{d\beta}{dt}=j\frac{C}{g}\sigma,$$

$$A\frac{d^2\beta}{dt^2}-H\frac{d\theta}{dt}=0, \qquad (1)$$

$$R\sigma+jC\frac{d\theta}{dt}=-\mu\beta.$$

Fig. 1. Scheme of a single-axis gyroscopic stabilizer

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Fig. 2. Schematic of a two-axis gyroscopic stabilizer (the stabilization amplifiers are not shown in the figure)

Figure 2: Fig. 2. Schematic of a two-axis gyroscopic stabilizer (the stabilization amplifiers are not shown in the figure)

Here θ is the angle of rotation of the outer gimbal ring of the stabilizer relative to a certain initial position; β is the angle of rotation of the stabilizer gyro casing from the mean position in which the axis of proper rotation of the gyro rotor is perpendicular to the plane of the outer ring; j is the transmission ratio of the reducer from the axis of the rotor of the stabilizing motor to the axis of the outer ring (the stabilization axis); Θ is the sum, reduced to the stabilization axis, of the moments of inertia of all moving parts of the stabilizer at $\beta = 0$; A is the sum of the moments of inertia of the gyro casing and rotor relative to the casing axis; H is the proper angular momentum of the gyro; R is the resistance of the output circuit of the amplifier; σ is the current in this circuit; C is the coefficient of the counter-electromotive force of the stabilizing motor; μ is the gain coefficient of the stabilizing amplifier.

circuit; g is a dimensional number equal to 9.81 $\mathrm{V \cdot a \cdot sec/kg \cdot m}$ (if the stray magnetic fields of the motor are not taken into account).

Since the angle θ enters equations (1) only through its derivatives, the characteristic equation of system (1)

$$S(\lambda) = \begin{vmatrix} \Theta\lambda^2 & H\lambda & -j\dfrac{C}{g} \\ -H\lambda & A\lambda^2 & 0 \\ jC\lambda & \mu & R \end{vmatrix} = 0 \tag{2}$$

has among its roots the root $\lambda = 0$, which is immaterial for what follows. Therefore, in order to judge the stability of a single-axis gyroscopic stabilizer, it is sufficient to investigate the roots of the cubic algebraic equation following from equation (2), namely:

Fig. 2. Schematic of a two-axis gyroscopic stabilizer (the stabilization amplifiers are not shown in the figure)

$$\lambda^3 + a\lambda^2 + b\lambda + d = 0, \tag{3}$$

whose coefficients

$$a = j^2C^2/Rg\Theta, \qquad b = H^2/A\Theta, \qquad d = \mu jCH/RgA\Theta \tag{4}$$

are positive quantities.

Equation (3) has roots with negative real parts under the condition (Routh–Hurwitz)

$$ab > d. \tag{5}$$

Substituting here the values of the coefficients a, b, and d according to the designations (4), we obtain the well-known stability condition for a single-axis gyroscopic stabilizer in the form

$$jC/\Theta > \mu/H. \tag{6}$$

Let us note that the stability condition of the stabilizer under consideration has exactly the same form also when the self-inductance of the output circuit of its amplifier is taken into account ([1]).

3. Let us now turn to a two-axis gyroscopic stabilizer (see its schematic in Fig. 2), consisting of two stabilizers mounted in a common suspension and identical to that just described.

In deriving the differential equations of small motions of a two-axis stabilizer, it should be borne in mind that the angle of rotation of each of its gyroscopes relative to fixed space, to within small quantities of the second order, consists of the sum of the angle of rotation of the stabilized

platform about the corresponding stabilization axis and the angle of rotation of the housing relative to the platform. It is precisely in this, in fact, that the two groups of equations (7) differ from the set of equations (1) for a single-axis stabilizer. We have:

$$\begin{gathered}
\Theta\frac{d^2\theta}{dt^2} + H\left(\frac{d\beta}{dt} + \frac{d\psi}{dt}\right) = \frac{jC}{g}\sigma, \qquad \Psi\frac{d^2\psi}{dt^2} + H\left(\frac{d\alpha}{dt} + \frac{d\theta}{dt}\right) = \frac{jC}{g}\tau, \\
A\left(\frac{d^2\beta}{dt^2} + \frac{d^2\psi}{dt^2}\right) - H\frac{d\theta}{dt} = 0, \qquad A\left(\frac{d^2\alpha}{dt^2} + \frac{d^2\theta}{dt^2}\right) - H\frac{d\psi}{dt} = 0, \\
R\sigma + jC\frac{d\theta}{dt} = -\mu\beta, \qquad R\tau + jC\frac{d\psi}{dt} = -\mu\alpha,
\end{gathered} \tag{7}$$

where, in addition to the notation $\theta, H, \beta, j, C, g, A, R$ and μ adopted in Sec. 1, now Θ is the sum (with the corresponding reductions) of the moments of inertia, with respect to the outer stabilization axis, of all moving parts of the stabilizer, but without the left gyroscope (Fig. 2); Ψ is the analogous sum of the moments of inertia of the stabilized platform with respect to the inner stabilization axis together with both gyroscopes, but with subtraction of the moment of inertia of the right gyroscope with respect to the axis of its housing; σ and τ are the current strengths in the output circuits of the stabilization amplifiers respectively along the outer and inner suspension axes; and, finally, ψ is the angle of rotation of the stabilized platform with respect to the outer ring.

The characteristic equation of the set (7)

$$\begin{vmatrix} \Theta\lambda^2 & H\lambda & -jC/g & H\lambda & 0 & 0 \\ -H\lambda & A\lambda^2 & 0 & A\lambda^2 & 0 & 0 \\ jC\lambda & \mu & R & 0 & 0 & 0 \\ H\lambda & 0 & 0 & \Psi\lambda^2 & H\lambda & -jC/g \\ A\lambda^2 & 0 & 0 & -H\lambda & A\lambda^2 & 0 \\ 0 & 0 & 0 & jC\lambda & \mu & R \end{vmatrix} = 0 \tag{8}$$

under the additional condition

$$\Psi = \Theta \tag{9}$$

can, as it turns out, be represented in the form

$$S^2(\lambda) + P^2(\lambda) = 0\ ^*, \tag{10}$$

where $S(\lambda)$ is the same determinant of the third order as in equation (2) for the single-axis stabilizer,

$$P(\lambda) = \frac{\mu jC}{g} A\lambda^2. \tag{11}$$

The left-hand side of equation (10) decomposes into the product of two conjugate factors

$$S(\lambda) + iP(\lambda) \quad \text{and} \quad S(\lambda) - iP(\lambda), \tag{12}$$

each of which is a fourth-degree polynomial in the variable λ. It follows from this that the question of stability of the two-axis gyroscopic stabilizer under consideration reduces to investigating the conditions for negativity of the real parts of the nonzero roots of two algebraic equations with complex coefficients

$$\lambda\left[\lambda^3 + a\lambda^2 + (b + ic)\lambda + d\right] = 0, \qquad \lambda\left[\lambda^3 + a\lambda^2 + (b - ic)\lambda + d\right] = 0. \tag{13}$$

Here the same notation (4) has been used and, in addition,

$$c = \mu j C / R g \Theta. \tag{14}$$

General methods for investigating the roots of algebraic equations with complex coefficients, set forth in detail in ([2]), lead in the present case to the following stability conditions:

$$a > 0, \qquad a^2 b - ad - c^2 > 0, \qquad a\left[(ab-d)^2 - bc^2\right] > 0. \tag{15}$$

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* In the original manuscript of this article (dating to 1961), an error in the sign had crept in here, noted by L. N. Slezkin.

Taking into account that the coefficients a, b, c, and d are positive, the second and third inequalities (15) may be written, respectively, in the form:

$$d/ab < 1 - c^2/a^2 b, \qquad d/ab < 1 - c/a\sqrt{b}. \tag{16}$$

It follows from this that

$$1 > c/a\sqrt{b} > c^2/a^2 b. \tag{17}$$

Thus, the second inequality (15) is a consequence of the third. The latter is conveniently transformed to the form:

$$ab > d + c\sqrt{b}. \tag{18}$$

Substituting into inequality (18)* the values of the coefficients a, b, c, and d according to their definitions (4) and (14), we obtain the stability condition for the two-axis gyroscopic stabilizer in the following simple form:

$$\frac{jC}{\Theta} > \frac{\mu}{H}\left(1 + \sqrt{\frac{A}{\Theta}}\right). \tag{19}$$

Inequality (19) is stronger than the corresponding inequality (6), which ensures the stability of a single-axis gyroscopic stabilizer. This explains the fact, observed in practice, of the lower stability of a two-axis stabilizer in comparison with a single-axis one, which was mentioned at the beginning of the article.

In conclusion we note that the comparison of the stability of three-axis and single-axis stabilizers was carried out at different times by L. N. Slezkin and, independently of him, by I. V. Novozhilov**.

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* Condition (18) can also be obtained from the following simple considerations. Equations (13) each have one complex coefficient. Therefore, they can have only complex roots (apart from roots equal to zero, which are immaterial). When the coefficients of the equations are varied, for example the coefficient d, the transition from stability to instability will evidently occur when one of the roots of the equations turns into a purely imaginary quantity. Put, in the second equation (13),

$$\lambda = i\omega.$$

Separating the real and imaginary quantities of its left-hand side, we obtain

$$\omega^3 - b\omega = 0, \qquad -a\omega^2 + c\omega + d = 0.$$

Eliminating the quantity ω from the second equality by means of the first, we arrive at the condition

$$ab = d + c\sqrt{b} \tag{*}$$

for the existence of periodic motion of the gyroscopic stabilizer. If we now increase the coefficient d somewhat, replacing it by the quantity $d + \delta$, then, with accuracy up to small quantities of second order with respect to δ, we obtain the following value of the root of the changed equation:

$$\lambda = \frac{2\delta b}{4b^2 + \left(c - 2a\sqrt{b}\right)^2} + i\left[\omega - \delta\,\frac{c - 2a\sqrt{b}}{4b^2 + \left(c - 2a\sqrt{b}\right)^2}\right].$$

It follows from this formula that the real part of the root of the changed equation is positive. Hence it is not difficult to conclude that, in order to obtain the stability condition for the gyroscopic stabilizer, equality (*) must be replaced by inequality (18).

** The results obtained are being published.

Note: Figure translations are in progress. See original paper for figures.